PLC

Presbyterian Ladies' College

SYDNEY

1888

Semester Two Examination, 2007

YEAR 11 MATHEMATICS EXTENSION 1

Student Name: ____________

Student Number: ____________

Teacher's Name: ____________

General Instructions:

- Reading time: 5 minutes
- Working time : 2 hours
- Write your student number at the top of every sheet of writing paper.
- Write working and answers on the front of the writing paper provided.
- Begin each Question on a new sheet of writing paper.
- Write using a blue or black pen. Diagrams may be drawn with pencil.
- Board-approved scientific calculators may be used.
- All necessary working should be shown in every question.
- Your work will be collected in 7 separately stapled bundles.

Question	Marks
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
Total	/84

Textbook references:

Topic
Basic Arithmetic
Algebra and Surds
Equations
Geometry
Functions and Graphs
Trigonometry
Straight Line Graphs
Introduction to Calculus
Quadratic Function
Polynomials

Question 1 **Start a new page** **Marks**

(a) If $P(x) = 2x^4 - 3x^3 + 6x^2 - 5x + 1$, find the remainder when $P(x)$ is divided by $(x + 1)$. 1

(b) If P divides the interval from $(-4,2)$ to $(2, -1)$ **externally** in the ratio $5 : 2$, find the coordinates of P. 2

(c) Differentiate $y = \sqrt[3]{x^4}$, with respect to x. 1

(d) Solve the inequality $\frac{2}{x-1} \leq 1 \qquad (x \neq 1)$ 3

(e) A polynomial $P(x)$ has a **leading term** of $-2x^3$ and $P(-2) = P(3) = P(1) = 0$. Write the equation for $P(x)$ in factor form. 2

(f) Find the **general solution** for $\cos\theta = \frac{-\sqrt{3}}{2}$ 3

End of Question 1

Question 2 **Start a new page** **Marks**

(a) (i) Divide the polynomial $P(x) = 4x^3 - 8x^2 + 8x - 1$ by $x^2 + 1$. **2**

(ii) Hence express $P(x)$ in the form $A(x)Q(x) + R(x)$ **1**

(b) Prove that $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$ **2**

(c) In the diagram $AB = AC$; $\angle BAC = \angle BPA = \angle CRA = 90^0$; $\angle BAP = \alpha$ **7**

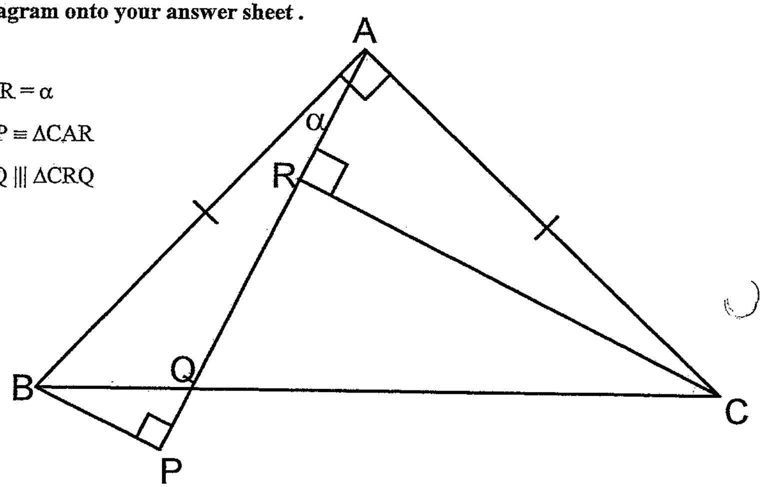
Copy the diagram onto your answer sheet .

