PLC

Presbyterian Ladies' College

SYDNEY

1888

Semester Two Examination, 2007

YEAR 11 MATHEMATICS (2 UNIT)

Student Name: ____________

Student Number: ____________

Teacher's Name: ____________

General Instructions:

- Reading time: 5 minutes
- Working time : 2.5 hours
- Write your student number at the top of every sheet of writing paper.
- Write working and answers on the front of the writing paper provided.
- Begin each Question on a new sheet of writing paper.
- Write using a blue or black pen. Diagrams may be drawn with pencil.
- Board-approved scientific calculators may be used.
- All necessary working should be shown in every question.
- Your work will be collected in 10 separately stapled bundles.

Question	Marks
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
Total	/100

Textbook references:

Topic
Basic Arithmetic
Algebra and Surds
Equations
Plane Geometry
Functions and Graphs
Trigonometry
Straight Line Graphs
Introduction to Calculus
Quadratic Function

Question 1 **Start a new page** **Marks**

a) Calculate $\dfrac{5\cdot3-6\cdot5}{5\cdot3\times6\cdot5}$, giving your answer to 3 decimal places. **1**

b) Expand and simplify $(2p-1)(4p+3)$ **1**

c) Draw the graph of $y=3^x$ **1**

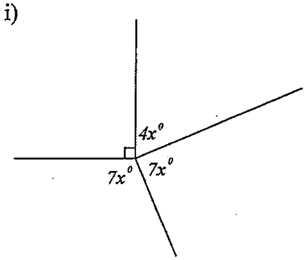
d) Factorise the following expression fully:
$2n^3+54m^3$ **1**

e) By rationalising the denominator, express $\dfrac{9}{3+\sqrt{7}}$ in the form $a+b\sqrt{7}$. **2**

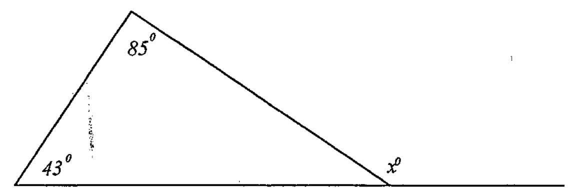
f) Solve $\dfrac{5x+1}{4}=1+2x$ **2**

g) Find the value of the x. NO reasons needed.

i) **1**



ii) **1**



End of Question 1

Question 2 **Start a new page** **Marks**

a) Find the exact value of

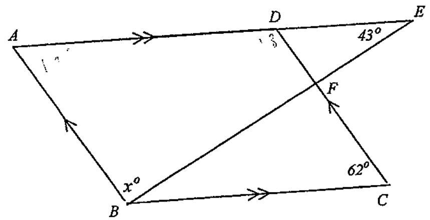
$$\sin 45^o + \cos 120^o$$ **2**

b) If $\tan\theta = -\dfrac{4}{3}$ and $\sin\theta > 0$, find the exact value of

i) $\cos\theta$ **1**

ii) $\operatorname{cosec}\theta$ **1**

c) Find the value of x, giving reasons **2**



d) Solve for x

$$|2x-4| \geq 6$$ **2**

e) Evaluate $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x^2-9}$ **2**

End of Question 2

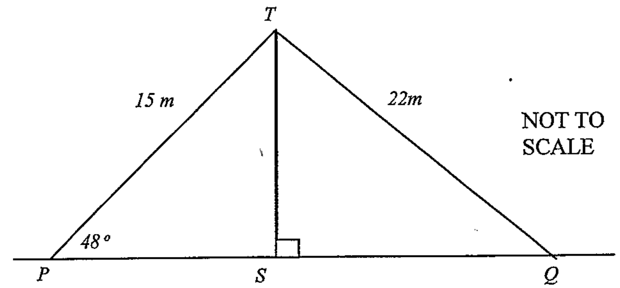
Question 3 **Start a new page** **Marks**

a) Consider the equation $x^2 + (k+1)x + 4 = 0$.
For what value(s) of k does the equation have:

i) equal roots; **1**

ii) distinct real roots? **2**

b) A flagpole on level ground is supported by two unequal lengths of wire, PT and QT, as shown. PT is 15 m long and is inclined at 48° to the horizontal and QT is 22 m long.



