POLYNOMIALS 1

1) For the polynomial $P(x) = 2x^3 - 3x^2 + 5x - 7$, find
   (a) $P(-2)$
   (b) the degree of $P(x)$
   (c) the leading coefficient
   (d) the constant term.

2) If $p(x) = x^3 + 5$ and $q(x) = 3x - 1$, find
   (a) $p(3) + q(-1)$
   (b) the degree of $p(x) + q(x)$
   (c) the degree of $p(x)\,q(x)$
   (d) the degree of $p^1(x)$

3) Find the zeros of $p(x) = x^3 - 2x^2 - 3x$.

4) Find the values of a, b and c in $p(x) = ax^2 + bx + c$ if $p(x)$ is monic, the constant term is -3 and the coefficient of x is -1.

5) (a) Show $P(x) = 2x^2 - x + 5$ has no zeros.
   (b) If $Q(x) = 5x - 3$, show $P(x) + Q(x)$ has one zero.

6) If $f(x) = x^3 - 4x^2 + x - 1$ and $q(x) = x - 2$, divide $f(x)$ by $q(x)$ and write $f(x)$ in the form $f(x) = q(x).p(x) + r(x)$

7) (a) Find the zeros of $p(x) = (x - 1)(x + 2)(x + 4)$.
   (b) Use part (a) to draw a rough sketch of the polynomial.

8) Find the remainder when $p(x) = x^2 - 3x + 4$ is divided by $x - 2$.

9) Find the value of k if $f(x) = 2x^3 + x^2 + kx - 2$ is divisible by $x + 1$.

10) (a) Write $P(x) = x^4 - 7x^3 + 13x^2 + 3x - 18$ as a product of its linear factors.
    (b) What are the roots of $P(x) = 0$?

11) (a) Show $f(x) = x^4 - 7x^3 + 10x^2$ has zeros 2 and 5.
    (b) Find the roots of $f(x) = 0$

12) A monic polynomial $f(x)$ of degree 2 has a double root at $x = -2$. Find the polynomial.

13) (a) A polynomial $P(x)$ of degree 3 has a double root at $x = 1$. Find $P(x)$. Is this a unique answer?
    (b) If $P(x)$ is monic and has a constant term of -2, does this give a unique polynomial? Write down $P(x)$ and find its zeros.

14) A polynomial is given by $p(x) = (x + 3)^4\,q(x)$. Show that $p(-3) = p^1(-3) = 0$

15) Find values of a and b if $f(x) = ax^3 + bx^2 - 2x + 5$ has a double root at $x = 1$.

16) If α, β and γ are the roots of $x^3 - 3x^2 + x - 5 = 0$, find
    (a) $\alpha + \beta + \gamma$
    (b) $\alpha\beta\gamma$
    (c) $\alpha\beta + \beta\gamma + \alpha\gamma$
    (d) $(\alpha + 1)(\beta + 1)(\gamma + 1)$

17) Given α, β, γ and δ are the roots of $x^4 + 2x^2 - 3x + 4 = 0$, find
    (a) $\alpha + \beta + \gamma + \delta$
    (b) $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$
    (c) $\alpha\beta\gamma + \alpha\gamma\delta + \alpha\beta\delta + \beta\gamma\delta$
    (d) $\alpha\beta\gamma\delta$
    (e) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

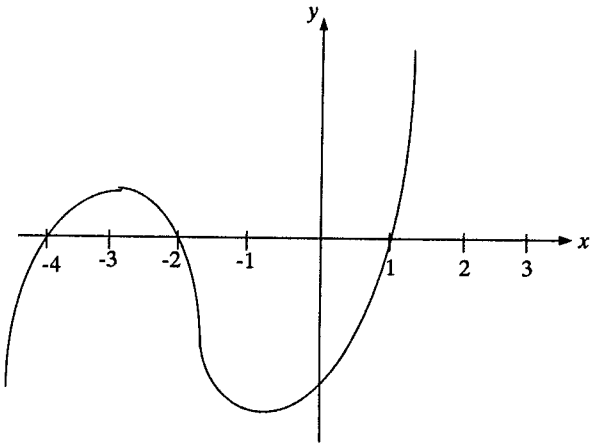
18) One root of $2x^3 + 7x^2 - x + 1 = 0$ is 3. If α and β are the other two roots, find
    (a) $\alpha + \beta$
    (b) $\alpha\beta$

19) If $x = 4$ is a double root of $x^4 - x^3 - 2x^2 + 5x + 3 = 0$, find the sum of the other two roots.

20) If two of the roots of $3x^3 + 11x^2 + 8x - 4 = 0$ are equal, find all the roots.

ANSWERS

1) (a) -45 (b) 3 (c) 2 (d) -7
2) (a) 28 (b) 3 (c) 4 (d) 2
3) 0, -1, 3
4) a = 1, b = -1, c = -3
5) (a) $\Delta = -39 < 0$ so P(x) has no zeros.
(b) $P(x) + Q(x) = 2x^2 + 4x + 2$
$\Delta = 0$ so P(x) has one zero.
6) $x^3 - 4x^2 + x - 1 = (x - 2)(x^2 - 2x - 3) - 7$
7) (a) 1, -2, -4
(b)



8) 2
9) k = -3
10) (a) $P(x) = (x - 2)(x + 1)(x - 3)^2$ (b) $x = -1, 2, 3$
11) (a) f(2) = f(5) = 0 (b) $x = 0, 2, 5$
12) $f(x) = (x + 2)^2$
13) (a) $P(x) = k(x - 1)^2 (x + a)$ This is not unique as k and a vary.
(b) $P(x) = (x - 1)^2 (x - 2)$ Zeros are 1, 2
14) $p(-3) = (-3 + 3)^4 q(-3) = 0$
$p^1(-3) = 4(-3 + 3)^3 q(-3) + (-3 + 3)^4 q^1(-3) = 0$
15) a = 8, b = -11
16) (a) 3 (b) 5 (c) 1 (d) 10
17) (a) 0 (b) 2 (c) 3 (d) 4 (e) $\frac{3}{4}$
18) (a) $-6\frac{1}{2}$ (b) $-\frac{1}{6}$
19) -7
20) $x = -2, \frac{1}{3}$