Prove that

(i) $\angle ACR = \alpha$

(ii) $\Delta ABP \equiv \Delta CAR$

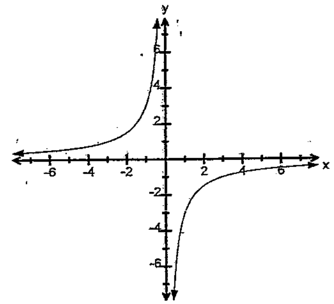
(iii) $\Delta BPQ \, ||| \, \Delta CRQ$



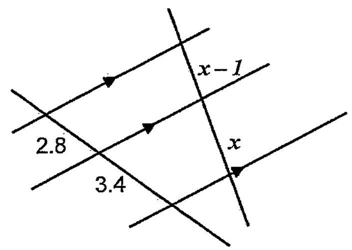
End of Question 2

Question 3 **Start a new page** **Marks**

(a) Sketch the **gradient function** for the following graph. **1**



(b) Find the **exact value** of x in the diagram below. Give a reason. **2**



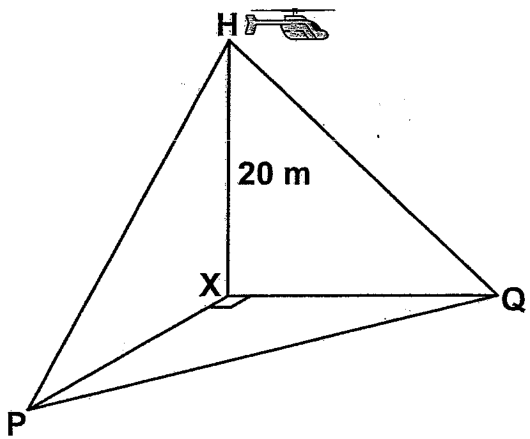
(c) Find the **acute angle** formed between the curves $y = x^2$ and $y = -x^3$ at the point $(-1,1)$. Give your answer correct to the nearest minute. **3**

(d) Show that the equation of the **tangent** to the curve $f(x) = \dfrac{3}{4\sqrt{x}}$ at the point $\left(4, \dfrac{3}{8}\right)$ is $3x + 64y - 36 = 0$. **3**

Question 3 continued on the next page.

Question 3

(e) From a helicopter H at a height of 20m above a point X in the ocean, two sailors are spotted in the water at positions P and Q with angles of depression of 64^0 and 55^0 respectively. Triangle **PQX** is right angled at X. How far apart are P and Q, correct to 3 significant figures? **3**



End of Question 3

Question 4 — Start a new page — Marks

(a) If α, β and λ are the roots of the equation $x^3 + 3x^2 - 4x - 5 = 0$, find the value of **3**

(i) $\alpha + \beta + \lambda$

(ii) $\alpha\beta + \alpha\lambda + \beta\lambda$

(iii) $\alpha^2 + \beta^2 + \lambda^2$ by considering the expansion of $(\alpha + \beta + \lambda)^2$

(b) (i) Sketch the graph of $y = |x - 3| - 2$, showing the x and y intercepts and the vertex. **5**

(ii) Hence, or otherwise, state the **domain** and **range** of $y = |x - 3| - 2$.

(iii) Find the value(s) of x for which the function is **not** differentiable.

Explain your answer.

(c) If $\tan A = -\frac{1}{7}$ where $90^0 < A < 180^0$ and $\cos B = \frac{2}{\sqrt{5}}$ where $270^0 < B < 360^0$, find the **exact value** of **4**

(i) $\sin A$

(ii) $\cos A$

(iii) $\sin B$

(iv) $\sin(A + B)$ in simplest surd form.

End of Question 4

Question 5 **Start a new page** **Marks**

(a) Show that the line $3x + 4y - 10 = 0$ is **not** a tangent to the circle $(x-1)^2 + (y+1)^2 = 4$ — **2**

(b) Show that the line $y = 3x - 7$ **is** a tangent to the parabola $y = 2x^2 - 5x + 1$ — **2**

(c) The point Q(0,3) divides the line segment joining C(–2,7) and D(x,y) **internally** in the ratio 2: 3. Find the coordinates of D. — **2**

(d) Find the value(s) of **k** in the equation $x^2 + 6kx - 4k + 4 = 0$ if one root is double the other root. — **3**

(e) A tangent to $y = x^2$ makes an angle of 60^0 with the positive direction of the x axis. — **3**

(i) Show that the tangent meets the curve at $\left(\frac{\sqrt{3}}{2}, \frac{3}{4}\right)$.