i) Find the height of the pole ST, correct to 1 decimal place. **2**

ii) Find the angle at which QT is inclined to the horizontal, correct to the nearest degree. **2**

c) If $f(x) = \sqrt{2x+3}$, evaluate:

i) $f(3)$ **1**

ii) $f'(3)$ **2**

End of Question 3

Question 4 **Start a new page** **Marks**

The line k passes through the points $A(2,1)$ and $B(1,3)$.

a) Show that the equation of line k is $2x+y-5=0$. **2**

b) Show that the point $(6,-7)$ lies on the line k. **1**

c) Find the co-ordinates of C such that A is the midpoint of CB. **2**

d) Show that $AB=OA$, where O is the origin. **2**

e) Find the equation of the line perpendicular to k, passing through the origin. **1**

f) Find the equation of the circle that passes through the points O, C and B. **2**

End of Question 4

Question 5 **Start a new page** **Marks**

a) Simplify $\dfrac{(3^{m+1})^n \times 3^m \times 3^n}{(3^{n+1})^m \times 3^{2m}}$ **2**

b) Differentiate with respect to x. Give your answers in simplest form.

i) $5x^3 - 7 + \dfrac{4}{x}$ **2**

ii) $\dfrac{1}{3\sqrt{x}}$ **1**

iii) $(5x+4)^3$ **1**

iv) $x^4(2x-1)^3$ **2**

v) $\dfrac{3x-4}{2x+7}$ **2**

End of Question 5

Question 6 **Start a new page** **Marks**

a) Solve $|3-x|=2x$. **2**

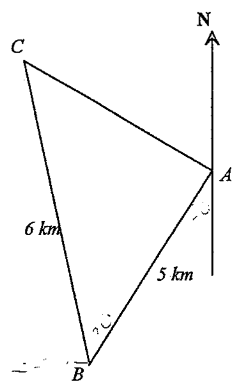
b) Jen runs 5 km from point A on a bearing of 210° to point B. She then walks 6 km on a bearing of 330° to point C.

i) Copy the diagram, showing all important information.

ii) Show that $\angle ABC = 60^\circ$. **1**

iii) Find the exact distance AC. **2**

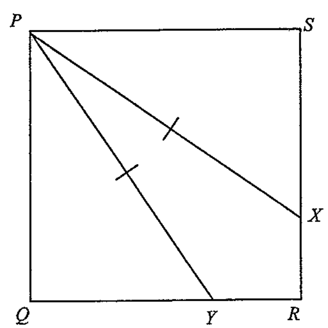
iv) Find the bearing of C from A, correct to the nearest degree. **2**



c) $PQRS$ is a square and $PX = PY$.

i) Prove that $\Delta PSX \equiv \Delta PQY$. **2**

ii) Hence, or otherwise, show that $SX=QY$. **1**



End of Question 6

Question 7 **Start a new page** **Marks**

a) i) Show that $\cot\theta\sin\theta = \cos\theta$. **1**

ii) Hence solve $\cot\theta\sin\theta = -\frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$. **1**

b) i) Solve the pair of simultaneous equations:

$y = x^2 - 3x$

$y = x + 5$ **2**

ii) Show the region of the number plane where the following hold simultaneously:

$y \le x^2 - 3x$ and

$y < x + 5$

Be sure to indicate any points of intersection clearly on your sketch. **3**

c) Find the equation of the tangent to the curve $y = 3x^2 - 2x + 1$ at the point where $x = 1$. Write your answer in general form. **3**

End of Question 7

Question 8 **Start a new page** **Marks**

a) Find, from first principles, the derivative of $x^2 + 3x$. **3**

b) Solve the equation $3\tan^2 x = 1$ for $0^0 \le x \le 360^0$. **2**

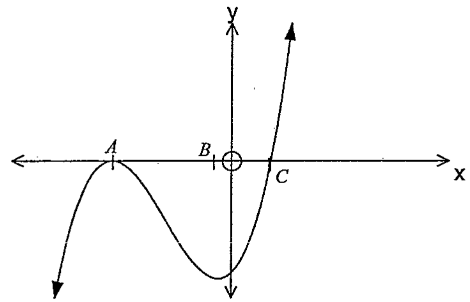
c) Solve $\left(1+\frac{1}{x}\right)^2 - 5\left(1+\frac{1}{x}\right) + 6 = 0$ **3**

d) Solve for $0^\circ \le x \le 360^\circ$:
$\tan(2x+10)^\circ = \cot(3x-20)^\circ$ **2**

End of Question 8

Question 9 **Start a new page** **Marks**

a) The diagram shows the graph of $y = f(x)$.



