Name: ______________________________

Teacher: ______________________________

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PYMBLE LADIES' COLLEGE
Year 12

MATHEMATICS EXTENSION 1

TRIAL EXAMINATION

August 2004

Time Allowed: **2 hours plus 5 minutes reading time**

Marking Guidelines **The marks for each part are indicated beside the question**

Instructions

- **All questions should be attempted.**
- **All necessary working must be shown**
- **Start each question on a new page.**
- **Write your name and your teacher's name on each page.**
- **Marks might be deducted for careless or untidy work.**
- **Only approved calculators may be used.**
- **All questions are of equal value**
- **Diagrams are not drawn to scale**
- **A standard integral sheet is attached**
- **DO NOT staple different questions together.**
- **All rough working paper must be attached to the end of the last question**
- **Staple a coloured sheet of paper to the back of each question**
- **Hand in this question paper with your answers.**
- **There are seven (7) questions in this paper and 10 pages**

Question 1 **(12 marks)**

a) Evaluate $\lim\limits_{x\to 0}\dfrac{\sin x}{4x}$. (1)

b) Find the acute angle between the lines $y = 2x - 4$ and $y = 6 - x$.
Answer to the nearest degree. (2)

c) Find the coordinates of the point P that divide the interval joining $(2,-3)$ and $(4,5)$ internally in the ratio 1:3. (2)

(d) α, β and γ are the roots of the equation $2x^3 - 6x + 1 = 0$

Find (without solving)

i) $\alpha + \beta + \gamma$ (1)

ii) $\alpha\beta\gamma$ (1)

iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ (2)

e) Solve $x - 3 \le \dfrac{10}{x}$. (3)

Question 2 Start a new page **(12marks)**

a) i) Sketch the graph of $y = \sin^{-1} 2x$. Clearly indicate the domain and range. (2)

ii) Find the gradient of the curve where $x = \frac{1}{4}$. (2)

b) i) If $\sin\frac{x}{2} = \frac{-3}{4}$ find the possible exact value(s) of $\sin x$. (2)

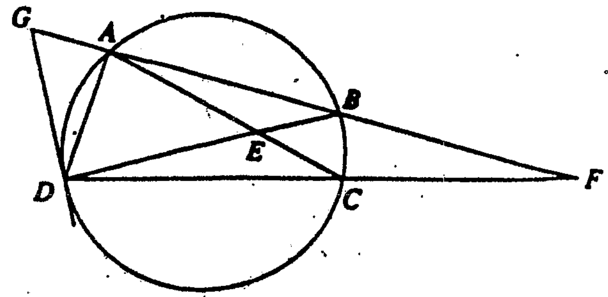
ii) Hence or otherwise find the exact value of $\sin(2\tan^{-1}\frac{-3}{\sqrt{7}})$. (1)

c) i) Express $\tan\theta$ in terms of $\tan\frac{\theta}{2}$. (1)

ii) Hence solve $\tan\theta - \cot\frac{\theta}{2} = 0 \quad (0 \le \theta \le 2\pi)$. (4)

Question 3 Start a new page **(12 marks)**

a)



In the figure above, DG is a tangent to the circle at D.
GABF and DCF are straight lines

Prove $2\angle ADG = \angle BEC + \angle BFC$ (3)

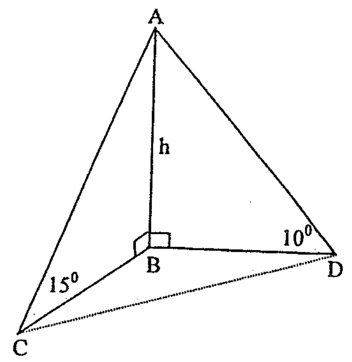
b) Using the substitution $x = u^2 - 1$ find

$\int \frac{x}{x+1}dx$ (3)

Question 3 continued on the next page

Question 3(*contd*)

c)



