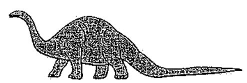
Probabilities

In maths, we assign a number to an event to describe how likely it is to occur.

All probabilities li e between _________ and ___________.
If an event has a probability of 0, it is _______________.
If an event has a probability of 1, it is _______________.

Choose a number that you think resembles the probability of the following events of occurring in your life.

- You will sit for the HSC _______
- There will be another world war _______
- You will get married _______
- You will be captain of the Australian cricket team _______
- A human being will be cloned _______
- You will turn into a dinosaur _______
- You will live in another country for a while _______
- John Howard will resign _______
- You will get a job sometime before you are 30 _______



A teacher is selecting a student from the class to go on an errand. For this situation, describe an event that has a probability of:

a) zero ____

____

b) one ____

____

Sample Space

In most situations there is more than one possible outcome. We call the set of possible outcome the ____________ ____________.

A simple example is tossing a coin where there are clearly 2 outcomes, either a head or a tail.

If H = getting a head

and T= getting a tail

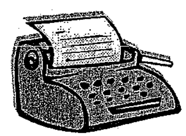
then the sample space for tossing a coin is {H, T}.

The curly brackets (called braces) are used when we are talking about a set of things.

The size of the sample space (*n*) is the total number of possible outcomes. This is the same as the number of elements in the sample space. For tossing a coin, this is 2.

Read the situations described below. For each one, list all the possible outcomes and hence find the size of the sample space.

1) A die is rolled.



2) An ape is given a simplified typewriter to type on. This typewriter only has the letters A, B and C on it. The ape types 3 letters.

3) Grace, Luke, Shani and George play a game of Monopoly. They decide that after 2 hours, they will stop and count their money and based on how much money they each have see who comes first second, third and fourth.

Relative Frequency

We can use data collected from an experiment to work out the ________________ _____________________ of each of the possible outcomes. If we rolled a die 20 times and recorded how many times each number was rolled, we could calculate the *relative frequencies* of each number. For example, if the number 3 was rolled 4 times, then the relative frequency of getting a 3 is $\frac{4}{20} = \frac{1}{5}$.

Language spot:

- each repeat of the experiment (eg. each time the die is rolled) is called a *trial*.
- in probability we use the word *event* to mean outcome. So rolling a 6 on a die is an *event* in probability.
- Relative frequency is sometimes called *experimental probability*

Relative frequency of event X = $\frac{\text{number of trials favourable to X}}{\text{total number trials}}$

Theoretical Probability

The theoretical probability of an event is the probability calculated without conducting an experiment. It is based on the number of ways a particular outcome could happen and the total number of outcomes possible.

Probability of event X = $\frac{\text{number of outcomes favourable to X}}{\text{total number of possible outcomes}}$

eg. If a die is to be rolled, the sample space is:

The theoretical probability of rolling an odd number is:

We write the probability of an event as P(E). eg. The probability of getting an odd number is P(odd).

Relative Frequency vs Theoretical Probability

Relative frequencies can be used to give a good indication of what the theoretical probability of an event is.

What is the theoretical probability of getting a head when a coin is tossed?

If I tossed a coin 2 times, in theory how many heads should I get?

Will I necessarily get this many heads?

Is the relative frequency always the same as the theoretical probability?

What should be done to ensure that the relative frequency is close to the theoretical probability?

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Coin Tossing

Use the coin provided for this experiment. You are going to toss the coin 20 times.

1. What is the theoretical probability of getting a tail? _______

2. How many tails would you expect to get from this experiment? _______

Toss the coin 20 times and use the table below to keep a tally of how many heads and tails you get.

	Heads	Tails
Tally		
Total		

3. How many tails did you get? _______ Is this the same as the expected number from question 1? _______

4. From your experiment, what is the relative frequency of getting a tail?

Does every group have the same answer for this question? Why or why not?

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What does this tell you about relative frequencies as opposed to theoretical probabilities? _______________________________________

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5. From your experiment, does it look like each of the two outcomes are equally likely? Why or why not? _______________________________

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Would you conclude that the coin is unfair based on your results? Why or why not? _______________________________________

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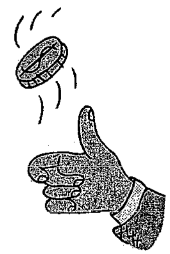

6. Why would tossing the coin 2 times not be enough to determine if the outcomes are equally likely?

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7. Why would tossing the coin 100 times be an even better test of whether the outcomes are equally likely than the experiment you have done? _______

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Spin The Spinner

Look at the spinner provided for this experiment.

1. Find

a) P(1)?

b) P(2)?

c) P(3)?

d) P(4)?

2. Add up your answers to parts (a) to (d) from Q1. What do you notice?

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3. Why must the total of probabilities for all outcomes equal 1? __________

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In your group, spin the spinner 40 times. Record your results in the table below.

	1	2	3	4
Tally				
Total				

4. Enter your results into the table on the board and calculate the relative frequencies of each of the colours first from your own experiment and secondly for the whole class results.

	1	2	3	4
Relative frequency from own experiment				
Relative frequency from class results				

5. How do the results compare with the theoretical probabilities from Q1?

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____

6. Do the class results give relative frequencies closer to the theoretical probabilities? Why might this be the case? __________

____

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Playing Cards

You are provided with an ordinary pack of playing cards for this experiment.

