Student's name: ___________ Student's number: _________ Teacher's name: ___________

Presbyterian Ladies' College, Sydney
Mathematics Department

HSC Assessment Task 1, 2008
Instruction, Notification and Reporting Sheet

Course:	Mathematics Extension 1
Topics:	Parametrics Further Curve Sketching Circle Geometry
Date:	Term 1, Week 4, 2008 (day to be advised)
Time allowed:	50 ~~40~~ minutes
Weighting:	20%

The outcomes being assessed are printed overleaf.

Instructions:

- Approved calculators may be used.
- Write your name and number on this question booklet.
- Write your student number on every page you hand in.
- All questions may be attempted.
- Start each Section on a new sheet of paper. The Sections will be collected separately.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- The question booklet will be collected with your answers.
- Your answers will be collected in 2 separately stapled bundles. **BRING A STAPLER.**

Marks awarded:

Section	Mark
1	/15
2	/15
Total	/30

Teacher's comment:

Student's comment:

This sheet will be re-printed as the front cover of the question booklet.

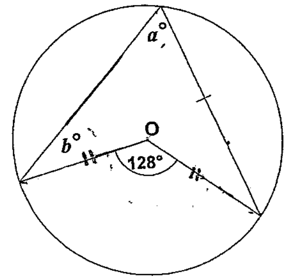
Year 12 2008 Extension 1 Mathematics Task 1:
Date printed: February 14, 2008

Marks

Section 1 (15 marks) **Start a new sheet of writing paper.**

(a) In each of the following, find the value of the pronumerals.
Reasons are not required.

(i)



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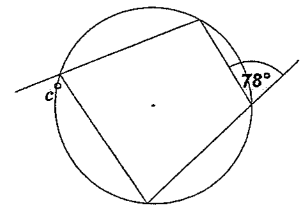

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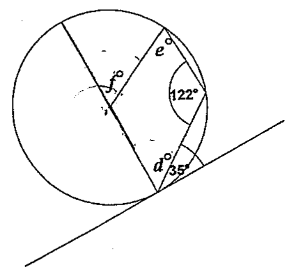

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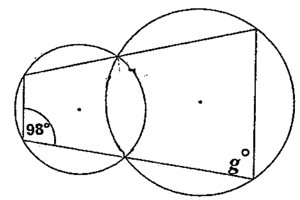

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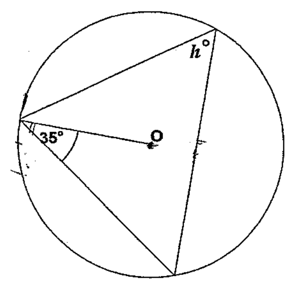

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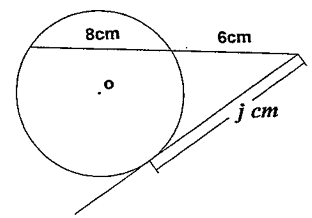

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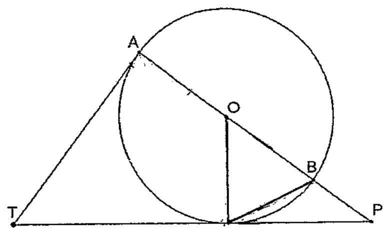
(b) **Please write your solution to this question on the removable page at the back of this examination. Attach the page to your answers.**

AB is a diameter of circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P. Join AC and CB.



(i) Why is $\angle TAO$ a right angle? **1**

(ii) Prove that $\angle CAT = 90^0 - \angle BCP$. **2**

(iii) hence, or otherwise, prove that $\angle ATC = 2\angle BCP$. **3**

End of Section 1

Section 2 (15 marks) Start a new sheet of writing paper.