Not to scale

AB is a hill height 'h' metres. From points C and D, in the same plane as the base of hill B, the angles of elevation of the top of the hill A are 15^0 and 10^0 respectively. From the base of the hill, the bearings of the points C and D are 230^0 and 100^0 respectively.

i) Find the size of angle CBD. (1)

ii) Show BD = h cot 10^0. (1)

iii) If CD is 450m, find the height 'h' metres of the hill. (4)

Question 4 Start anew page **(12 marks)**

a) Find $\int \sin^2 2x \; dx$ (2)

b) i) Show $\sqrt{2}\sin x + \sqrt{2}\cos x$ can be expressed as $2\sin(x + \frac{\pi}{4})$. (2)

ii) What is the maximum and minimum value of $\sqrt{2}\sin x + \sqrt{2}\cos x$?
State the values of 'x' when these occur $(0 \le x \le 2\pi)$. (2)

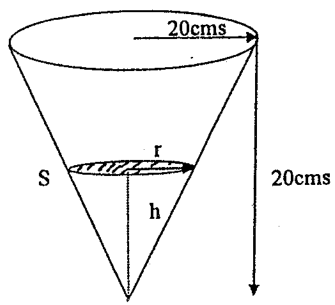
iii) Using i) and ii) or otherwise sketch
$y = \sqrt{2}\sin x + \sqrt{2}\cos x$. $(0 \le x \le 2\pi)$ (2)

iv) Solve algebraically
$\sqrt{2}\sin x + \sqrt{2}\cos x = 1$ $(0 \le x \le 2\pi)$ (2)

v) Using (iii) and (iv), solve
$\sqrt{2}\sin x + \sqrt{2}\cos x \ge 1$ $(0 \le x \le 2\pi)$ (2)

Question 5 Start anew page **(12 marks)**

a) Water flows from a conical vessel of radius 20cm, height 20cm. The water flows out at a constant rate of 18cm^3/second. The depth of water is 'h' cm at time't' seconds.



Not to scale

i) 'S' is the shaded area of the surface of the water remaining in the vessel. Given 'r' is the radius of this area 'S', show that the radius 'r' is decreasing at $\dfrac{18}{\pi r^2}$ cm/sec

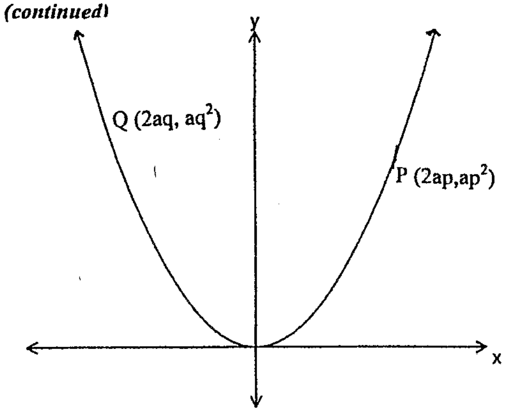
(Volume of a cone $= \frac{1}{3}\pi r^2 h$) (3)

ii) Hence, find the rate at which the surface area 'S' of the water is changing when the depth of the water is 8cm. (3)

Question 5 continued next page

Question 5 *(continued)*

b)



P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$

i) Find the coordinates of M, the mid-point of PQ in terms of p and q. (1)

ii) If PQ subtends a right angle at the origin, show $pq = -4$. (2)

iii) Hence, or otherwise show the locus of M is a parabola. (3)

Question 6 **(12 marks)**

a) i) Prove $\dfrac{d^2x}{dt^2} = \dfrac{d\left(\frac{1}{2}v^2\right)}{dx}$ (2)

ii) Hence given $\dfrac{d^2x}{dt^2} = 10x - 2x^3$ and $v = 0$ at $x = 1$
find 'v' in terms of 'x'. (3)

