

TUTORIAL PROBLEMS (2) – WEEK 1

For the exercises below, do only every second question (the even numbered questions). The answers to all these questions are at the end of this handout – so if you are having difficulty getting correct answers for a particular exercise, go on and do all the questions in that exercise.

1. Basic Algebra

Algebraic Factors

1. Write down the expansion of:

- | | | |
|-------------------------|-------------------------|-------------------|
| (a) $(x+5)(x+1)$ | (b) $(x-2)(x-3)$ | (c) $(a-3)(a+4)$ |
| (d) $(p-7)(p-3)$ | (e) $(2x+3)(x+5)$ | (f) $(3x-4)(x-2)$ |
| (g) $(3y-2)(4y+3)$ | (h) $(x-14)(6x-1)$ | (i) $(3x+4)^2$ |
| (j) $(4p-5)^2$ | (k) $(3x^2-5x+2)(2x-4)$ | (l) $(2a+b)^3$ |
| (m) $(x^2+5)(x^2-2x-3)$ | (n) $(x-2)(x+2)(x+2)$ | (o) $(5m-2n)^3$ |

2. Factorise

- | | | |
|------------------|------------------------|---------------------|
| (a) $bx+by$ | (b) $3n+3m$ | (c) $6y+12$ |
| (d) $18a-6b$ | (e) y^2-4y | (f) a^2-ax |
| (g) $12xy+16yz$ | (h) $17a^2x^5+2a^4x^3$ | (i) $bx+by-bz$ |
| (j) $3x^3-x^2+x$ | (k) $am^2+2am+3m$ | (l) $3xy-3yz+21y$ |
| (m) $a(x+y)-az$ | (n) $10a^2+a(c+b)$ | (o) $a(x+y)+b(x+y)$ |

3. Factorise

- | | | |
|-------------------------|-------------------------|-----------------------|
| (a) $xy+xz+yw+zw$ | (b) $5a+5b+xa+xb$ | (c) $3a+12c+ab+4bc$ |
| (d) $2xy-2xz+7y-7z$ | (e) $a^2+ab+ac+bc$ | (f) $5a+ab+5b+b^2$ |
| (g) $3ax-bx-3ay+by$ | (h) $15ac-10ad+3bc-2bd$ | (i) ab^2+ac-b^2d-cd |
| (j) $a(x-y)+b(y-x)$ | (k) $2a-2b+ax-xb$ | (l) x^4+x^3+2x+2 |
| (m) $x^2(x+2y)-y(x+2y)$ | (n) $xz+z^2-x-z$ | (o) $2x^3-x^2-12x+6$ |

4. Factorise

- | | | |
|-----------------------|--------------------|----------------------|
| (a) x^2-y^2 | (b) a^2-36 | (c) x^2-48 |
| (d) $25-n^2$ | (e) x^2-1 | (f) $9-4a^2$ |
| (g) x^2-16y^2 | (h) $49a^2-121b^2$ | (i) $25a^2b^2-4c^2$ |
| (j) $a^2x^2-16b^2y^2$ | (k) $1-(a-b)^2$ | (l) $49x^2-64y^2z^2$ |
| (m) $100-x^4$ | (n) a^4-y^4 | (o) $(x+y)^2-x^2$ |

5. Factorise

- | | | |
|------------------|-------------------|------------------|
| (a) x^2+3x+2 | (b) $x^2+7x+12$ | (c) $x^2+13x+40$ |
| (d) $x^2-7x+12$ | (e) $x^2-13x+36$ | (f) $x^2-18x+56$ |
| (g) x^2-2x-3 | (h) $x^2-10x-11$ | (i) $x^2+15x+36$ |
| (j) $x^2-10x+25$ | (k) x^2+6x+9 | (l) x^2+2x+1 |
| (m) x^2+6x+5 | (n) $x^2-22x+120$ | (o) x^2-x-42 |