Copy this sketch onto your writing paper. On the same set of axes, draw a sketch of $y = f'(x)$. **2**

b) If α and β are the roots of the equation $x^2 + 5x - 8 = 0$, find the value of:

i) $\alpha + \beta$ **1**

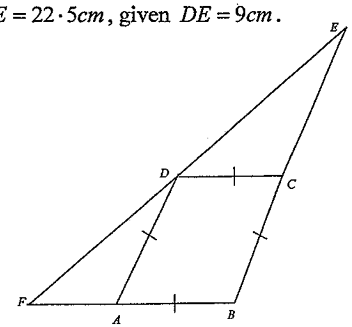
ii) $\alpha\beta$ **1**

iii) $\alpha^2\beta + \alpha\beta^2$. **1**

c) *ABCD* is a rhombus within the triangle *EFB*.

i) Prove that ΔFAD is similar to ΔDCE. **2**

ii) If $2FA = 3AB$, show that $FE = 22\cdot 5cm$, given $DE = 9cm$. **1**



c) State the domain and range of:
$$(x-2)^2 + (y+4)^2 = 25$$ **2**

End of Question 9

Question 10 **Start a new page** **Marks**

a) Prove that
$$\frac{(\cos x \cot x - \sin x \tan x)\sin x \cos x}{\cos x - \sin x} = 1 + \sin x \cos x$$ **3**

b) If $x = -2$ is one root of $x^2 + kx - 2k = 0$, find the other root **2**

c) The equation of a tangent to a circle centre $(4,5)$ is $3x + 4y + 18 = 0$.
Find the length of the radius of the circle. **2**

d) Find the point(s) on the curve $y = \frac{x^3}{3} + x^2 - x$ where the slope of the normal is $-\frac{1}{2}$. **3**