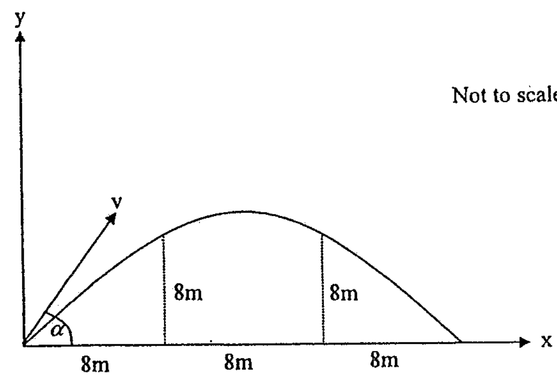
iii) Describe the motion in terms of displacement and velocity. Is it simple harmonic motion? Explain your answer. (3)

b) The velocity 'v' of a particle along the 'x' axis is given by $v = \sqrt{25 - x^2}$ where 'x' is the displacement of the particle from O. Initially the particle is 5cm to the right of O.

Find an expression for 'x' in terms of 't' (4)

Question 7 **(12 marks)**

A particle is projected to just clear two walls of height 8 metres and distant 8 metres and 16 metres respectively from the point of projection. The particle's initial velocity is 'v' and the angle of projection is α. (assume $g = 10\text{m/sec}^2$)



i) Given $\dfrac{d^2x}{dt^2} = 0$ and $\dfrac{d^2y}{dt^2} = -10$
show $x = vt\cos\alpha$ and $y = -5t^2 + vt\sin\alpha$ (4)

ii) Hence or otherwise find the value of α. (5)

iii) Prove that if the 2 walls are 'h' metres high and distant 'a' and 'b' metres respectively from the point of projection, then

$\tan\alpha = \dfrac{h(a+b)}{ab}$ *(you may assume the results from part (i) only)* (3)

END OF PAPER

Answers

Ext 1 Trial 2004

Q1

(a) $\frac{1}{4}$ (1)

(b) $m_1 = 2 \quad m_2 = -1$

$\tan\theta = \left|\frac{2-(-1)}{1+2(-1)}\right|$

$= \left|\frac{3}{-1}\right|$

$= 3$

$\theta = 71.56^\circ$ (2)

$= 72$ (nd)

(c) $2, -3, \qquad 4, 5$

$1:3$

$x = \frac{6+4}{4} \qquad y = \frac{-9+5}{4}$

$= 2\frac{1}{2} \qquad = -1$ (2)

$(2\frac{1}{2}, -1)$

OR

$(4,5)$

$(2,-3)$ $\quad 2 + \frac{1}{4} = \frac{1}{2} \quad 8 + \frac{1}{4} = 2$

$2\frac{1}{2} \quad -1$

(d)

(i) $\alpha + \beta + \gamma = 0$ (1)

(ii) $\alpha\beta\gamma = -\frac{1}{2}$ (1)

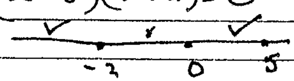
(iii) $\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-3}{-\frac{1}{2}} = 6$ (2)

(e) $x - 3 = \frac{10}{x} \qquad x \neq 0$

$x^2 - 3x = 10$

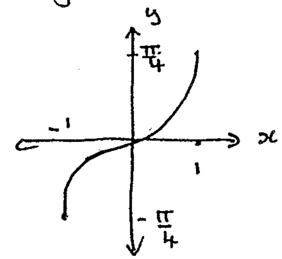
$x^2 - 3x - 10 = 0 \qquad x \leq -2$

$(x-5)(x+2) = 0 \qquad 0 < x \leq 5$ (3)



Q2

(a) (i) $y = \sin^{-1} 2x$



(2)

(ii) $f(x) = \sin^{-1} 2x$

$f'(x) = \frac{2}{\sqrt{1-4x^2}}$ (2)

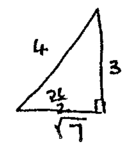
$f'\left(\frac{1}{4}\right) = \frac{2}{\sqrt{\frac{3}{4}}}$