Marks

(a) Find the cartesian equation of the curve whose parametric equations are:

$$x = t + 2$$
$$y = t^2 - 1$$

2

(b) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

(i) Show that the normal to the parabola at P has equation $x + py = ap^3 + 2ap$. **3**

(ii) The normal at P meets the parabola's axis of symmetry at N. If M is the midpoint of NP, show the locus of M is $x^2 = a(y - a)$. **3**

(c) $$f(x) = \frac{x^2}{x^2 - 4}$$

(i) Show that $f(x)$ is an even function. **1**

(ii) Describe the behaviour of the curve as x approaches infinity. **1**

(iii) Determine the equations of all vertical asymptotes. **1**

(iv) Find the coordinates of the stationary point(s) on the curve $y = f(x)$ and determine their nature. **2**

(v) Sketch the graph of the curve showing the above features. **2**

End of Section 2

tion 1

, $a = 64°$ $b = 32°$

) $c = 102$

i) $d = 55°$ $e = 67°$ $f = 64$

v) $g = 82°$

v) $h = 55°$

vi) $\sqrt{84} = 2\sqrt{21}$. or $9.16\ldots$

(i) tangent is perpendicular to radius drawn to point of contact.

i) $\angle BCP = \angle CAO$
ngle tangent makes with chord is qual to angle in opposite segment).

$\angle CAT = \angle TAO - \angle CAO$

$= 90 - \angle BCP$

i) $\angle ATC + \angle CAT + \angle TCA = 180°$
(anglesum of $\triangle TAC$ is equal to $180°$)

$\triangle TAC$ is isosceles
($TA = TC$, tangents equal drawn from external pt).

$\therefore$ $\angle CAT = \angle TCA$ (base angles of an isosceles equal.

ten

$\angle ATC + 2\angle CAT = 180$

$\angle ATC + 2(90 - \angle BCP) = 180$

$\angle ATC + 180 - 2\angle BCP = 180$

$\angle ATC = 2\angle BCP$.

Please see supplement attached

Section 2.

(a) $t = x - 2$.

$y = (x-2)^2 - 1$

$= x^2 - 4x + 4 - 1$

$y = x^2 - 4x + 3$

(b) (i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$ $m = p$

Eqn of Normal

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = ap^3 + 2ap$

(ii) Axis of symmetry $x = 0$

$py = ap^3 + 2ap$

$y = ap^2 + 2a$

$= a(p^2+2)$

$N(0, a(p^2+2))$

$M\left(ap, a(p^2+1)\right)$

$x = ap$ $y = a(p^2+1)$

$x^2 = a^2p^2$

$x^2 = a\,ap^2$

$x^2 = a(y-a)$.

) (i) $f(x) = \frac{x^2}{x^2-4}$

$f(-x) = \frac{(-x)^2}{(-x)^2-4}$

$= \frac{x^2}{x^2-4}$

$= f(x)$

$\therefore$ $f(x)$ is an EVEN fn.

(ii). $\lim_{x\to\infty} f(x) = 1$

(iii) $x = -2$ $x = 2$.

(iv) $U = x^2$ $V = x^2 - 4$

$U' = 2x$ $V' = 2x$

$f'(x) = \frac{(x^2-4)2x - 2x^3}{(x^2-4)^2}$

$= \frac{2x^3 - 8x - 2x^3}{(x^2-4)^2}$

$= \frac{-8x}{(x^2-4)^2}$

Stat pts occur when $f'(x) = 0$

ie $-8x = 0$

$x = 0$

$y = 0$

$\therefore$ Stat pt at $(0,0)$

$U = -8x$ $V = (x^2-4)^2$

$U' = -8$ $V' = 4x(x^2-4)$

$f''(x) = \frac{(x^2-4)^{\not 2}\cdot -8 + 32x^2\,\cancel{(x^2-4)}}{(x^2-4)^{\not 4\,3}}$

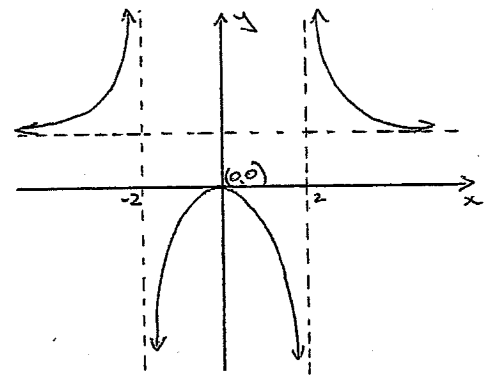
$= \frac{-8x^2 + 32 + 32x^2}{(x^2-4)^3}$

$= \frac{24x^2 + 32}{(x^2-4)^3}$

when $x = 0$ $f''(0) = \frac{32}{-4^3} < 0$ concave down

$\therefore$ $(0,0)$ is a MAX.