End of Paper

Question 1 $\boxed{10}$ 11590

a) $\frac{5\cdot3 - 6\cdot5}{5\cdot3 \times 6\cdot5}$

$= -0.035$ 1

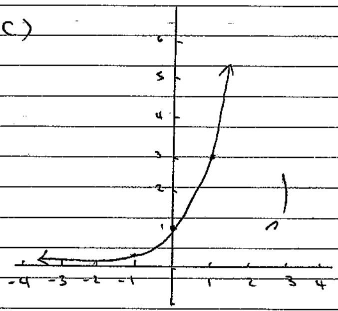
b) $(2p-1)(4p+3)$

$= 8p^2 + 6p - 4p - 3$

$= 8p^2 + 2p - 3$ 1

c) $y = 3^x$

x	-1	0	1	2
y	$\frac{1}{3}$	1	3	9



1

d) $2n^3 + 54m^3$

$= 2(n^3 + 27m^3)$

$= 2(n+3m)(n^2 - 3mn + 9m^2)$ 1

e) $\frac{9}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}}$

$= \frac{9(3-\sqrt{7})}{9-7} = \boxed{\frac{27 - 9\sqrt{7}}{2}}$

$\left(\therefore a = \frac{27}{2},\ b = -\frac{9}{2}\right)$ 2

$= \frac{27}{2} - \frac{9}{2}\sqrt{7}$

-1-

11590

f) $\frac{5x+1}{4} = 1 + 2x$

$5x + 1 = 4(1+2x)$

$5x + 1 = 4 + 8x$

$-3 = 3x$

$\therefore x = -1$ 2

g)(i) $7x + 7x + 4x + 90° = 360°$

$18x = 270$

$x = \frac{270}{18}$ $\therefore x = 15°$

1

(ii) $x = 85 + 43$

$x = 128°$

1

-2-

Question 2 11590 10

a) $\sin 45 = \frac{1}{\sqrt{2}}$ ✓

$\cos 120 = -\cos 60$

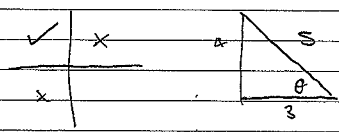
$= -\frac{1}{2}$ ✓

$\therefore \sin 45 + \cos 120$

$= \frac{1}{\sqrt{2}} - \frac{1}{2}$

$= \frac{2 - \sqrt{2}}{2\sqrt{2}}$ rationalise denom. 2/2

b) $\tan\theta = -\frac{4}{3}$ $\sin\theta > 0$



(i) $\cos\theta = -\frac{3}{5}$ ✓

(ii) $\text{cosec}\,\theta = \frac{1}{\frac{4}{5}} = 1 \div \frac{4}{5}$

$= \frac{1}{1} \times \frac{5}{4}$

$= \frac{5}{4}$ ✓ 2/2

c) ~~∠ADC + ∠DCB = 62~~ $\angle ADC + \angle DCB = 180°$ (cointerior angles on parallel lines) ✓

$\therefore \angle ADC = 118°$ ✓

$\angle ADC + \angle BAD = 180°$ (cointerior angles on parallel lines)

$\therefore \angle BAD = 62°$ ✓

-3-

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$\angle BAD + x° + 43° = 180°$ (angle sum of Δ, 180°)

$\therefore x = 180° - 43 - 62$ ✓

$x = 75°$ ✓ 2/2

d) $|2x - 4| \geq 6$

(+) $2x - 4 \geq 6$ when $2x - 4 > 0$

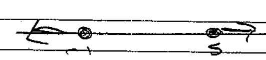
$2x \geq 10$

$x \geq 5$ ✓

(−) $-2x + 4 \geq 6$ when $2x - 4 < 0$

$-2x \geq 2$

$x \leq -1$ ✓ 2/2



$\therefore x \leq -1,\ x \geq 5$

e) $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 - 9}$

$= \lim_{x \to 3} \frac{(x-3)(x+1)}{(x-3)(x+3)}$ ✓

$= \lim_{x \to 3} \frac{x+1}{x+3}$

$= \frac{2}{3}$ ✓ 2/2

-4-

Question 3 (10) 11590

Well done!

a) $x^2 + (k+1)x + 4 = 0$

i) $\Delta = 0 \rightarrow$ show

$\Delta = (k+1)^2 - 4(1)(4)$

$= k^2 + 2k + 1 - 16$

$= k^2 + 2k - 15$

$k^2 + 2k - 15 = 0$

$(k-3)(k+5) = 0$

$\therefore k = 3, \quad k = -5$ ✓ $\frac{1}{1}$

(ii) $\Delta > 0$

$\Delta = k^2 + 2k - 15$

$\therefore k^2 + 2k - 15 > 0$

$(k-3)(k+5) > 0$

$k = 3, \; k = -5$

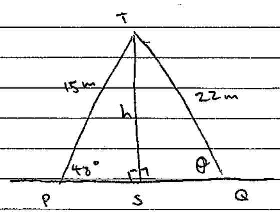
$+ \quad - \quad +$

$-5 \quad 3$

$k > 3$

$k < -5$ ✓ $\frac{2}{2}$

b)