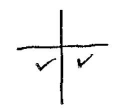
$= \frac{4}{\sqrt{3}}$

(b) (i) $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 2 \times -\frac{3}{4} \times \pm\frac{\sqrt{7}}{4}$

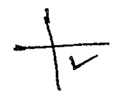
$= \mp\frac{3\sqrt{7}}{8}$





(2)

(ii) $\sin\left(2\tan^{-1} - \frac{3}{\sqrt{7}}\right)$



$\sin\left(2\left(\frac{x}{2}\right)\right)$

$= \sin x$

$= -\frac{3\sqrt{7}}{8}$ (1)

(c) $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} \qquad$ let $\tan\frac{\theta}{2} = t$

$\frac{2t}{1-t^2} - \frac{1}{t} = 0$

$2t^2 - 1 + t^2 = 0$

$3t^2 = 1$

$t = \mp\frac{1}{\sqrt{3}}$

$\therefore \frac{\theta}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \pi/3, 5\pi/3$

Check π ✓

$\therefore \theta = \pi/3, 5\pi/3, \pi$ (4)

Q3

(a) Let $\angle ADG = x$

$\angle ABD = x$ [$\angle ABD = \angle ADG$ ∠ in alt seg)

$\angle ACD = \angle ABD = x$ ($\angle$ at circum. standing on same segment)

$\angle EBF = \angle ECF = 180 - x$ (st $\angle$)

$\angle BEC + \angle EBF + \angle BFC + \angle ECF = 360°$ ($\angle$ + quad)

$\therefore \angle BEC + \angle BFC = 360 - [\angle EBF + \angle ECF]$

$= 360 - 2(180 - x)$

$= 2x$

$= 2 \times \angle ADG$ (3)

(b) $\int \frac{x}{\sqrt{x+1}}\, dx$ $\quad x = u^2 - 1$

$\frac{dx}{du} = 2u\, du$

$= 2\int \frac{u^2-1}{u^2}\, du . u$

$= 2\int \frac{u^2-1}{u}\, du$

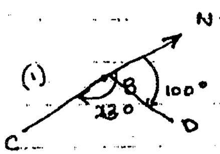
$= 2\int u - \frac{1}{u}\, du$

$= 2\left[\frac{u^2}{2} - \ln u\right]$

$= u^2 - \ln u^2$

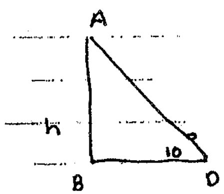
$= x+1 - \ln(x+1) + C$ (3)

(c) (i)



$\angle CBD = 130°$ (1)

(ii)

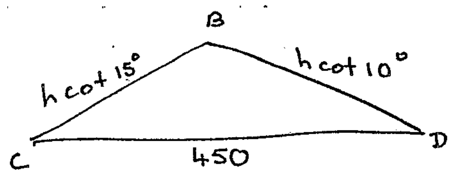


$\tan 10° = \frac{h}{BD}$

$BD = \frac{h}{\tan 10°}$

$BD = h \cot 10°$ (1)

(c) $BC = h\cot 15$



$$450^2 = h^2\cot^2 15 + h^2\cot^2 10 - 2\times h^2\cot 10^\circ\cot 15^\circ\cos 130^\circ$$

$$450^2 = h^2\left[\cot^2 15 + \cot^2 10 - 2\cot 10\cot 15\cos 130^\circ\right]$$

$$450^2 = h^2 \times 73.3008\ldots$$

$$h^2 = 2762.589\ldots$$

$$h = 52.56 \text{ m.}$$

(4)

Q 4

(a) $\cos 4x = 1 - 2\sin^2 2x$

$2\sin^2 2x = 1 - \cos 4x$

$\sin^2 2x = \frac{1}{2}\left[1 - \cos 4x\right]$

$\int \sin^2 2x\,dx = \frac{1}{2}\int 1 - \cos 4x\,dx$

$= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right] + C$

(2)

(b) (i) $\sqrt{2}\sin x + \sqrt{2}\cos x = R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$\cos\alpha = \frac{\sqrt{2}}{R}$ $\quad R^2 = 4$