(ii) Hence show that the equation of the **normal** to the curve at this point is

$4x + 4\sqrt{3}y - 5\sqrt{3} = 0$

End of Question 5

Question 6 **Start a new page** **Marks**

(a) Prove that $\dfrac{\sin(180-\theta)\cos(90-\theta)\tan(-\theta)}{\cos(360-\theta)\tan(180+\theta)} = -\tan\theta\sin\theta$ — **3**

(b) Determine $\lim\limits_{x\to 1}\dfrac{x^3 + 5x^2 - 7x + 1}{x - 1}$ — **2**

(c) For which x values is the tangent to the curve $y = 2x^3 - 3x^2 - 35x$ **parallel** to the line $y = x$? — **2**

(d) $P(x) = 5x^3 - 24x^2 + 9x + 54$ has a double root at $x = \alpha$. — **5**

(i) Find $P'(x)$

(ii) Hence find the value of α.

(iii) Determine the value of the other root.

End of Question 6

Question 7 **Start a new page** **Marks**

(a) Use the t formulae to solve the equation $7\sin x - 4\cos x = 4$ where $0^0 \le x \le 180^0$. **4**

Give your answer correct to the nearest minute.

(b) Consider the function $f(x) = \dfrac{x^2 - 4}{x(x-3)}$ **8**

(i) Determine all the x intercepts.

(ii) Determine the equations of all the vertical asymptotes.

(iii) Determine $\lim\limits_{x \to \infty} f(x)$.

(iv) Hence, sketch the graph of $f(x)$.

(v) State the domain and range of the function.

End of Paper

Question 1 — 12/12 — 11590

(a) $P(-1) = 17$

$\therefore$ remainder $= 17$ — $\frac{1}{1}$

(b) $5 : -2$ $\quad (-4, 2), (2, -1)$

$x = \dfrac{5(2) - 2(-4)}{5-2}$ $\qquad y = \dfrac{5(-1) - 2(2)}{5-2}$

$= 6$ $\qquad = -3$

$P(6, -3)$ — $\frac{2}{2}$

(c) $y = x^{4/3}$

$y' = \dfrac{4}{3} x^{1/3}$ — $\frac{1}{1}$

$= \dfrac{4\sqrt[3]{x}}{3}$

(d) $\dfrac{2}{x-1} \leq 1$ $\qquad x \neq 1$

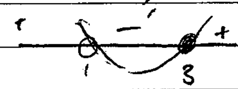
$2(x-1) \leq (x-1)^2$

$2x - 2 \leq x^2 - 2x + 1$ — $\frac{3}{3}$

$0 \leq x^2 - 4x + 3$

$x^2 - 4x + 3 \geq 0$

$(x-1)(x-3) \geq 0$ $\qquad x = 3 \quad x = 1$

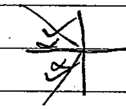


$\therefore x < 1, \quad x \geq 3$

(e) — $\frac{2}{2}$

$P(x) = -2(x+2)(x-3)(x-1)$

(f) $\cos\theta = -\dfrac{\sqrt{3}}{2}$



$\alpha = 30°$

$\therefore \theta = 150°, 210°$

$\therefore \theta = 150° + 360n$

$210° + 360n$

$\frac{3}{3}$

Question 2 $6\frac{1}{2}$ 11590

(a)

(i)

$$\begin{array}{r} 4x - 8 \\ x^2+1 \,\overline{)\, 4x^3 - 8x^2 + 8x - 1} \\ -(4x^3 + 0x^2 + 4x) \\ \hline -8x^2 + 4x - 1 \\ -(-8x^2 + 0x - 8) \\ \hline 4x\ \cancel{49}+7 \end{array}$$

$1\frac{1}{2}$ / 2

(ii) $P(x) = (x^2+1)(4x-8) + (4x-9)$ 1

CFPA / 1

(b) $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$

LHS $= \dfrac{\cos\theta}{1+\sin\theta} + \tan\theta$

$\cancel{= \dfrac{\cos\theta + \tan\theta(1+\sin\theta)}{1+\sin\theta}}$

$= \dfrac{\cos\theta}{1+\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos\theta + \sin\theta\cos\theta}$

