Exercise 2.2

1. Use the remainder theorem to find the remainder when each of the following polynomials is divided by the linear polynomial given.
   (a) $x^3 + 4x^2 - 3x + 2$; $x - 2$
   (b) $x^4 - 3x^3 + 2x - 1$; $x + 2$
   (c) $x^5 - x^3 + 6$; $x + 1$
   (d) $x^3 + 2x + 1$; $2x - 1$
   (e) $x^3 - 2x^2 + x + 1$; $2x - 3$
   (f) $x^3 + 15x - 1$; $3x + 1$
2. Determine whether each of the following linear polynomials is a factor of the polynomial given.
   (a) $x + 2$; $x^4 + 4x^3 + 4x^2$
   (b) $x - 1$; $x^5 + 3x^2 - 6x + 3$
   (c) $x + 1$; $x^3 - 2x^2 + 6x + 9$
   (d) $x - 2$; $x^3 - 4x^2 + 3x + 2$
   (e) $2x + 1$; $2x^3 - 3x^2 + 2x + 2$
   (f) $2x - 1$; $x^3 + 4x^2 - 3x - 1$
3. Factorise each of the following polynomials.
   (a) $2x^3 - 3x^2 + 1$
   (b) $3x^3 - 2x^2 - 7x - 2$
   (c) $x^4 - x^2 - 72$
   (d) $x^5 + x^3 + x$
   (e) $4x^3 - 13x + 6$
   (f) $4x^4 - 4x^3 - 9x^2 + x + 2$
4. If $(x - 2)$ is a factor of $ax^3 + 3x^2 - 2x + a$, find the value of a.
5. Show that $(x - a)$ is a factor of $x^3 + (1 - a)x^2 + (3 - a)x - 3a$.
6. When the polynomial $x^2 + ax + b$ is divided by $(x - 1)$ and $(x + 2)$, its remainder is 4 and 5 respectively. Find the values of a and b.
7. When $x^3 + px^2 + qx + 1$ is divided by $(x - 2)$, its remainder is 9; when it is divided by $(x + 3)$, its remainder is 19. Find the values of p and q.
8. The expression $2x^3 + ax^2 + b$ can be divided exactly by $(x + 1)$, and its remainder is 16 when divided by $(x - 3)$. Find the values of a and b.
9. The expression $ax^3 - 8x^2 + bx + 6$ can be divided exactly by $x^2 - 2x - 3$. Find the values of a and b.
10. When the polynomials $x^3 + 4x^2 - 2x + 1$ and $x^3 + 3x^2 - x + 7$ are divided by $(x - p)$, the remainders are equal. Find the possible values of p.
11. The expressions $x^3 - 4x^2 + x + 6$ and $x^3 - 3x^2 + 2x + k$ have a common factor. Find the possible values of k.
12. If $(x - a)$ is a factor of the expression $ax^3 - 3x^2 - 5ax - 9$, find the possible values of a. Factorise the expression for each of the values of a.
13. Given that $f(x) \equiv x^3 + kx^2 - 2x + 1$ has a remainder k when it is divided by $(x - k)$, find the possible values of k.
14. Find the value of k if $(x + 1)$ is a factor of $2x^3 + 7x^2 + kx - 3$. Using this value of k, solve the equation $2x^3 + 7x^2 + kx - 3 = 0$.
15. Find the values of a and b if $f(x) \equiv ax^3 + bx^2 + 12$ can be divided exactly by both $(x + 1)$ and $(x - 2)$ respectively. With these values of a and b, solve the equation $f(x) = 0$.
16. The expression $x^3 + ax^2 + bx - 8$ is divisible by $(x + 1)$. When it is divided by $(x - 2)$, its remainder is 42. Find the values of a and b and the value of the expression when $x = 1$.

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1. (a) 20 (b) 35 (c) 6
   (d) $2\frac{1}{8}$ (e) $1\frac{3}{8}$ (f) $-6\frac{1}{27}$
2. (a) Yes (b) No (c) Yes
   (d) Yes (e) Yes (f) No
3. (a) $(x - 1)^2(2x + 1)$
   (b) $(x + 1)(x - 2)(3x + 1)$
   (c) $(x + 3)(x - 3)(x^2 + 8)$
   (d) $x(1 - x + x^2)(1 + x + x^2)$
   (e) $(x + 2)(2x - 1)(2x - 3)$
   (f) $(x + 1)(x - 2)(2x + 1)(2x - 1)$
4. $-\frac{8}{9}$
6. $a = \frac{2}{3}$, $b = \frac{7}{3}$
7. $p = 3$, $q = -6$
8. $a = -5$, $b = 7$
9. $a = 3$, $b = -5$
10. 3, −2
11. $k = 6, 0, -6$
12. $a = 3$, $3(x - 3)(x + 1)^2$; $a = -3$, $-3(x + 3)(x - 1)^2$
13. 1, $\frac{-1 \pm \sqrt{3}}{2}$
14. 2; $\frac{1}{2}$, −1, −3
15. $a = 3$, $b = -9$, $x = -1, 2, 2$
16. $a = 10$, $b = 1$; 4