$\sin\alpha = \frac{\sqrt{2}}{R}$

$\tan\alpha = 1 = \frac{\pi}{4}$ $\quad R = 2$

$\sqrt{2}\sin x + \sqrt{2}\cos x = 2\sin(x + \pi/4)$

OR

$2\sin(x + \pi/4) = 2\left[\sin x\cos\pi/4 + \sin\pi/4\cos x\right]$

$= 2\left[\sin x \times \frac{1}{\sqrt{2}} + \cos x \times \frac{1}{\sqrt{2}}\right)$

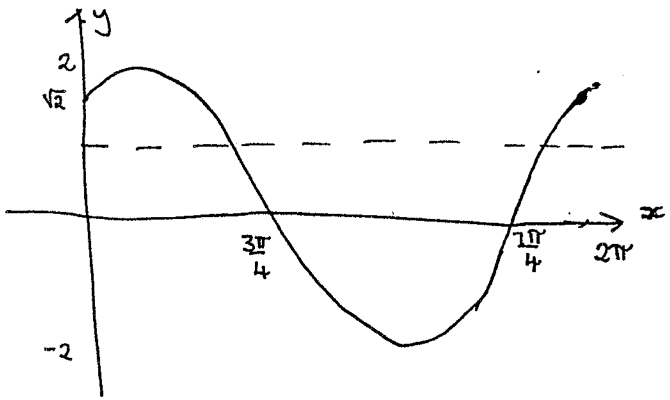
$= \frac{2}{\sqrt{2}}\sin x + \frac{2}{\sqrt{2}}\cos x$

$= \sqrt{2}\sin x + \sqrt{2}\cos x$

(ii) Max value $= 2$ when $x = \pi/4$

Min value $= -2$ when $x = \frac{5\pi}{4}$

(iii)



(2)

(iv) $2\sin(x+\frac{\pi}{4})=1$

$\sin(x+\pi/4)=1/2$

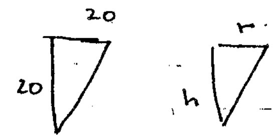
$x+\pi/4=\pi/6,\ 5\pi/6,\ 13\pi/6$

$x=\frac{7\pi}{12},\ \frac{23\pi}{12}$

(v) $0\le x\le\frac{7\pi}{12}$ $\qquad \frac{23\pi}{12}\le x\le 2\pi$

Q5

(a) (i) $\frac{dV}{dt}=-18$



$\frac{20}{h}=\frac{20}{r}$

$r=h.$

$V=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi r^3$

$\frac{dV}{dr}=\pi r^2$

$\frac{dV}{dt}=\frac{dV}{dr}\cdot\frac{dr}{dt}$

$-18=\pi r^2\cdot\frac{dr}{dt}$

$\frac{dr}{dt}=\frac{-18}{\pi r^2}$

$\therefore$ decreasing at $\frac{18}{\pi r^2}$ cm/sec (3)

(ii) $SA=\pi r^2$

$\frac{dS}{dr}=2\pi r$

$\frac{dS}{dt}=\frac{dS}{dr}\cdot\frac{dr}{dt}$

$=2\pi r\times\frac{-18}{\pi r^2}$

$=\frac{-36}{r}$ $\qquad$ when $r=8$

$=-4\frac{1}{2}$ cm²/sec (3)

[or decreasing at $4\frac{1}{2}$ cm²/sec)

Q5 (cont)

(b) (i) M $x = \frac{2ap+2aq}{2}$

$x = a(p+q)$ (1)

$y = \frac{a(p^2+q^2)}{2}$

(ii) $mPO \times mQO = -1$

$\frac{ap^2}{2ap} \times \frac{aq^2}{2aq} = -1$

$\frac{pq}{4} = -1$ (1)

$pq = -4$

(iii) $p+q = \frac{x}{a}$

$y = \frac{a}{2}\left[(p+q)^2 - 2pq\right]$ (3)