6. Factorise

(a) $3x^2 + 7x + 4$

(b) $4x^2 - 11x + 6$

(c) $9x^2 + 24x + 16$

(d) $4 - 3x - x^2$

(e) $12x^2 - 7xy + y^2$

(f) $2x^2 - 9x - 18$

(g) $18x^2 - 9x - 2$

(h) $3x^2 - 13x + 14$

(i) $2 + x - 10x^2$

7. Factorise

(a) $x^3 - y^3$

(b) $a^3 + 27$

(c) $x^3 - 64$

(d) $b^3 - 1$

(e) $x^3 + 125$

(f) $a^3 - 1000$

(g) $4a^3 + 32$

(h) $5m^3 - 5$

(i) $64 + y^3$

(j) $x^6 - y^6$

(k) $8x^3 + 27$

(l) $x^7y^4 - x^4y^7$

(m) $5y^3 - 5000$

(n) $a^3 + b^3 + a + b$

(o) $(x+5)^3 + (x-2)^3$

Algebraic Fractions

1. Simplify the following fractions.

(a) $\frac{2ab}{3ac}$

(b) $\frac{5a^2}{15ab}$

(c) $\frac{2x+4}{3x+6}$

(d) $\frac{a^2 - a}{a^2 + a}$

(e) $\frac{x^3 + 8}{x^2 - 2x + 4}$

(f) $\frac{x^2 + 2x + 1}{x^2 + 3x + 2}$

(g) $\frac{3x^2y - 6xy}{2x^2y - 4xy^2}$

(h) $\frac{a^2 - 5a}{a^2 - 4a - 5}$

(i) $\frac{3x^2 - 12}{10 - 5x}$

(j) $\frac{6x^2 - 150}{3x + 15}$

(k) $\frac{8a^2 - 8}{4a^2 + 8a + 4}$

(l) $\frac{a^3 - 27}{a^2 - 9}$

(m) $\frac{(a+b)^2 - c^2}{3a + 3b - 3c}$

(n) $\frac{1 - \frac{y^2}{x^2}}{1 + \frac{y^2}{x^2}}$

(o) $\frac{\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}}{\frac{9}{x} - x}$