(i) $\frac{h}{\sin 48} = \frac{15}{\sin 90}$

$\therefore h = \frac{15 \times \sin 48}{1}$

$= 11 \cdot 1 \text{ m}$ ✓

(ii) $\frac{\sin\theta}{11 \cdot 1} = \frac{\sin 90}{22}$ $\frac{4}{4}$

$\sin\theta = 0.5045$ ✓

$\therefore \theta = 30°$

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11590

c) $f(x) = \sqrt{2x+3}$

(i) $f(3) = \sqrt{2(3)+3}$

$= \sqrt{9}$

$= 3$ ✓ $\frac{1}{1}$

(ii) $f(x) = (2x+3)^{1/2}$

$f'(x) = \frac{1}{2}(2x+3)^{-1/2} \times 2$

$= \frac{1}{\sqrt{2x+3}}$

$\therefore f'(3) = \frac{1}{\sqrt{2(3)+3}}$

$= \frac{1}{\sqrt{9}} \quad = \frac{1}{3}$ ✓

$\frac{2}{2}$

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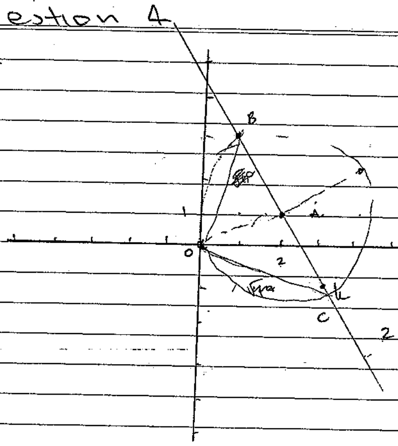
Question 4 (10) 11590

Excellent!



$2x + y - 5$

$y = -2x + 5$

a) Sub in point A (2,1)

LHS $= 2(2) + 1 - 5$

$= 0 =$ RHS $\therefore$ true ✓ $\frac{2}{2}$

Sub in point B (1,3) ✓

LHS $= 2(1) + 3 - 5$

$= 0 =$ RHS $\therefore$ true

$\therefore$ the equation of line k is $2x + y - 5 = 0$ as two points that lie on the line satisfy it ✓ ✓

b) Sub in $x = 6$ and $y = -7$

$2x + y - 5 = 0$ ✓

LHS $= 2(6) + -7 - 5$

$= 0 \quad =$ RHS ✓ $\frac{1}{1}$

$\therefore (6,-7)$ lies on the line.

c) $2 = \frac{x+1}{2} \qquad 1 = \frac{y+3}{2}$ $\frac{2}{2}$

$4 = x + 1 \qquad 2 = y + 3$

$x = 3 \qquad y = -1$

$C(3, -1)$ ✓

-7-

11590

d) $AB = \sqrt{(2-1)^2 + (1-3)^2}$ ✓

$= \sqrt{5}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$OA = \sqrt{(2-0)^2 + (1-0)^2}$ ✓

$= \sqrt{5}$ $\frac{2}{2}$

$\therefore AB = OA = \sqrt{5}$ ✓

e) ~~$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$~~

~~$d = |2(0) + 0 - 5|$~~

$m_1(k) = -2$ ✓

$\therefore m_2 = \frac{1}{2}$ ✓

Point (0,0)

$y - 0 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x$

$x + 2y = 0$ ✓ $\frac{1}{1}$

f) $r = \sqrt{5} \qquad C(2,1)$

$(x-2)^2 + (y-1)^2 = 5$ ✓ ✓ $\frac{2}{2}$

-8-

Question 5 (10) Well done! 11590

a) $\frac{(3^{m+1})^n \times 3^m \times 3^n}{(3^{n+1})^m \times 3^{2m}}$

$= \frac{3^{mn+n+m+n}}{3^{mn+m+2m}}$

$= \frac{3^{mn+m+2n}}{3^{mn+3m}}$

$= 3^{mn+m+2n} \div 3^{mn+3m}$

$= 3^{2n-2m}$

$= (3^2)^{n-m}$

$= 9^{n-m}$

2/2

b) (i) $y = 5x^3 - 7 + 4x^{-1}$

$y' = 15x^2 - \frac{4}{x^2}$ 2/2

(ii) $y = \frac{1}{3} x^{-1/2}$

$= \frac{-1}{6} x^{-3/2}$

$= \frac{-1}{6\sqrt{x^3}}$ 1/1

(iii) $y = (5x+4)^3$

$y' = 3(5x+4)^2 \times 5$

$= 15(5x+4)^2$ 1/1

11590

(iv) $y = x^4(2x-1)^3$

$u = x^4 \quad u' = 4x^3$

$v = (2x-1)^3 \quad v' = 6(2x-1)^2$

$u'v + v'u = y'$

$\therefore y' = 4x^3(2x-1)^3 + 6x^4(2x-1)^2$

$= 2x^3(2x-1)^2[2(2x-1)+3x]$

$= 2x^3(2x-1)^2(4x-2+3x)$

$= 2x^3(2x-1)^2(7x-2)$

2/2

(v) $\frac{3x-4}{2x+7}$

$u = 3x-4 \quad u' = 3$

$v = 2x+7 \quad v' = 2$

$y' = \frac{3(2x+7) - 2(3x-4)}{(2x+7)^2}$

$= \frac{6x+21-6x+8}{(2x+7)^2}$

$= \frac{29}{(2x+7)^2}$

2/2

Question 6 (10) 11590

Excellent!