$= \dfrac{1 - \cancel{\sin^2\theta} + \cancel{\sin^2\theta} + \sin\theta}{\cos\theta(1+\sin\theta)}$

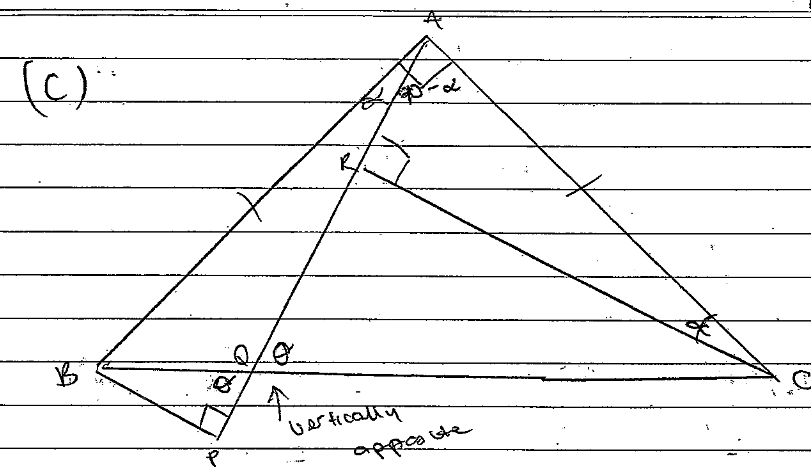
$= \dfrac{\cancel{1+\sin\theta}}{\cos\theta\,(\cancel{1+\sin\theta})}$

$= \dfrac{1}{\cos\theta} = \sec\theta$

= RHS 2 / 2

$\therefore \dfrac{\cos\theta}{1+\sin\theta} + \tan = \sec\theta$

(c)



(i) In ΔARC, $\angle RAC = 90 - \alpha$

$\therefore \angle ACR = 180° - (90° + 90° - \alpha)$ ... (sum of ∠s of Δ)

$= 180° - 180° + \alpha$

$= \alpha$ as req^d.

(ii) $\angle APB = \angle CRA = 90°$ (given)

$AB = CA$ (given) 2 / 2

~~$\angle BAP = \angle CAR$ $\angle BAP = \angle ACR$~~

$\angle ACR = \angle BAB = \alpha$ (previously shown)

~~$\therefore \Delta ABP \,|||\, \Delta CAR$~~

$\therefore \Delta ABP \equiv \Delta CAR$

(SAS)

(iii) In ΔBPQ, ΔCRQ

$\angle BPQ = \angle CRQ = 90°$ (given)

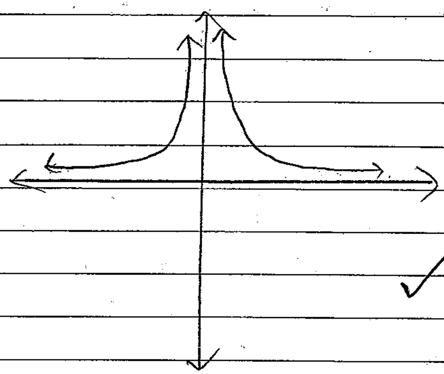
$\angle PQB = \angle RQC$ (Vert. opp. ∠s)

$\therefore \Delta BPQ \,|||\, \Delta CRQ$ (Equiangular)

Question 3 11590

(12) Wow!

(a)



$\frac{1}{1}$

(b) $\frac{3\cdot 4}{2\cdot 8} = \frac{x}{x-1}$ (equal ratios of intercepts ~~on~~ of parallel lines) SOS

$\therefore 3\cdot 4(x-1) = 2\cdot 8x$

$3\cdot 4x - 3\cdot 4 = 2\cdot 8x$

$0\cdot 6x = 3\cdot 4$

$x = 5\frac{2}{3}$

$\therefore x = 5\frac{2}{3}$

$\frac{2}{2}$

(c) $y = x^2$

$y' = 2x$

$m_1 = 2(-1)$

$\therefore m_1 = -2$

$y = -x^3$

$y' = -3x^2$

$m_1 = -3(-1)^2$

$m_2 = -3$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$

$= \left|\frac{-3+2}{1+(-3)(-2)}\right|$

$\therefore \theta = 8°08'$

$\frac{3}{3}$

11590

(d) Find gradient (of tangent).