(p) $\frac{2x^2 - x - 6}{\frac{4}{x^2} - 1}$

2. Simplify the following fractions.

(a) $\frac{5}{x} \times \frac{15}{y}$

(b) $\frac{3x}{5} \times \frac{1}{6x}$

(c) $\frac{2m}{3n} \times \frac{6}{m^2}$

(d) $\frac{18}{7x} \div \frac{9}{28x}$

(e) $\frac{3ab}{4a} \times \frac{8a^2}{12b}$

(f) $\frac{10x - 15}{6} \times \frac{1}{8x - 12}$

(g) $\frac{3m - 6}{4} \times \frac{8m}{m^2 - 2m}$

(h) $\frac{4a - 6}{5} \div \frac{6a - 9}{15}$

(i) $\frac{x - y}{4} \times \frac{8}{x^2 - y^2}$

(j) $\frac{m^2 - mn}{n^2 - n} \times \frac{n - 1}{m - n}$

(k) $\frac{1 - a}{b + b^2} \div \frac{1 - a^2}{1 - b^2}$

(l) $\frac{x^2 - 4}{2x - 4} \times \frac{2}{x + 2}$

(m) $\frac{a^2 + 5a + 6}{a^2 - 25} \div \frac{a + 3}{a - 5}$

(n) $\frac{x - 3y}{x^3y} \div \frac{3y - x}{xy^3}$

(o) $\frac{x^2 - y^2}{x^2 - 2xy + y^2} \times \frac{xy - y^2}{xy + y^2}$

3. Express each one of the following as single fraction.

(a) $\frac{x}{3} + \frac{x}{5}$

(b) $\frac{m}{2} + \frac{2m}{3}$

(c) $\frac{x+7}{3} + \frac{2x-1}{2}$

(d) $\frac{3x-1}{3} - \frac{x+2}{6}$

(e) $\frac{1}{x} + \frac{2}{x+y}$

(f) $\frac{2}{a+1} + \frac{1}{a-3}$

(g) $\frac{a}{b-c} - \frac{3a}{5b-5c}$

(h) $\frac{5c}{a^2+ab} - \frac{c}{a+b}$

$$(i) \frac{x^2}{x^2 + 3x + 2} - \frac{2x}{x + 2}$$

$$(k) \frac{x-1}{2} + \frac{x+3}{5} + \frac{x+7}{10}$$

$$(m) \frac{1}{x+2} + \frac{1}{x+3}$$

$$(o) \frac{1}{2x^2 - x - 1} - \frac{3}{6x^2 - x - 2}$$

$$(j) \frac{m}{m^2 + mn} + \frac{n}{n^2 + mn}$$

$$(l) \frac{2a^2 - b^2}{a^2} - \frac{b^2 - c^2}{b^2} - \frac{c^2 - a^2}{c^2}$$

$$(n) \frac{1}{x^2 - 9x + 20} + \frac{1}{x^2 - 11x + 30}$$

$$(p) \frac{x-3}{x+2} - \frac{x-2}{x+3} + \frac{1}{x-1}$$

Surds

1. Express each of the surds in the simplest form.

$$(a) \sqrt{8} \quad (b) \sqrt{12} \quad (c) \sqrt{32} \quad (d) 3\sqrt{63}$$

$$(e) \sqrt{72} \quad (f) \sqrt{96} \quad (g) \sqrt{288} \quad (h) \frac{1}{6}\sqrt{18}$$

$$(i) \frac{3}{2}\sqrt{192} \quad (j) \sqrt{320} \quad (k) \sqrt{\frac{10}{25}} \quad (l) \sqrt{\frac{72}{25}}$$