a) $|3-x| = 2x$

(+) $3 - x = 2x$ when $3 - x > 0$

$3 = 3x$

$x = 1$

(−) $-3 + x = 2x$ when $3 - x < 0$

$-3 = x$

$x = -3$

Check

(+) $|3-x| = 2x$ sub in $x = 1$

LHS $= |3-1|$

$= 2$

RHS $= 2(1)$

$= 2$

LHS = RHS $\therefore$ $x = 1$ is a solution

(−) $|3-x| = 2x$ sub in $x = -3$

LHS $= (3+3)$

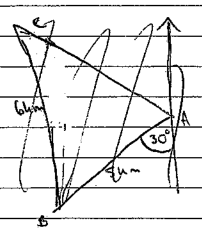
$= 6$

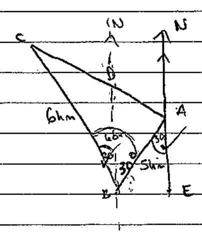
RHS $= 2(-3)$

$= -6$

LHS $\neq$ RHS $\therefore$ $x = -3$ is not a solution

2/2

b) 

(i) 

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(ii) Construct point D on AC

$\angle CBD = 30°$

$(360° - 330° = 30°)$

Construct point E

$\angle EAB = 30°$

$(210° - 180°)$

$\angle DBA = \angle EAB$

(alternate angles on parallel lines)

$\angle CBD + \angle DBA = 30° \; \angle ABC$

$\therefore 30° + 30° = 60°$

$\therefore \angle ABC = 60°$

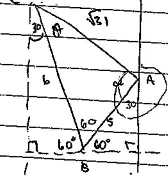
1/1

(iii) $c^2 = a^2 + b^2 - 2ab \cos C$

$\therefore AC^2 = 6^2 + 5^2 - 2(6)(5)\cos 60°$

$\therefore AC = \sqrt{31}$

2/2

(iv) 

$\frac{\sin\theta}{5km} = \frac{\sin 60}{\sqrt{31}}$

$\therefore \theta \approx 51°$

$\therefore \alpha = 180 - 60 - 51$

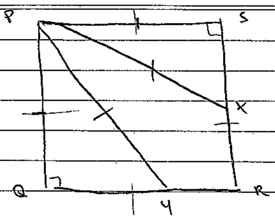
$= 69°$

$180 + 30 + 69 = 279°$

279°T

2/2

11590

c) 

(i) $PS = SR = RQ = QP$ ✓
(sides of a square are equal)
$\angle PQY = \angle PSX = 90°$ ✓
(interior angles of a square are 90°)
$PY = PX$ (given) ✓
$\therefore \triangle PSX \equiv \triangle PQY$ (RHS) ✓

2

(ii) Through Pythagoras Theorem:
$a^2 + b^2 = c^2$
$c^2 - a^2 = b^2$
If $PQ = PS$ (shown) and
If $PY = PX$ (given)
$PY^2 - PQ^2 = PQ^2 - PS^2$
$\therefore SX^2 = QY^2$
$\therefore SX = QY$ ✓

1

or: corresponding sides in congruent triangles

Question 7 (9) 11590

a) (i) $\cot\theta \sin\theta = \cos\theta$
LHS $= \cot\theta \sin\theta$
$= \frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta}$
$= \cos\theta$ = RHS
$\therefore \cot\theta \sin\theta = \cos\theta$

1

(ii) $\cot\theta \sin\theta = -\frac{1}{2}$
$\therefore \cos\theta = \frac{-1}{2}$