$f(x) = \frac{3}{4}x^{-1/2}$

$f'(x) = \frac{-3}{8}x^{-3/2}$

$m = \frac{-3}{8}(4)^{-3/2}$

$= \frac{-3}{64}$

Use point gradient formula to find equation of tangent.

$y - y_1 = m(x - x_1)$

$y - \frac{3}{8} = \frac{-3}{64}(x-4)$

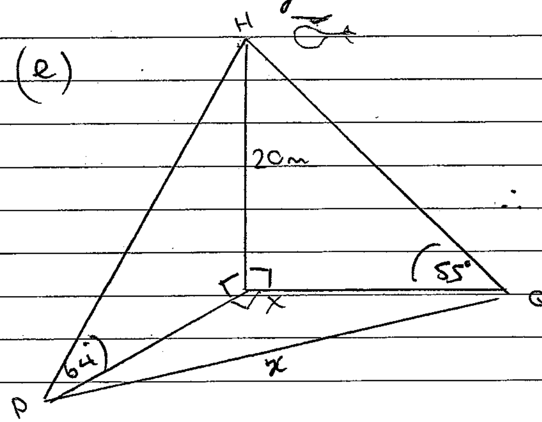
$y = \frac{-3x}{64} + \frac{3}{16} + \frac{3}{8}$

$3x + 64y + 36 = 0$

$\frac{3}{3}$

$\therefore$ the equation of the tangent is $3x + 64y - 36 = 0$

(e)



~~tan 55~~ =

$\tan 55 = \frac{20}{XQ}$

$\therefore XQ = \frac{20}{\tan 55}$

$\tan 64 = \frac{20}{XP}$

$\therefore XP = \frac{20}{\tan 64}$

11590

$\therefore \quad XQ = 14.00$ (to 2 d.p.)

$XP = 9.75$ (to 2 d.p.s)

find x using cos rule

$$x^2 = 14^2 + 9.75^2 - 2(14)(9.75)\cos 90$$
$$= 291.0625$$

$\therefore \quad x = 17.1$ m (to 3 sig fig's.)

3

3

Question 4 $\boxed{10\frac{1}{2}}$ 11590

(a) (i) -3

(ii) -4

(iii) $\alpha^2 + \beta^2 + \lambda^2 \rightarrow$

$(\alpha + \beta + \lambda)^2$

$= (\alpha + \beta + \lambda)(\alpha + \beta + \lambda)$

$= \cancel{\alpha^2 + \alpha\beta + \alpha\lambda + \alpha\beta + \beta^2 + \beta\lambda}$

$= \alpha^2 + \beta^2 + \lambda^2 + \cancel{2(\alpha\beta + \alpha\lambda + \alpha\beta}$

$\alpha\beta + 2\alpha\lambda + \alpha\beta + 2\beta\lambda$

$= \alpha^2 + \beta^2 + \lambda^2 + 2(\alpha\beta + \alpha\lambda + \beta\lambda)$

