Polynomials

1 Giving your answers in descending powers of x, simplify

    a $(x^3 + 2x) + (x^3 + 3x^2 + 5x)$ **b** $(x^5 + 4x^4 + 2x^3) + (3x^5 + x^4 + 5x^3)$

    c $(7x^3 + x^2 + 4x) - (x^3 + 3x^2 + x)$ **d** $(x^4 + x^3 - 5x^2 + 4) + (x^3 - 3x^2 - 6)$

    e $(x^8 - 3x^6 + x^5) - (3x^7 - x^6 - 5x^5)$ **f** $(9x^3 - x^2 - 4) - (^-5x^3 + x^2 - x - 3)$

    g $(x^2 + 7x - 2 + x^{-1}) + (x - 5 + x^{-1})$ **h** $5(x^3 - 4x^2 + x) + (x^3 + 7x^2 - 3x)$

2 Giving your answers in ascending powers of x, simplify

    a $(7x + x^2 - 2x^4 - 5x^5 + x^6) + (5 - x - x^3 + x^5 + 3x^6) + (1 + 9x^2 - 6x^3 + 2x^4)$

    b $8(x^4 + 5x^3 - x^2 - 11x + 4) - 3(2x^4 - 5x^3 - x^2 + 16x + 9)$

    c $(8x^3 + x^7 - 5x - 2x^2 + x^6 - x^5) + (7x^2 - x^5 - x^4 + 12x) - (x^7 - x - x^3 - x^6)$

    d $(7x^{-2} - 2x^{-1} + 7 - x + 5x^2 - x^3) - (x^{-2} - 3x^{-1} - x + 5x^2 + x^3 - 9x^4)$

3 Giving your answers in descending powers of x, simplify

    a $(x + 2)(x^3 + 4x^2 + 8)$ **b** $(3x - 5)(2x^3 + 7x^2 + 5x - 10)$

    c $(x - 5)(1 - 2x + x^2 - 3x^3)$ **d** $(2x^2 + 3)(x^4 - 3x^3 + x + 2)$

    e $(x^2 + 3x + 1)(x^2 + x + 4)$ **f** $(x^2 - 2x + 6)(x^3 + 5x - 7)$

    g $(4x^2 + x - 3)(2x^3 - 8x^2 - x + 5)$ **h** $(2x^2 + 3 - x^{-1})(4x - 2x^{-1} + x^{-2})$

4 Find the values of the constants A and B.

    a $A(x - 3) + B(x - 4) \equiv 6x - 19$

    b $A(x + 2) + B(x - 1) \equiv 3x - 9$

    c $A(x + 2)(x + 1) + B(x + 4)(x - 1) \equiv 5x^2 + 15x - 2$

    d $A(x - 3)(x - 5) + B(2x + 1)(x - 4) \equiv 136 - 36x$

    e $A(3x - 2)(2x + 1) + Bx(4x - 6) \equiv 26x^2 - 7x - 8$

    f $A(x^2 + 5x + 3) + B(2x^2 - 3) \equiv 9x^2 + 5x - 9$

    g $A(x^2 - 3x + 2) + B(4x^2 - x + 5) \equiv 3(1 - 8x - x^2)$

    h $A(x - 1)(x^2 + 2) + B(x^3 - 5x + 1) \equiv x^3 - 5x^2 + 30x - 14$

    i $A(4x + 2 - \frac{5}{x}) + B(x - 3 + \frac{2}{x}) \equiv \frac{24}{x} - 25 - x$

    j $A(x^2 - 3)^2 + B(x - 1)^3 \equiv 4x^4 + 2x^3 - 30x^2 + 6x + 34$

    k $x(x + A)^2 + (x^2 - 4x + 2) \equiv (x - 2)^3 + B(x^2 - 10)$

5 Find the values of the constants A, B and C.

    a $Ax(x - 2) + B(x + 1)(x + 3) + C(x + 3)(x - 2) \equiv 6x^2 - 6x - 27$

    b $A(x + 3)(x - 1) + B(x + 2)(x - 1) + C(x + 2)(x - 4) \equiv 4x^2 + x - 23$

    c $A(x^2 + 1) + B(x^2 - 2x - 3) + C(x^2 + 3x) \equiv 10x^2 + 7x - 11$

    d $A(x + 2)(x - 5) + B(x + 2)(x - 1) + C(2x + 1)(x + 7) \equiv 4x^2 + 27x - 7$

    e $A(x + 1) + (Bx + C)(x - 2) \equiv 3(x^2 - 3x - 3)$

    f $A(x - 5)(2x + 1) + (Bx + C)(2x - 1) \equiv 10x^2 - 43x - 17$

    g $A(x^3 - x + 4) + (Bx + C)(x^2 - 1) \equiv 5x^2 - 13$

    h $A(x + B)^2 + C \equiv x^2 - 4x + 1$

    i $A(x + B)^2 + C \equiv 3(x + 2)(x - 5)$

    j $Ax^6 + Bx^4 + Cx^2 \equiv x^3(5x - 3x^{-1})(x^2 + 2)$

    k $Ax^4 + Bx^2 + C \equiv x^2(x - 2x^{-1})^2$

    l $(x^2 + Ax)(Bx - 2) \equiv x(5x + C)(x - 1)$

6 Divide

a $(x^3 + 4x^2 + 5x + 2)$ by $(x + 1)$ **b** $(2x^3 - 11x^2 + 18x - 9)$ by $(x - 3)$

c $(x^3 + 2x^2 - 7x + 6)$ by $(x - 1)$ **d** $(x^3 + 3x^2 + 2x + 24)$ by $(x + 4)$

e $(10 - 7x + 3x^2 + x^3)$ by $(x + 2)$ **f** $(x^3 - 18x + 35)$ by $(x + 5)$

g $(x^4 + x^3 - 2x^2 + 3x - 3)$ by $(x - 1)$ **h** $(28 - 15x^2 - 5x^3 + x^4)$ by $(x - 7)$

i $(4x^3 + 12x^2 - 3x - 4)$ by $(2x + 1)$ **j** $(6x^4 - 4x^3 - 3x^2 + 5x - 1)$ by $(3x - 2)$