2. Simplify each of the following surds.

$$(a) 4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3} \quad (b) 3\sqrt{5} + 5\sqrt{5} - \sqrt{5} \quad (c) 3\sqrt{45} - \sqrt{20} + 7\sqrt{5}$$

$$(d) \sqrt{12} + \sqrt{27} \quad (e) \sqrt{18} + \sqrt{50} \quad (f) 4\sqrt{48} - 5\sqrt{27}$$

$$(g) \sqrt{128} + 3\sqrt{18} - \sqrt{162} \quad (h) 3\sqrt{32} + 2\sqrt{75} - 5\sqrt{162} \quad (i) 2\sqrt{363} - 5\sqrt{320} - \sqrt{192}$$

$$(j) -2\sqrt{a} - 4\sqrt{a} + 3\sqrt{a} \quad (k) 5\sqrt{y} - \sqrt{x} - 2\sqrt{y} \quad (l) 2\sqrt{x} + 2\sqrt{y} - 3\sqrt{x}$$

3. Express each of the surds in the simplest form.

$$(a) \sqrt{3} \times \sqrt{3} \quad (b) 4\sqrt{5} \times 2\sqrt{2} \quad (c) (\sqrt{5})^2$$

$$(d) (2\sqrt{7})^2 \quad (e) \sqrt{8} \times \sqrt{5} \times \sqrt{125} \quad (f) \sqrt{2}(\sqrt{3} + 2\sqrt{2})$$

$$(g) (2\sqrt{7} - 1)^2 \quad (h) 2\sqrt{5}(3\sqrt{3} - 4\sqrt{2}) \quad (i) \sqrt{2}(\sqrt{18} + \sqrt{8} - 4\sqrt{2})$$

$$(j) (\sqrt{3} + 4)(\sqrt{3} - 2) \quad (k) (2\sqrt{5} - 3)(2\sqrt{5} + 3) \quad (l) (3\sqrt{5} - 2\sqrt{2})(2\sqrt{5} + 3\sqrt{2})$$

$$(m) (2\sqrt{x} + \sqrt{y})(\sqrt{x} - 3\sqrt{y}) \quad (n) (3\sqrt{a} - \sqrt{b})^2 \quad (o) \sqrt{x}(6\sqrt{y} - 3\sqrt{x})$$

4. Multiply each surd by its conjugate and write down the answer.

$$(a) \sqrt{2} - 1 \quad (b) \sqrt{11} - \sqrt{5} \quad (c) 2\sqrt{5} + \sqrt{3} \quad (d) 2\sqrt{11} - 3\sqrt{5}$$

5. Express each of the surds in the simplest form with a rational denominator.

$$(a) \frac{1}{\sqrt{7}} \quad (b) \frac{3}{\sqrt{5}} \quad (c) \frac{4}{3\sqrt{7}} \quad (d) \frac{3\sqrt{2}}{\sqrt{8}}$$

$$(e) \frac{\sqrt{12}}{5\sqrt{3}} \quad (f) \frac{4\sqrt{2}}{3\sqrt{8}} \quad (g) \frac{\sqrt{2} + 3}{\sqrt{5}} \quad (h) \frac{1 - \sqrt{3}}{\sqrt{3}}$$

$$(i) \frac{\sqrt{5} - \sqrt{2}}{\sqrt{3}} \quad (j) \frac{4\sqrt{3} + 2\sqrt{2}}{3\sqrt{2}} \quad (k) \frac{\sqrt{x}}{\sqrt{y}} \quad (l) \frac{2\sqrt{a}}{\sqrt{ab}}$$