$(\alpha + \beta + \lambda)^2 - 2(\alpha\beta + \alpha\lambda + \beta\lambda)$

$= (-3)^2 - 2(-4)$

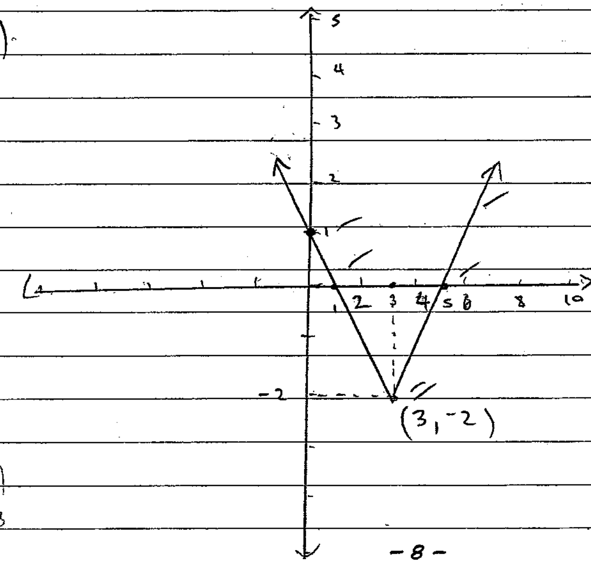
$= 17$

3

3

17

(b) (i)



(ii) $2 = |x - 3|$

$2 = x - 3$

$x = 5$

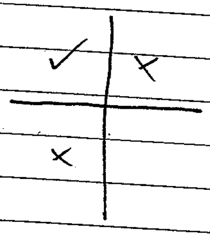
(ii) Domain: All real x ✓

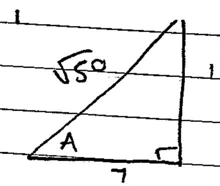
Range: $y \geq -2$ ✓

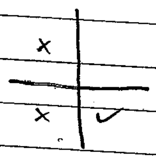
(iii) A $x = 3$, because there, the gradient is changing from positive to negative meaning the tangent at that point has a zero gradient which is not differentiable. (because of a sharp corner or critical point)

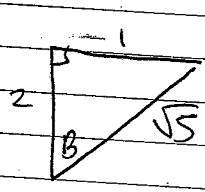
$4\frac{1}{2}$ / 5

(c)









(i) $\sin A = \frac{1}{\sqrt{50}}$ $= \frac{1}{5\sqrt{2}}$ $= \frac{\sqrt{2}}{10}$

(ii) $\cos A = -\frac{7}{\sqrt{50}} = \frac{7}{5\sqrt{2}}$

$= \frac{7\sqrt{2}}{10}$

(iii) $\sin B = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

(iv) $\sin(A+B)$

$= \sin A \cos B + \cos A \sin B$

$= \left(\frac{\sqrt{2}}{10} \times \frac{2}{\sqrt{5}}\right) + \left(\frac{-7\sqrt{2}}{10} \times -\frac{\sqrt{5}}{5}\right)$

$= \frac{2\sqrt{2}}{5\cancel{10}\sqrt{5}} + \frac{7\sqrt{10}}{50}$

$= \frac{\sqrt{2}}{5\sqrt{5}} + \frac{7\sqrt{10}}{50}$ ecf

$= \frac{\sqrt{10}}{25} + \frac{7\sqrt{10}}{50}$

$= \frac{2\sqrt{10} + 7\sqrt{10}}{50}$

$= \frac{9\sqrt{10}}{50}$

3 / 4

Question 5 $\boxed{11}$ 11590

(a) radius of circle = 2
c (1, -1)
∴ show that from point (1, -1), the perpendicular distance of line $3x+4y-10 \neq 2$

LHS

$$d = \frac{|3(1)+4(-1)-10|}{\sqrt{1+1}}$$

$$= \frac{\pm 11}{\sqrt{2}}$$

$1\frac{1}{2}$ / 2

LHS $\frac{11}{\sqrt{2}} \neq 2$

∴ $3x+4y-10=0$ is not a tangent to the circle.

(b) $y = 2x^2 - 5x + 1$
$y = 3x - 7$

~~line $y = 3x - 7$ has a gradient of 3~~ $\rightarrow m_1 = 3$

Solve simultaneously to find a possible point of intersection.