1. How many cards are in a pack?

2. How many red cards are there?

3. How many cards of each suit (clubs, hearts, diamonds, spades) are there?

4. If a card is drawn at random from the pack, what is the probability that it will be:

a) the 3 of spades?

b) a black card?

c) a club?

d) a picture card (Jack, Queen or King)?

e) a 7?

f) a red Ace?

5. Shuffle the pack and have someone choose a card at random. Design a table to record whether the chosen card is a number card (including the Ace) or a picture card. Take turns at repeating this experiment and continue until 40 trials have been done.

6. From your experiment, what is the experimental probability of choosing a picture card at random? ____________________

Is your answer close to the theoretical probability calculated in Q4? ________

____

What might account for the difference between your experimental probability and the theoretical probability? ____________________

____

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7. Comment on this statement: "When choosing a card from a pack, since you either get a number or a picture card, the probability of getting a number card is 50-50." ____________________

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8. Two friends each have a pack of 52 cards. They each draw a card at random from the pack. What is the probability that they have drawn:

a) the same suit?

b) the same number?

c) the same card?

Dropping Toothpicks

For this activity you will need to use the sheet marked with parallel lines. The distance between the lines is exactly the length of the toothpick. Drop the toothpick onto the sheet from a height of at least 1 metre. Record in a frequency distribution table the number of times the toothpick landed touching a line and the number of times it landed not touching a line.

1. From your results, calculate your experimental probability of the toothpick landing on a line.

2. What is $2 \div \pi$? ____________

Compare this with your answer to Q1. What do you suspect? ____________

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PIN Numbers

Peter has forgotten the PIN code for his mobile phone, but he remembers that it contains the digits 8, 7, 5, 2 in some order.

You have been provided with a bag containing 4 cards, each with one of the digits 8, 7, 5, 2 on it. You are to pull the digits out one at a time without looking to form a 4-digit PIN number.

1. List all the possible 4-digit PIN numbers that could be formed in this way (this is the sample space).

2. How many possible PIN numbers could be made using each of these digits once? ____________

3. How many of these are even? ____________

4. If Peter guesses his PIN, what is the probability of him getting it right on his first guess?

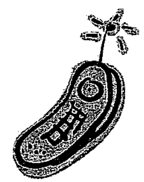
____

Mobile phones give you 3 chances to put in the correct PIN before the SIM card locks. What are Peter's chances of guessing correctly without his SIM card locking?

____

5. What is the probability that the number:

a) begins with 5?



b) is less than 4000?

c) is greater than 4000?

d) is at least 9000?

e) is divisible by 4? (A number is divisible by 4 if the last 2 digits are divisible by 4).

6. What do you notice about the sum of answers (b) and (c) from Q5? Why is this the case? ______________________________

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7. Complete 20 trials of the experiment and record your results in the table below.

TRIAL	RESULT	TRIAL	RESULT
1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

8. What is your experimental probability of getting a number divisible by 4?

Even or Odd

Children often use the game "scissors, paper, rock" to make a decision such as who goes first in a game. In "scissors, paper, rock", however, it is possible to draw (eg. You both do rock, nobody wins so you have to do it again). To avoid this problem, a simpler game can be played, "even or odd". This game works in a very similar way to scissors paper rock, except for a few differences:

- Instead of doing scossors, paper or rock, each person does either a closed fist or has their index finger out
- To decide who wins, the total number of fingers are counted. A closed fist counts as 0, while the index finger counts as 1.
- Before starting, it is agreed who will be even (ie. if the result is even you win) and who will be odd (ie. if the result is odd you win). [NOTE: a total of zero is considered even].

Try the game out to see how it works.

1. List the possible outcomes for Person 1 and Person 2 playing the game "even or odd".

2. Is the game fair? Does each person have an equal chance of winning?

____

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3. Play 10 trials of the game and keep a record of who wins each time.

Does each person win roughly half of the time? ______________

Octahedral Die

You are provided with an octahedral die for this experiment. What is the probability of rolling any particular number?

Roll the die 40 times and record your results in a table. Think what your column headings should be.

Collect the results from another group and record their results and the combined results in this table:

Score	Other Group	Total For Both Groups	Relative Frequency
1			
2			
3			
4			
5			
6			
7			
8			

Are your relative frequencies closer to the theoretical probabilities after combining results with another group? Why or why not? ____________________

____

Balls in a Bag

A bag contains 20 balls. When a ball is drawn at random we know that:

P(pink) = $\frac{1}{2}$ P(green) = $\frac{2}{5}$ P(white) = $\frac{1}{20}$ P(black) = $\frac{1}{20}$

Design a bag that fits this description, and write how many of each colour are in the bag.

Pink ____________________

Green ____________________

White ____________________

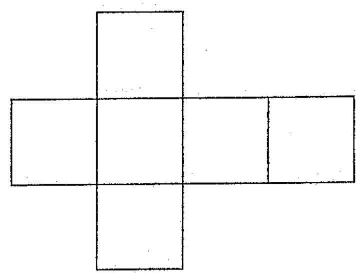
Black ____________________

Cube

You are provided with a net of a cube and some coloured pencils. Colour the faces of the cube so that when thrown, it has:

P(purple) = $\frac{1}{2}$ P(yellow) = $\frac{1}{3}$ P(orange) = $\frac{1}{6}$

Cut out and assemble the cube, and copy your colouring scheme onto the miniature net below.



Design a Spinner

Using the circle below, design a spinner with coloured sectors with the following probabilities:

P(blue) = $\frac{1}{3}$ P(red) = $\frac{1}{4}$ P(yellow) = $\frac{1}{4}$ P(green) = $\frac{1}{6}$