6. Simplify the following surds by rationalizing the denominator.

$$(a) \frac{1}{\sqrt{3} + 1} \quad (b) \frac{1}{\sqrt{3} + \sqrt{7}} \quad (c) \frac{1}{5 + 3\sqrt{2}} \quad (d) \frac{2}{2\sqrt{2} - 1}$$

$$(e) \frac{3}{\sqrt{5} - 2} \quad (f) \frac{\sqrt{3}}{4\sqrt{3} + 5} \quad (g) \frac{6}{7 + 2\sqrt{5}} \quad (h) \frac{\sqrt{5}}{\sqrt{5} - 1}$$

$$(i) \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}} \quad (j) \frac{2\sqrt{3} - 1}{2\sqrt{3} + 1} \quad (k) \frac{\sqrt{5} + \sqrt{3}}{2\sqrt{10} - \sqrt{6}} \quad (l) \frac{5\sqrt{2} - 1}{5\sqrt{2} + 1}$$

Answers to Exercises

1. Basic Algebra

Algebraic Factors

- | | | |
|------------------------------------|------------------------------|--|
| (a) $x^2 + 6x + 5$ | (b) $x^2 - 5x + 6$ | (c) $a^2 + a - 12$ |
| (d) $p^2 - 10p + 21$ | (e) $2x^2 + 13x + 15$ | (f) $3x^2 - 10x + 8$ |
| (g) $12y^2 + y - 6$ | (h) $6x^2 - 85x + 14$ | (i) $9x^2 + 24x + 16$ |
| (j) $16p^2 - 40p + 25$ | (k) $6x^3 - 22x^2 + 24x - 8$ | (l) $8a^3 + 12a^2b + 6ab^2 + b^3$ |
| (m) $x^4 - 2x^3 + 2x^2 - 10x - 15$ | (n) $x^3 + 2x^2 - 4x - 8$ | (o) $125m^3 - 150m^2n + 60mn^2 - 8n^3$ |
- | | | |
|-----------------------|----------------------------|----------------------|
| (a) $b(x + y)$ | (b) $3(n + m)$ | (c) $6(y + 2)$ |
| (d) $6(3a - b)$ | (e) $y(y - 4)$ | (f) $a(a - x)$ |
| (g) $4y(3x + 4z)$ | (h) $a^2x^3(17x^2 + 2a^2)$ | (i) $b(x + y - z)$ |
| (j) $x(3x^2 - x + 1)$ | (k) $m(am + 2a + 3)$ | (l) $3y(x - z + 7)$ |
| (m) $a(x + y - z)$ | (n) $a(10a + c + b)$ | (o) $(x + y)(a + b)$ |
- | | | |
|-------------------------|-------------------------|-------------------------|
| (a) $(y + z)(x + w)$ | (b) $(a + b)(5 + x)$ | (c) $(a + 4c)(3 + b)$ |
| (d) $(y - z)(2x + 7)$ | (e) $(a + b)(a + c)$ | (f) $(5 + b)(a + b)$ |
| (g) $(3a - b)(x - y)$ | (h) $(3c - 2d)(5a + b)$ | (i) $(b^2 + c)(a - d)$ |
| (j) $(x - y)(a - b)$ | (k) $(a - b)(2 + x)$ | (l) $(x + 1)(x^3 + 2)$ |
| (m) $(x + 2y)(x^2 - y)$ | (n) $(x + z)(z - 1)$ | (o) $(2x - 1)(x^2 - 6)$ |
- | | | |
|----------------------------|---------------------------------|--------------------------------------|
| (a) $(x + y)(x - y)$ | (b) $(a + 6)(a - 6)$ | (c) $(x + \sqrt{48})(x - \sqrt{48})$ |
| (d) $(5 + n)(5 - n)$ | (e) $(x + 1)(x - 1)$ | (f) $(3 + 2a)(3 - 2a)$ |
| (g) $(x + 4y)(x - 4y)$ | (h) $(7a + 11b)(7a - 11b)$ | (i) $(5ab + 2c)(5ab - 2c)$ |
| (j) $(ax + 4by)(ax - 4by)$ | (k) $(1 + a - b)(1 - a + b)$ | (l) $(7x + 8yz)(7x - 8yz)$ |
| (m) $(10 + x^2)(10 - x^2)$ | (n) $(a^2 + y^2)(a + y)(a - y)$ | (o) $y(2x + y)$ |
- | | | |
|----------------------|------------------------|-----------------------|
| (a) $(x + 1)(x + 2)$ | (b) $(x + 4)(x + 3)$ | (c) $(x + 8)(x + 5)$ |
| (d) $(x - 3)(x - 4)$ | (e) $(x - 4)(x - 9)$ | (f) $(x - 4)(x - 14)$ |
| (g) $(x + 1)(x - 3)$ | (h) $(x - 11)(x + 1)$ | (i) $(x + 12)(x + 3)$ |
| (j) $(x - 5)^2$ | (k) $(x + 3)^2$ | (l) $(x + 1)^2$ |
| (m) $(x + 5)(x + 1)$ | (n) $(x - 10)(x - 12)$ | (o) $(x + 6)(x - 7)$ |
- | | | |
|------------------------|------------------------|------------------------|
| (a) $(3x + 4)(x + 1)$ | (b) $(4x - 3)(x - 2)$ | (c) $(3x + 4)^2$ |
| (d) $(4 + x)(1 - x)$ | (e) $(3x - y)(4x - y)$ | (f) $(2x + 3)(x - 6)$ |
| (g) $(6x + 1)(3x - 2)$ | (h) $(3x - 7)(x - 2)$ | (i) $(2 + 5x)(1 - 2x)$ |
- | | | |
|-------------------------------|------------------------------|---------------------------------|
| (a) $(x - y)(x^2 + xy + y^2)$ | (b) $(a + 3)(a^2 - 3a + 9)$ | (c) $(x - 4)(x^2 + 4x + 16)$ |
| (d) $(b - 1)(b^2 + b + 1)$ | (e) $(x + 5)(x^2 - 5x + 25)$ | (f) $(a - 10)(a^2 + 10a + 100)$ |
| (g) $4(a + 2)(a^2 - 2a + 4)$ | (h) $5(m - 1)(m^2 + m + 1)$ | (i) $(4 + y)(16 - 4y + y^2)$ |