$2x^2 - 5x + 1 = 3x - 7$
$2x^2 - 8x + 8 = 0$
$\frac{(2x-4)(2x-4)}{2} = 0$

$(x-2)(2x-4) = 0$ ∴ $x = 2$,
when $x = 2$, $y = -1$
∴ point of intersection = $(2, -1)$

~~Sub both this point into both~~

LHS $y = 3x - 7$ has a gradient of 3 $\rightarrow m_1 = 3$

find gradient of $y = 2x^2 - 5x + 1$
$y' = 4x - 5$
$m_2 = 4(2) - 5$
$= 3$

$m_1 = m_2$ at point of intersection ∴ $y = 3x - 7$ is a tangent to $2x^2 - 5x + 1$ ✓ 2/2

c) C(-2, 7) D(x, y) 2 : 3

$$0 = \frac{2(x) + 3(-2)}{5} \qquad 3 = \frac{2(y) + 3(7)}{5}$$

$2x - 6 = 0$ $\qquad$ $2y + 21 = 15$
$2x = 6$ $\qquad$ $2y = -6$
$x = 3$ $\qquad$ $y = -3$

∴ D is $(3, -3)$ 2/2

(d) Roots : α, 2α
★ $\alpha + 2\alpha = -6k$
∴ $3\alpha = -6k$
$\alpha = -2k$ ①
★ $-4k + 4 = 2\alpha^2$
$2\alpha^2 = -4k + 4$
$\alpha^2 = -2k + 2$ ②
sub ① into ②
$(-2k)^2 = -2k + 2$

$4h^2 = -2h + 2$

$4h^2 + 2h - 2 = 0$

$(4h-2)\left(\frac{4h+4}{4}\right) = 0$

$(4h-2)(h+2) = 0$

$h = \frac{2}{4}, -2$

$\therefore h = \frac{1}{2}, -2$

$2\frac{1}{2}$

3

(e) (i) x axis : $m = 0$ $y = x^2$

$y' = 2x$

$\tan 60° = \left|\frac{2x-0}{1}\right|$

$\tan 60° = 2x$

$\tan 60° = \sqrt{3}$

$\therefore 2x = \sqrt{3}$

$\therefore x = \frac{\sqrt{3}}{2}$

When $x = \frac{\sqrt{3}}{2}$:

$y = x^2$

$y = \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4}$

$\therefore$ tangent meets the curve at $\left(\frac{\sqrt{3}}{2}, \frac{3}{4}\right)$

(ii) ~~m =~~

$m_1 = 2x$

$= 2\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$ (m of normal)

$\therefore m_2 = \frac{-1}{\sqrt{3}}$

Using point gradient formula:

$y - \frac{3}{4} = \frac{-1}{\sqrt{3}}\left(x - \frac{\sqrt{3}}{2}\right)$

$y = \frac{-1}{\sqrt{3}}x + \frac{\sqrt{3}}{2\sqrt{3}} + \frac{3}{4}$

$y = -\frac{1}{\sqrt{3}}x + \frac{5}{4}$ $\times \sqrt{3}$

~~$\sqrt{3}y$~~

$x + \sqrt{3}y = \frac{5}{4}\sqrt{3}$

$4x + 4\sqrt{3}y - 5\sqrt{3} = 0$

$\therefore$ equation of normal to the curve is

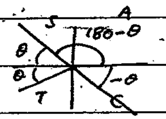
$4x + 4\sqrt{3}y - 5\sqrt{3} = 0$

3

3

Question 6 $\frac{8}{12}$ 11590

(a) $\frac{\sin(180-\theta)\cos(90-\theta)\tan(-\theta)}{\cos(360-\theta)\tan(180+\theta)} = -\tan\theta\sin\theta$



$LHS = \frac{\sin\theta \,.\, \sin\theta \,.\, (-\cancel{\tan\theta})}{\cos\theta \,.\, \cancel{\tan\theta}}$

$= -\sin\theta \,.\, \tan\theta = R.H.S.$

(b) $\lim_{x\to 1} \frac{x^3+5x^2-7x+1}{x-1}$

$$\begin{array}{r} x^2+6x-1 \\ x-1 \overline{\big) x^3+5x^2-7x+1} \\ -(x^3-x^2) \\ \hline 6x^2-7x \\ -(6x^2-6x) \\ \hline -x+1 \\ -(-x+1) \\ \hline 0 \end{array}$$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+6x-1)}{\cancel{x-1}}$

