PRODUCT QUOTIENT AND CHAIN RULES

- If $y = f(x)g(x)$ then $\dfrac{dy}{dx} = f'(x)g(x) + g'(x)f(x)$
- If $y = \dfrac{f(x)}{g(x)}$ then $\dfrac{dy}{dx} = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$
- If $y = f(g(x))$ then $\dfrac{dy}{dx} = f'(g(x))g'(x)$
- If $f''(x) > 0$ for all x in an interval (a, b) then f is concave up on (a, b) .
- If $f''(x) < 0$ for all x in an interval (a, b) then f is concave down on (a, b) .

201. Find $\dfrac{dy}{dx}$ for each of the following functions:

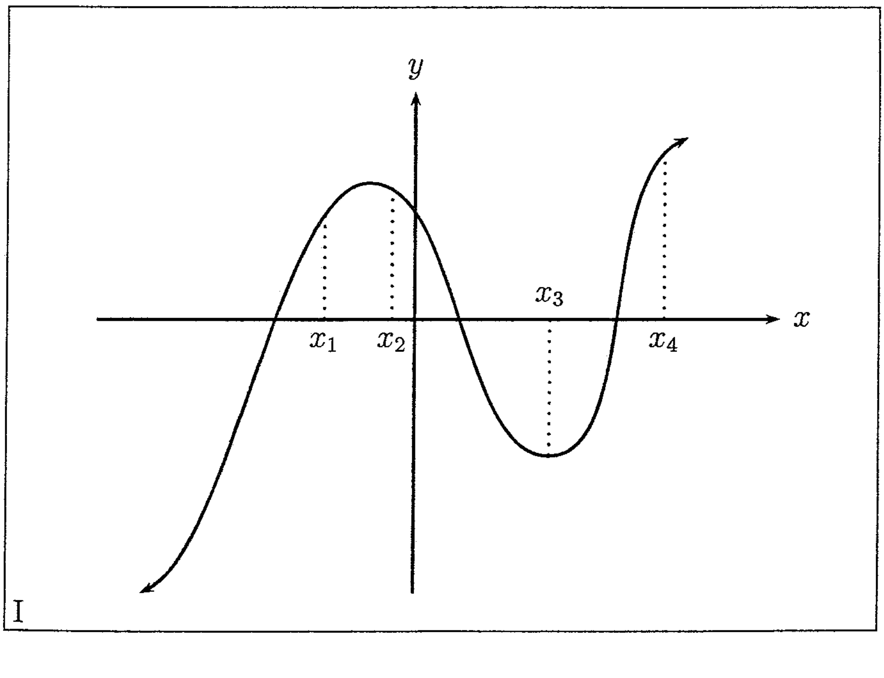
a) $y = \dfrac{x-1}{\sqrt{x}}$ b) $y = \dfrac{2x+1}{3x-2}$