$= \frac{a}{2}\left[\frac{x^2}{a^2} - 2(-4)\right]$

$= \frac{a}{2}\left[\frac{x^2}{a^2} + 8\right]$

$y = \frac{x^2}{2a} + 4a$ or $x^2 = 2ay - 8a^2$

$x^2 = 2a(y-4a)$

Q6

(a) (i) $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$

$= \frac{dv}{dx}\cdot v$

$= \frac{dv}{dx}\cdot\frac{d\frac{1}{2}v^2}{dv}$

$= \frac{d\frac{1}{2}v^2}{dx}$

or

$\frac{d\frac{1}{2}v^2}{dx} = \frac{d\frac{1}{2}v^2}{dv}\cdot\frac{dv}{dx}$

$= v\cdot\frac{dv}{dx}$

$= \frac{dx}{dt}\cdot\frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \frac{d^2x}{dt^2}$ 2

(ii) $\frac{d\frac{1}{2}v^2}{dx} = 10x - 2x^3$

$\frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} + c$

$v = 0 \quad x = 1$

$0 = 5 - \frac{1}{2} + c$

$c = -4\frac{1}{2}$

$\therefore \frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} - 4\frac{1}{2}$

$v^2 = 10x^2 - x^4 - 9$ 3

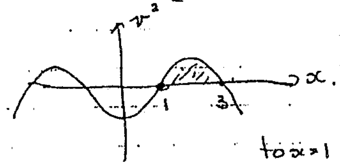
$\therefore v^2 = -\left[x^4 - 10x^2 + 9\right]$

$v = \mp\sqrt{-(x^4-10x^2+9)}$

$\left(\pm\sqrt{-(x-3)(x+3)(x+1)(x-1)}\right)$

[at $x=1$, $v=0$, $a \geq 0$ $\rightarrow$; $x=3$, $v=0$, $a<0$ $\leftarrow$] so $v = +$ and $-$

(iii) $v^2 = -(x-3)(x+3)(x+1)(x-1)$



Particle is at rest at $x=1$ moves to right to $x=3$ where it stops. = moves back to $x=1$ oscillates between $x=1$ and $x=3$ as $\ddot{x}$ is >0 at $x=1$ and $\ddot{x}<0$ at $x=3$ 3

$\ddot{x} = 2x(5-x^2)$ Not SHM

(b)

$$\frac{dx}{dt} = \sqrt{25 - x^2}$$

$$\frac{dt}{dx} = \frac{1}{\sqrt{25-x^2}}$$

$$t = \sin^{-1}\frac{x}{5} + C$$

$t = 0 \quad x = 5$

$$0 = \sin^{-1} 1 + C$$

$$C = -\frac{\pi}{2}$$

$$\therefore t = \sin^{-1}\frac{x}{5} - \pi/2$$

$$t + \frac{\pi}{2} = \sin^{-1}\frac{x}{5}$$

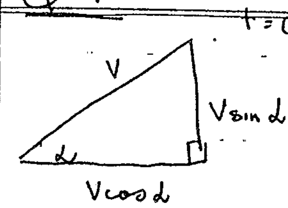
$$\frac{x}{5} = \sin\left[t + \pi/2\right)$$

$$x = 5\sin\left[t + \pi/2\right]$$

(4)

Q7

(i)