$= \lim_{x\to 1} x^2+6x-1$

$= 6$

$\frac{2}{2}$

(c) $y = 2x^3 - 3x^2 - 35x$

$y' = 6x^2 - 6x - 35$

$y = x$

$y' = 1$

$\cancel{6x - 6x - 35 = 1}$

$6x^2 - 6x - 35 = 1$

$6x^2 - 6x - 36 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\therefore x = 3, \quad x = -2$

$\frac{2}{2}$

11590

(d) (i) $P(x) = 5x^3 - 24x^2 + 9x + 54$

$P'(x) = 15x^2 - 48x + 9$ ✓ $\frac{1}{1}$

(ii) $15x^2 - 48x + 9 = 0$

$5x^2 - 16x + 3 = 0$

$\frac{(5x - 15)(5x - 1)}{5} = 0$ $\frac{2}{3}$

$(x - 3)(5x - 1) = 0$ ✓

$x = 3$ ✓ , $x = 1/5$

$P(3) = 5(3^3) - 24(3^2) + 9(3) + 54$

$= 135 - 216 + 27 + 54 = 0$

$\therefore$ 3 is the double root i.e. $\alpha = 3$

(iii) $P(x) = (x-3)^2(5x + 6)$ by inspection

$\therefore 5x + 6 = 0$

$x = -\frac{6}{5}$ is the other root.

Question 7 11590 (9)

(a) $7\sin x - 4\cos x = 4$

where $t = \tan\frac{\theta}{2}$

$7\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right) = 4$

$\frac{14t - 4 + 4t^2}{1+t^2} = 4$

$\therefore 14t = 8$

$t = \frac{4}{7}$

$\therefore \tan\frac{x}{2} = \frac{4}{7}$

$\frac{x}{2} = 29^\circ 45'$

$\therefore x = 59^\circ 29'$

Also need to test $x = 180^\circ$

$\therefore 7\sin 180^\circ - 4\cos 180$

$= 0 - 4(-1) = 4$ = RHS.

$\therefore x = 59^\circ 29', 180^\circ$.

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(b) (i) $0 = \dfrac{(x-2)(x+2)}{x(x-3)}$

$(x-2)(x+2) = 0$ ✓ 1/1

$x = 2, \quad x = -2$

$(2,0) \quad (-2,0)$

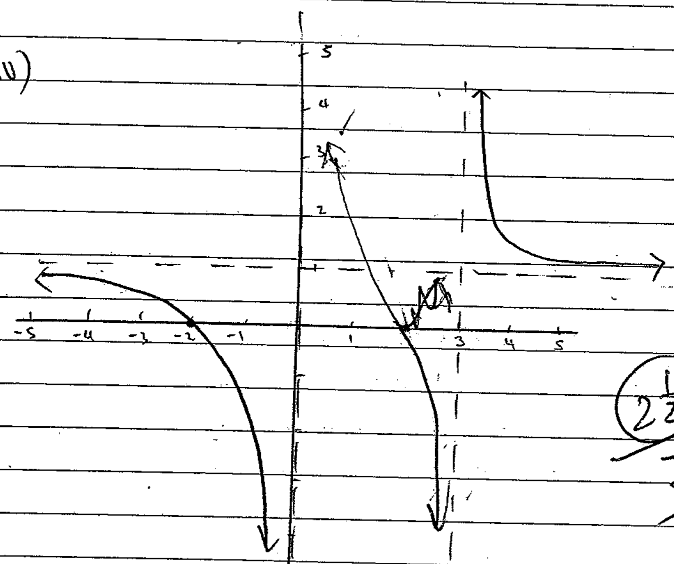
(ii) $y = \dfrac{(x-2)(x+2)}{x(x-3)}$ ✓ 1/1

$x = 3, \quad x = 0$

(iii) $\lim_{x \to \infty} \dfrac{x^2 - 4}{x^2 - 3x}$ ✓ 2/2

$= 1$

(iv)



$2\frac{1}{2}$/3

(v) D: All real x except $x \neq 0, 3$

R: All real y except $y \neq 1$

(v) NA 0/1