1

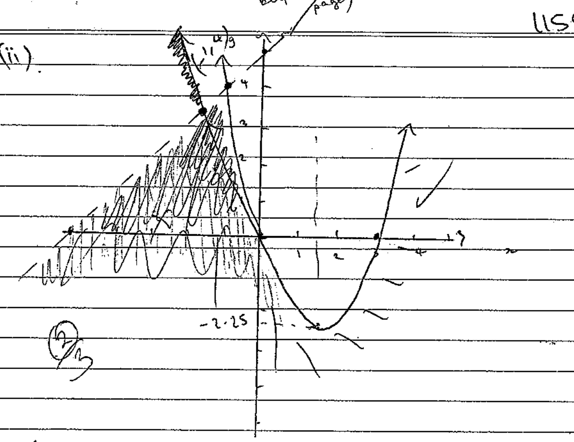
$\alpha = 60°$
$\therefore \theta = 120°, 240°$

b) (i) $y = x^2 - 3x$ ①
$y = x + 5$ ②

Sub ② into ①
$x + 5 = x^2 - 3x$
$0 = x^2 - 4x - 5$
$x^2 - 4x - 5 = 0$
$(x-5)(x+1) = 0$
$\therefore x = 5$ $x = -1$
when $x = 5$, when $x = -1$,
$y = 10$ $y = 4$
$x = 5, y = 10$ or $x = -1, y = 4$

2

11590

(ii) 

2/3

$y \leq x^2 - 3x$ $y < x + 5$
$x^2 - 3x = 0$ $y = x + 5$
$x(x-3) = 0$
$x = 0, x = 3$
$\frac{-b}{2a}$ $\frac{3}{2} = 1\frac{1}{2}$ $0 < 0 + 5$
$0 < 5$
vertex
$0 \leq 0$
$x^2 - 3x$

c) $y = 3x^2 - 2x + 1$
$y' = 6x - 2$
sub in $x = 1$
$y' = 4$ point $(1, 2)$

3/3

$y - 2 = 4(x - 1)$
$y = 4x - 2$
$4x - y - 2 = 0$

Question 8 $\frac{9\frac{1}{2}}{10}$ 11590

a) ~~$\lim_{h \to 0} x^2 + 3x$~~
~~$= \lim_{h \to 0}$~~

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - x^2 - 3x}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{x^2} - \cancel{3x}}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}}$

$= \lim_{h \to 0} 2x + h + 3$

$= 2x + 3$

3/3

b) $3\tan^2 x = 1$
$\tan^2 x = \frac{1}{3}$
$\tan x = \pm\frac{1}{\sqrt{3}}$
$x = 30°, 150°, 210°, 330°$

2/2

11590

c) $\left(1+\frac{1}{x}\right)^2 - 5\left(1+\frac{1}{x}\right) + 6 = 0$

Let $1 + \frac{1}{x} = u$

$u^2 - 5u + 6 = 0$

$(u-3)(u-2) = 0$

$u = 3, \quad u = \cancel{0}\ 2$

$\therefore 1 + \frac{1}{x} = 3 \qquad 1 + \frac{1}{x} = \cancel{0}\ 2$

$\frac{x+1}{x} = 3 \qquad \frac{x+1}{x} = \cancel{0}\ 2$

$x + 1 = 3x \qquad x + 1 = \cancel{0}\ 2x$

$1 = 2x \qquad \cancel{x = -1}$

$x = \frac{1}{2} \qquad x = 1$

$2\frac{1}{2}/3$

$x = \frac{1}{2}, \ x = -1$ CFPW

d) $\tan(2x+10)^\circ = \cot(3x-20)^\circ$

$\tan\theta = \cot(90 - \theta)$

$\therefore \tan(2x+10)^\circ = \cot(90 - 2x - 10)^\circ$

$\therefore 3x - 20 = 90 - \cancel{2x} - 10$

$5x = 100$

$x = 20$

$x = 20^\circ, \ 200^\circ$

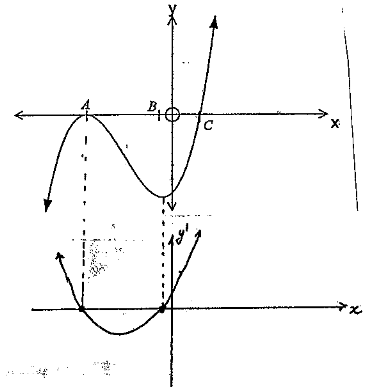
$x/2$

-0.51

$340 \quad 1.19$

Question 9 (8) 11590

a)