7 Find if $(x - 2)$ is a factor of

a $x^3 - 3x - 2$ **b** $x^3 + 5x^2 - x + 7$ **c** $3x^3 - 10x^2 + 9x - 2$

8 Find if $(x + 3)$ is a factor of

a $x^3 + 4x^2 + 7$ **b** $x^3 + 2x^2 + 3x + 18$ **c** $5x^3 + 17x^2 + 13x + 21$

9 Evaluate $f(^-2)$, $f(^-1)$, $f(1)$ and $f(2)$ in each case.
Hence write down any expressions that you can deduce are factors of $f(x)$.

a $f(x) \equiv x^3 + 4x^2 + 11x + 8$ **b** $f(x) \equiv x^3 + 5x^2 + 2x - 8$

c $f(x) \equiv 12 - 11x^2 + 2x^4$ **d** $f(x) \equiv 2x^4 - 4x^3 - 5x^2 + 6x + 5$

10 By first searching for a linear factor, fully factorise

a $x^3 + x^2 + 2x - 4$ **b** $x^3 + 2x^2 - 9x - 18$ **c** $x^3 + 2x^2 - 14x - 3$

d $8 + 2x - 5x^2 + x^3$ **e** $2x^3 - 11x^2 + 17x - 6$ **f** $3x^3 - 2x^2 - 37x - 12$

g $x^3 - 8$ **h** $1 - 5x + 5x^2 - x^3$ **i** $x^4 + x^3 - x - 1$

Exercise 6S   Skills Practice

1   **a** $2x^3+3x^2+7x$ **b** $4x^5+5x^4+7x^3$
   c $6x^3-2x^2+3x$ **d** $x^4+2x^3-8x^2-2$
   e $x^8-3x^7-2x^6+6x^5$ **f** $14x^3-2x^2+x-1$
   g $x^2+8x-7+2x^{-1}$ **h** $6x^3-13x^2+2x$

2   **a** $6+6x+10x^2-7x^3-4x^5+4x^6$
   b $5-136x-5x^2+55x^3+2x^4$
   c $8x+5x^2+9x^3-x^4-2x^5+2x^6$
   d $6x^{-2}+x^{-1}+7-2x^3+9x^4$

3   **a** $x^4+6x^3+8x^2+8x+16$
   b $6x^4+11x^3-20x^2-55x+50$
   c $-3x^4+16x^3-7x^2+11x-5$
   d $2x^6-6x^5+3x^4-7x^3+4x^2+3x+6$
   e $x^4+4x^3+8x^2+13x+4$
   f $x^5-2x^4+11x^3-17x^2+44x-42$
   g $8x^5-30x^4-18x^3+43x^2+8x-15$
   h $8x^3+8x-2-6x^{-1}+5x^{-2}-x^{-3}$

4   **a** $A=5, B=1$ **b** $A=-2, B=5$
   c $A=3, B=2$ **d** $A=8, B=-4$
   e $A=4, B=\frac{1}{2}$ **f** $A=1, B=4$
   g $A=9, B=-3$ **h** $A=5, B=-4$
   i $A=-2, B=7$ **j** $A=4, B=2$
   k $A=-4, B=-1$

5   **a** $A=3, B=-1, C=4$
   b $A=3, B=-1, C=2$
   c $A=1, B=4, C=5$
   d $A=4, B=-6, C=3$
   e $A=-5, B=3, C=2$
   f $A=4, B=1, C=-3$
   g $A=-2, B=2, C=5$
   h $A=1, B=-2, C=-3$
   i $A=3, B=-^3/_2, C=-36^3/_4$
   j $A=5, B=7, C=-6$
   k $A=1, B=-4, C=4$
   l $A=-1, B=5, C=-2$

6   **a** x^2+3x+2 **b** $2x^2-5x+3$
   c x^2+3x-4 r2   **d** x^2-x+6
   e x^2+x-9 r28   **f** x^2-5x+7
   g x^3+2x^2+3 **h** x^3+2x^2-x-7 r-21
   i $2x^2+5x-4$ **j** $2x^3-x+1$ r1

7   **a** yes   **b** no   **c** yes

8   **a** no   **b** yes   **c** yes

9   **a** $-6, 0, 24, 54$; $(x+1)$
   b $0, -6, 0, 24$; $(x+2)(x-1)$
   c $0, 3, 3, 0$; $(x+2)(x-2)$
   d $37, 0, 4, -3$; $(x+1)$

10   **a** $(x-1)(x^2+2x+4)$ **b** $(x+2)(x+3)(x-3)$
   c $(x-3)(x^2+5x+1)$ **d** $(x-2)(x-4)(x+1)$
   e $(x-2)(2x-1)(x-3)$ **f** $(x+3)(3x+1)(x-4)$
   g $(x-2)(x^2+2x+4)$ **h** $(1-x)(x^2-4x+1)$
   i $(x-1)(x+1)(x^2+x+1)$