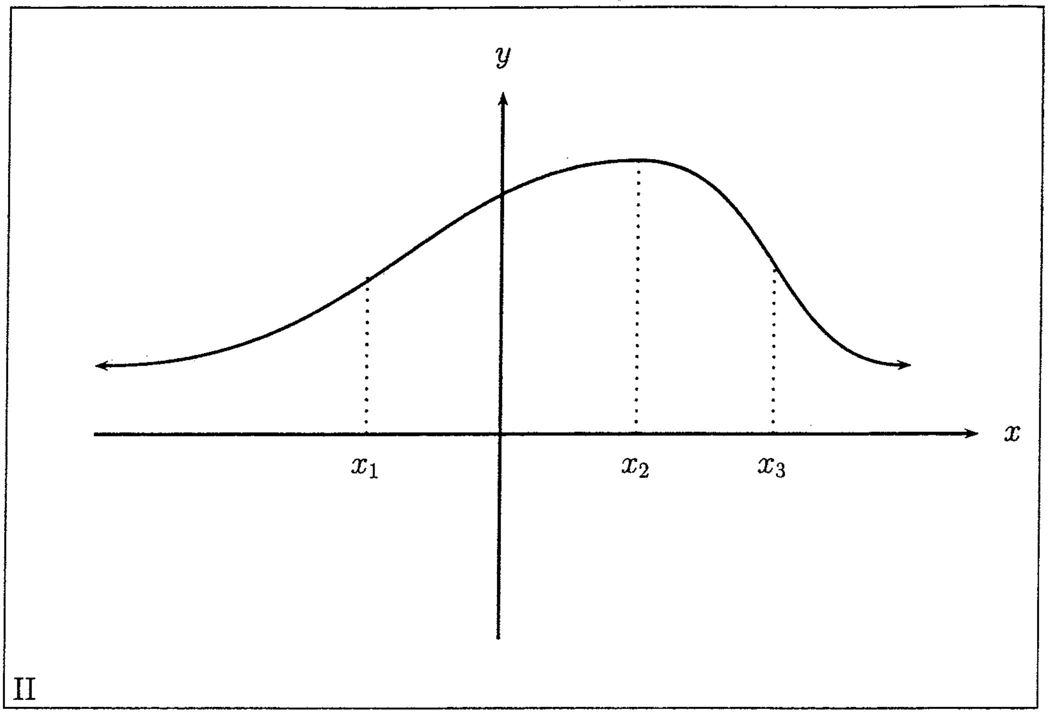
c) $y = (x^2+1)^5$ d) $y = \sqrt{3x-1}$

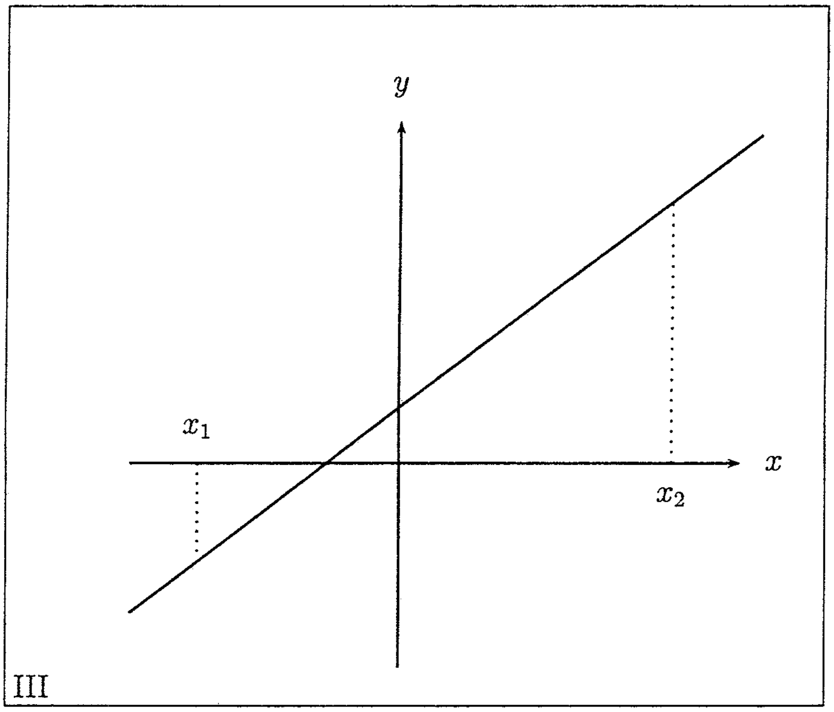
e) $y = x\sqrt{1-x}$ (f) $y = x^2(x^2-1)^{13}$

202. Evaluate $\dfrac{d^2y}{dx^2}$ for (a) and (c) above.

203. For the following graphs of $y = f(x)$ state the **sign** of y, $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at each of the indicated values of x :







204. A function $y = f(x)$ is defined for $0 \leq x \leq 3$ and has the property that $\dfrac{dy}{dx} > 0$ and $\dfrac{dy^2}{dx^2} < 0$ for all $0 < x < 3$. Draw a possible sketch of the graph of f .

205. (*) Two functions f and g are both defined over $[0,2]$ and have the following properties for all $x \in (0,2)$:

a) $f(x) < g(x)$ b) $f'(x) > g'(x)$

c) $f''(x) \geq 0$ d) $g''(x) \leq 0$

Draw a possible sketch of the graphs of f and g on the same set of axes.

SOLUTIONS

Product, quotient and chain rules

201. (a) $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ (b) $\frac{-7}{(3x-2)^2}$ (c) $10x(x^2+1)^4$

(d) $\frac{3}{2\sqrt{3x-1}}$ (e) $\frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$ (f) $26x^3(x^2-1)^{12} + 2x(x^2-1)^{13}$

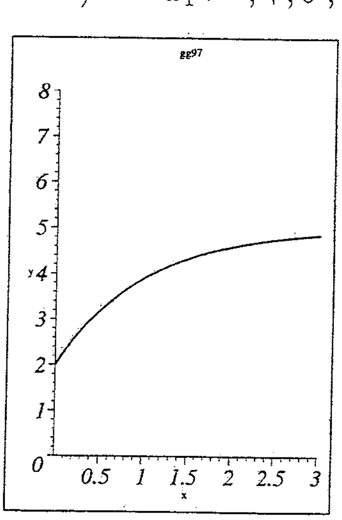
202. $-\frac{1}{4}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$, $80x^2(x^2+1)^3 + 10(x^2+1)^4$

203. For I) $x_1 : +, +, -$, $x_2 : +, -, -$, $x_3 : -, 0, +$, $x_4 : +, +, -$.

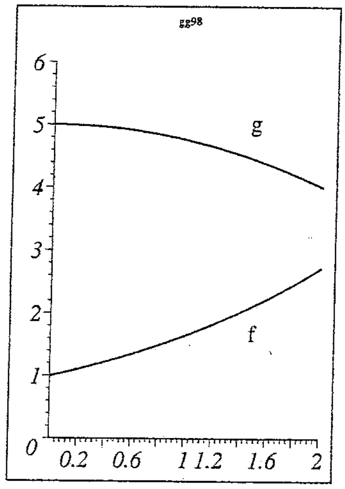
For II) $x_1 : +, +, 0$, $x_2 : +, 0, -$, $x_3 : +, -, 0$.

For III) $x_1 : -, +, 0$, $x_2 : +, +, 0$.

204.



205.



206. It is linear.

207. (a) 80 ° C (b) $\frac{dT}{dt} = \frac{-260}{(4t+1)^2}$ (c) $-0.24°$ /sec (d) After 3.78 seconds.

(e) 15° (f) 0

208. (a) $t = \frac{1}{2}$: $R = 5$, $L \approx 4.44$ $t = 1$: $R = 7.5$, $L = 10$.

(b) After 3/5 of a month. (c) $R \to 15$ m ^{3} , $L \to 40$ m ^{3} .

(d) 15/4 m ^{3} /month (e) After 3/13 months.

209. (a) $A = \frac{100t}{t+50}$ for Emmy, $A = \frac{100t}{t+20}$ for Amir.

(b) Proof (Hint: Consider $\lim_{t\to\infty} A$).

(c) Amir: 30 years, Emmy: 75 years.

(d) Amir: 5/9 % per year, Emmy: 50/81 % per year.

(e) 31.62 years.