(v)



PLC
Year 12, Ext. 1, Task 1, 2008

Tip: Annotate the diagram!

Section 1

(a) (i) $a = 64$, $b = 32$

(ii) $c = 102$

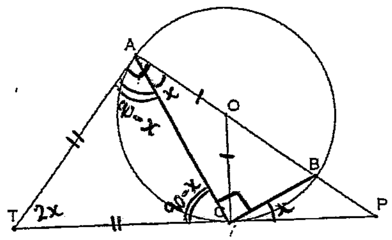
marks) (iii) $d = 55$, $e = 67$, $f = 64$

(iv) $g = 82$

(v) $h = 55$

(vi) $j = \sqrt{84}$ or $2\sqrt{21}$

(b)



(b) (i) TA is a tangent and OA is a radius

1 mark) The angle between a tangent and a radius (at the point of contact) is always 90°.

$\therefore \angle TAO$ is a right angle.

marks) (ii) Let $\angle BCP = x$

$\therefore \angle OAC = x$ (the angle between a tangent and a chord is equal to the angle in the alternate segment)

Tip: This will save time

Also, $\angle TAO = 90°$ (from part i)

$\therefore \angle CAT = 90 - x$ (angles in a right angle)

$\therefore \angle CAT = 90 - \angle BCP$, as required.

(3 marks) (iii) (Method 1)

$TA = TC$ (tangents drawn from an external point are equal in length)

$\therefore \angle TAC = \angle TCA$ (equal sides of ΔATC are opposite equal angles)

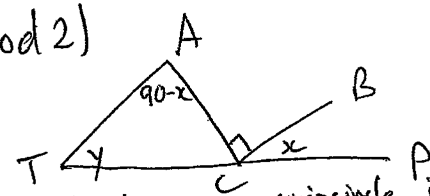
$\therefore \angle ATC + 90 - x + 90 - x = 180$ (angle sum of ΔATC equals 180°)

$\therefore \angle ATC = 2x$

$\therefore \angle ATC = 2 \times \angle BCP$, as required.

PTO for 2 different methods, produced by PLC students

(iii) (Method 2)



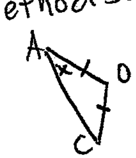
$\angle ACB = 90°$ (angle in a semi-circle is 90°)

$y + 90 - x = 90 + x$ (exterior angle of ΔATC is equal to the sum of the 2 opposite interior angles)

$\therefore y = 2x$

$\therefore \angle ATC = 2 \times \angle BCP$, as required

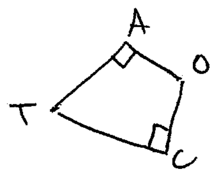
(iii) (Method 3)



$OA = OC$ (equal radii)

$\therefore \angle OCA = \angle OAC = x$

$\therefore \angle AOC = 180 - 2x$



$\angle TAO + \angle OCT = 180°$

$\therefore$ Quad AOTC is a cyclic quadrilateral

$\therefore \angle ATC + \angle AOC = 180$ (opposite angles in a cyclic quadrilateral are supplementary)

$\therefore \angle ATC + 180 - 2x = 180$

$\therefore \angle ATC = 2x$

$\therefore \angle ATC = 2 \times \angle BCP$, as required

Note to Year 12:

Many students got 6/6 for 1(b), but could have written a lot less than they did, had they begun with: "Let $\angle BCP = x$" and then expressed other angles in terms of x.