$\ddot{x} = 0$

$\dot{x} = C_1$ when $t=0$ $\dot{x} = V\cos\alpha$

$C_1 = V\cos\alpha$

$\dot{x} = V\cos\alpha$

$x = Vt\cos\alpha + C_2$ when $t=0$ $x=0$

$C_2 = 0$

$x = Vt\cos\alpha$

$\ddot{y} = -10$

$\dot{y} = -10t + C_3$ $t=0$: $C_3 =$

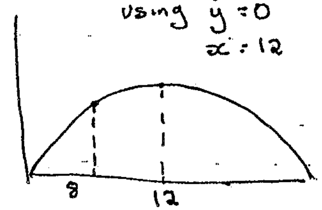
$\dot{y} = -10t + V\sin\alpha$

$y = -5t^2 + Vt\sin\alpha + C_4$ $t=0$; C_4

$y = -5t^2 + Vt\sin\alpha$

(ii) Method 1

using $\dot{y} = 0$, $x = 12$ and $x = 8$, $y = 8$



$\dot{y} = -10t + V\sin\alpha$

$x = Vt\cos\alpha$

$12 = Vt\cos\alpha$

$t = \frac{12}{V\cos\alpha}$

sub in

$0 = -10t + V\sin\alpha$

$0 = -\frac{10.12}{V\cos\alpha} + V\sin\alpha$

$V^2\sin\alpha\cos\alpha = 120$

$V^2 = \frac{120}{\sin\alpha\cos\alpha}$ $\xrightarrow{\text{sub in}}$

$x = Vt\cos\alpha$

$8 = Vt\cos\alpha$

$t = \frac{8}{V\cos\alpha}$

sub in

$y = -5t^2 + Vt\sin\alpha$

$8 = -5t^2 + Vt\sin\alpha$

$8 = -5\left(\frac{8}{V\cos\alpha}\right)^2 + V\left(\frac{8}{V\cos\alpha}\right)$

$8 = -\frac{5\times 64}{V^2\cos^2\alpha} + 8\tan\alpha$

$8 = -\frac{5\times 64}{120\cos^2\alpha}\sin\alpha\cos\alpha + 8$

$\div 8$

$1 = -\frac{1}{3}\tan\alpha + \tan\alpha$

$\tan\alpha = 3/2 \qquad \alpha = 56°$

alternate answer using

$x = 8$ $y = 8$

$x = 16$ $y = 8$

$x = Vt\cos\alpha$

$8 = Vt\cos\alpha$

$t = \frac{8}{V\cos\alpha}$ sub in

$y = -5t^2 + Vt\sin\alpha$

$8 = -5\left(\frac{8}{V\cos\alpha}\right)^2 + V\left(\frac{8}{V\cos\alpha}\right)\sin\alpha$

$8 = -5\cdot\frac{64}{V^2\cos^2\alpha} + 8\tan\alpha$

$(\div \text{by } 8)$

$1 = \frac{-40}{V^2\cos^2\alpha} + \tan\alpha$

$\frac{-40}{V^2\cos^2\alpha} = 1 - \tan\alpha$

$\frac{-160}{V^2\cos^2\alpha} = 4(1-\tan\alpha)$

$x = Vt\cos\alpha$

$16 = Vt\cos\alpha$

$t = \frac{16}{V\cos\alpha}$

sub in

$y = -5t^2 + Vt\sin\alpha$ $y = 8$

$8 = -5\left(\frac{16}{V\cos\alpha}\right)^2 + V\cdot\frac{16}{V\cos\alpha}\sin\alpha$

$8 = \frac{-5\cdot 16^2}{V^2\cos^2\alpha} + 16\tan\alpha$

$(\div \text{by } 8)$

$1 = \frac{-5\cdot 2\cdot 16}{V^2\cos^2\alpha} + 2\tan\alpha$

$1 = \frac{-160}{V^2\cos^2\alpha} + 2\tan\alpha$

$1 = 4(1-\tan\alpha) + 2\tan\alpha$

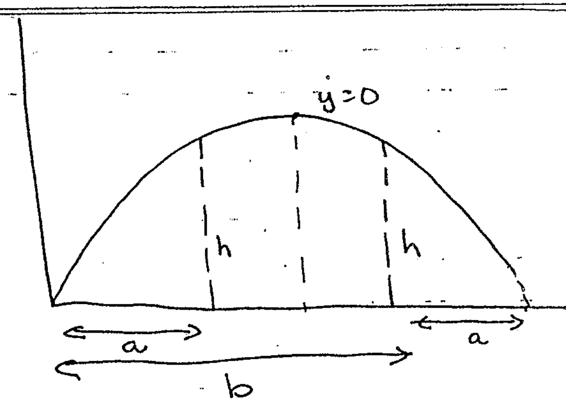
$1 = 4 - 4\tan\alpha + 2\tan\alpha$

$2\tan\alpha = 3$

$\tan\alpha = \frac{3}{2}$

$\alpha = 56°$ (5)