b) (i) $\alpha + \beta = \frac{-b}{a}$

$= -5$

(ii) $\alpha\beta = \frac{c}{a} = -8$

(iii) $\alpha^2\beta + \alpha\beta^2$

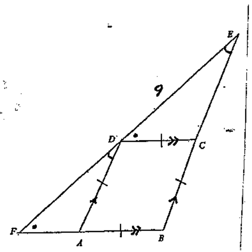
$= \alpha\beta(\alpha + \beta)$

$= -8(-5)$

$= 40$

3/3

(9)(c)



(i) In $\triangle FAD$, $\triangle DCE$

$\angle DFA = \angle EDC$ (Corr. $\angle$s $DA \parallel CB$; parallel sides of rhombus)

$\angle ADF = \angle CED$ (" ")

$\therefore \triangle FAD \parallel\!\mid \triangle DCE$ (Equiangular)

(ii) Given that $2FA = 3AB$

$\therefore \frac{FA}{AB} = \frac{FA}{DC} = \frac{3}{2}$

From (i)

$\frac{FA}{DC} = \frac{FD}{DE}$

$\frac{3}{2} = \frac{FE - DE}{DE} = \frac{FE - 9}{9}$

$\therefore FE = \frac{3}{2} \times 9 + 9$

$= \frac{27}{2} + 9$

$= 13\frac{1}{2} + 9$

$= 22.5$ as req^d.

(c) $D: -3 \le x \le 7$

$R: -9 \le y \le 1.$

Question 10 8 11590

a) $\frac{(\cos x \cot x - \sin x \tan x)\sin x\cos x}{\cos x - \sin x} = 1 + \sin x \cos x$

LHS $= \frac{(\cos x \cot x - \sin x \tan x)\sin x \cos x}{\cos x - \sin x}$

$= \frac{\left(\frac{\cos x \times \cos x}{\sin x} - \frac{\sin x \times \sin x}{\cos x}\right)\sin x \cos x}{\cos x - \sin x}$

$= \frac{\left(\frac{\cos^3 x - \sin^3 x}{\cancel{\sin x \cos x}}\right)\cancel{\sin x \cos x}}{\cos x - \sin x}$

$= \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$

$= \frac{\cancel{(\cos x - \sin x)}(\cos^2 x + \sin x \cos x + \sin^2 x)}{\cancel{\cos x - \sin x}}$

$= \cos^2 x + \sin x \cos x + \sin^2 x$

$= 1 - \cancel{\sin^2 x} + \sin x \cos x + \cancel{\sin^2 x}$

$= 1 + \sin x \cos x \quad = $ RHS

$\therefore \frac{(\cos x \cot x - \sin x \tan x)\sin x \cos x}{\cos x - \sin x} = 1 + \sin x \cos x$

3/3

b) $x^2 + kx - 2k = 0$

$\alpha - 2 = -k$ ① $\qquad \alpha = 2 - k$

$-2\alpha = -2k$ ② $\qquad \alpha = \frac{-2k}{-2}$

$= k$

$\therefore 2 - k = k$

$2 = 2k$

$\therefore k = 1$

11590

~~the other root is~~

If $k = 1$,

$\alpha - 2 = -1$

$\alpha = 1$

the other root is 3

2

2

(c) Perp. distance : $r = \dfrac{|3(4) + 4(5) + 18|}{\sqrt{3^2+4^2}}$

$= \dfrac{|50|}{5}$

$r = 10$

4

d) $y = \dfrac{x^3}{3} + x^2 - x$

if $m_2 = \dfrac{-1}{2}$, $m_1 = 2$

$y = \frac{1}{3}x^3 + x^2 - x$

$y' = x^2 + 2x - 1$

$\therefore 2 = x^2 + 2x - 1$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$ $\therefore x = 1$, $x = -3$

$(1, \frac{1}{3})$ and $(-3, 3)$

3

3