$$(j) (x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$$

$$(l) x^4y^4(x-y)(x^2+xy+y^2)$$

$$(n) (a+b)(a^2-ab+b^2+1)$$

$$(k) (2x+3)(4x^2-6x+9)$$

$$(m) 5(y-10)(y^2+10y+100)$$

$$(o) (2x-3)(x^2+3x+39)$$

Algebraic Fractions

$$1. (a) \frac{2b}{3c}$$

$$(b) \frac{a}{3b}$$

$$(c) \frac{2}{3}$$

$$(d) \frac{a-1}{a+1}$$

$$(e) x+2$$

$$(f) \frac{x+1}{x+2}$$

$$(g) \frac{3(x-2)}{2(x-2y)}$$

$$(h) \frac{a}{a+1}$$

$$(i) \frac{-3(x+2)}{5}$$

$$(j) 2(x-5)$$

$$(k) \frac{2(a-1)}{a+1}$$

$$(l) \frac{a^2+3a+9}{a+3}$$

$$(m) \frac{a+b+c}{3}$$

$$(n) \frac{x^2-y^2}{x^2+y^2}$$

$$(o) -\frac{(x+1)}{x^2(3+x)}$$

$$(p) -\frac{x^2(2x+3)}{2+x}$$

$$2. (a) \frac{75}{xy}$$

$$(b) \frac{1}{10}$$

$$(c) \frac{4}{mn}$$

$$(d) 8$$

$$(e) \frac{a^2}{2}$$

$$(f) \frac{5}{24}$$

$$(g) 6$$

$$(h) 2$$

$$(i) \frac{2}{x+y}$$

$$(j) \frac{m}{n}$$

$$(k) \frac{1-b}{b(1+a)}$$

$$(l) 1$$

$$(m) \frac{a+2}{a+5}$$

$$(n) -\frac{y^2}{x^2}$$

$$(o) 1$$

$$3 (a) \frac{8x}{15}$$

$$(b) \frac{7m}{6}$$

$$(c) \frac{8x+11}{6}$$

$$(d) \frac{5x-4}{6}$$

$$(e) \frac{3x+y}{x(x+y)}$$

$$(f) \frac{3a-5}{(a+1)(a-3)}$$

$$(g) \frac{2a}{5(b-c)}$$

$$(h) \frac{c(5-a)}{a(a+b)}$$

$$(i) \frac{-x}{x+1}$$

$$(j) \frac{2}{m+n}$$

$$(k) \frac{4(x+1)}{5}$$

$$(l) \frac{a^4b^2-b^4c^2+a^2c^4}{a^2b^2c^2}$$

$$(m) \frac{2x+5}{(x+2)(x+3)}$$

$$(n) \frac{2}{(x-4)(x-6)}$$

$$(o) \frac{1}{(x-1)(2x+1)(3x-2)}$$

$$(p) \frac{x^2+11}{(x-1)(x+2)(x+3)}$$

Surds - ANSWERS

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $4\sqrt{2}$ (d) $9\sqrt{7}$
 (e) $6\sqrt{2}$ (f) $4\sqrt{6}$ (g) $12\sqrt{2}$ (h) $\frac{1}{2}\sqrt{2}$
 (i) $12\sqrt{3}$ (j) $8\sqrt{5}$ (k) $\frac{\sqrt{10}}{5}$ (l) $\frac{6\sqrt{2}}{5}$

2. (a) $5\sqrt{3}$ (b) $7\sqrt{5}$ (c) $14\sqrt{5}$
 (d) $5\sqrt{3}$ (e) $8\sqrt{2}$ (f) $\sqrt{3}$
 (g) $8\sqrt{2}$ (h) $10\sqrt{3} - 33\sqrt{2}$ (i) $14\sqrt{3} - 40\sqrt{5}$
 (j) $-3\sqrt{a}$ (k) $3\sqrt{y} - \sqrt{x}$ (l) $2\sqrt{y} - \sqrt{x}$

3. (a) 3 (b) $8\sqrt{10}$ (c) 5
 (d) 28 (e) $50\sqrt{2}$ (f) $\sqrt{6} + 4$
 (g) $29 - 4\sqrt{7}$ (h) $6\sqrt{15} - 8\sqrt{10}$ (i) 2
 (j) $2\sqrt{3} - 5$ (k) 11 (l) $18 + 5\sqrt{10}$
 (m) $2x - 5\sqrt{xy} - 3y$ (n) $9a - 6\sqrt{ab} + b$ (o) $6\sqrt{xy} - 3x$

4. (a) 1 (b) 6 (c) 17 (d) -1

5. (a) $\frac{\sqrt{7}}{7}$ (b) $\frac{3\sqrt{5}}{5}$ (c) $\frac{4\sqrt{7}}{21}$ (d) $\frac{3}{2}$
 (e) $\frac{2}{5}$ (f) $\frac{2}{3}$ (g) $\frac{\sqrt{10} + 3\sqrt{5}}{5}$ (h) $\frac{\sqrt{3}}{3} - 1$
 (i) $\frac{\sqrt{15} - \sqrt{6}}{3}$ (j) $\frac{2\sqrt{6} + 2}{3}$ (k) $\frac{\sqrt{xy}}{y}$ (l) $\frac{2\sqrt{b}}{b}$

6. (a) $\frac{\sqrt{3} - 1}{2}$ (b) $\frac{\sqrt{7} - \sqrt{3}}{4}$ (c) $\frac{5 - 3\sqrt{2}}{7}$ (d) $\frac{4\sqrt{2} + 2}{7}$
 (e) $3(\sqrt{5} + 2)$ (f) $\frac{12 - 5\sqrt{3}}{23}$ (g) $\frac{42 - 12\sqrt{5}}{29}$ (h) $\frac{5 + \sqrt{5}}{4}$
 (i) $\frac{5 - \sqrt{21}}{2}$ (j) $\frac{13 - 4\sqrt{3}}{11}$ (k) $\frac{13\sqrt{2} + 3\sqrt{30}}{34}$ (l) $\frac{51 - 10\sqrt{2}}{49}$