(iii)



Using $\dot{y} = 0$ when $x = \frac{a+b}{2}$

$x = Vt\cos\alpha$

$\frac{a+b}{2} = Vt\cos\alpha$

sub $t = \frac{a+b}{2V\cos\alpha}$ in

$\dot{y} = -10t + V\sin\alpha$

$0 = -\frac{10(a+b)}{2V\cos\alpha} + V\sin\alpha$

$V^2\sin\alpha\cos\alpha = 5(a+b)$

$V^2 = \frac{5(a+b)}{\sin\alpha\cos\alpha}$

$y = h$ $x = a$

$x = Vt\cos\alpha$

$a = Vt\cos\alpha \therefore t = \frac{a}{V\cos\alpha}$

$y = -5t^2 + Vt\sin\alpha$

$h = \frac{-5a^2}{V^2\cos^2\alpha} + \frac{Va}{V\cos\alpha}\sin\alpha$

$h = \frac{-5a^2}{V^2\cos^2\alpha} + a\tan\alpha$

but $V^2 = \frac{5(a+b)}{\sin\alpha\cos\alpha}$

$\therefore h = \frac{-5a^2\sin\alpha\cos\alpha}{5(a+b)\cos^2\alpha} + a\tan\alpha$

$h = \frac{-a^2}{(a+b)}\tan\alpha + a\tan\alpha$

$h(a+b) = -a^2\tan\alpha + a(a+b)\tan\alpha$

$h(a+b) = -a^2\tan\alpha + a^2\tan\alpha + ab\tan\alpha$

$h(a+b) = ab\tan\alpha$

$\tan\alpha = \frac{h(a+b)}{ab}$

alternate way

using $x = a$, $y = h$ — $x = b$, $y = h$

$Vt\cos\alpha = a \quad t = \frac{a}{V\cos\alpha}$ — $Vt\cos\alpha = b \quad t = \frac{b}{V\cos\alpha}$

$-5t^2 + Vt\sin\alpha = h$ — $-5t^2 + Vt\sin\alpha = h$

$-5\left(\frac{a}{V\cos\alpha}\right)^2 + \frac{Va\sin\alpha}{V\cos\alpha} = h$ — $-5\left(\frac{b^2}{V^2\cos^2\alpha}\right) + \frac{Vb\sin\alpha}{V\cos\alpha} = h$

$-\frac{5a^2}{V^2\cos^2\alpha} + a\tan\alpha = h$ — $-\frac{5b^2}{V^2\cos^2\alpha} + b\tan\alpha = h$

$$\frac{-5}{V^2\cos^2\alpha} = \frac{h - a\tan\alpha}{a^2}$$

$$b^2\left(\frac{h - a\tan\alpha}{a^2}\right) + b\tan\alpha = h$$

$$\frac{b^2 h}{a^2} - \frac{b^2\tan\alpha}{a} + b\tan\alpha = h$$

$$\tan\alpha\left[b - \frac{b^2}{a}\right] = h - \frac{b^2 h}{a^2}$$

$$\tan\alpha\left[\frac{ab - b^2}{a}\right] = \frac{a^2 h - b^2 h}{a^2}$$

$$\tan\alpha = \frac{h\left[a^2 - b^2\right] \times a}{b\left[a - b\right] a^2}$$

$$= \frac{h\,[\cancel{(a-b)}](a+b)\,a}{b\,\cancel{(a-b)}\,a^2}$$

$$\tan\alpha = \frac{h(a+b)}{ab}$$