ST ALOYSIUS' COLLEGE
HSC Assessment Task (Component B): 29th May 2012
Extension 1 Mathematics

Time allowed: 40 minutes (1 Period)

Maximum marks: 24

Booklet 1: Questions 1-5
Booklet 2: Question 6

Instructions:
- All necessary working is to be shown.
- Approved calculators may be used.
- Marks may be deducted for careless or poorly arranged work.
- A summary sheet may be used in the assessment.
- Maximum marks will be awarded for showing depth of understanding and clear explanation.

Question 1

Let α, β and γ be the roots of $x^3-3x+2=0$.

What is the value of $\alpha^2+\beta^2+\gamma^2$?

(A) -3 (B) 7 (C) 6 (D) 8

Question 2

A particle is projected from a horizontal plane at an angle of elevation of 30° with a speed of 100m/s. Assume the acceleration due to gravity is 10 m/s^{-2}. What is the equation of the trajectory?

(A) $y=-\frac{x}{\sqrt{3}}-\frac{x^2}{500}$ (B) $y=-\frac{x}{\sqrt{3}}-\frac{x^2}{1500}$

(C) $y=\frac{x}{\sqrt{3}}-\frac{x^2}{500}$ (D) $y=\frac{x}{\sqrt{3}}-\frac{x^2}{1500}$

Question 3

It is known that two of the roots of the equation $3x^3+x^2-kx+6$ are reciprocals of each other. What is the value of k?

(A) -2 (B) 6 (C) 7 (D) 17

Question 4

A particle is projected with a speed of 20 m/s and passes through a point P whose horizontal distance from the point of projection is 30 m and whose vertical height above the point of projection is $8\frac{3}{4}$ m. What is the angle of elevation θ? Take $g=10$ m/s^2.

(A) $\theta=\tan^{-1}(\frac{3}{4})$ (B) $\theta=\tan^{-1}(\frac{4}{3})$ (C) $\theta=\tan^{-1}(\frac{2}{3})$ (D) $\theta=\tan^{-1}(\frac{3}{2})$

Question 5

The cubic polynomial $6x^3-ax^2+2x-b$ has a remainder of ninety nine when divided by $(x-2)$, and is exactly divisible by $(x+1)$.

(i) Show that the values of a and b are -13 and 5 respectively. **4**

(ii) Show that $(2x-1)$ is also a factor of the polynomial and find the remaining factor. **4**

Question 6

A particle is projected from a point (0, 1) at an angle of 45° with a velocity of V metres per second. The equations of motion are

$$\ddot{x}=0 \quad \text{and} \quad \ddot{y}=-g$$

(i) Show that the expressions for the positions of the particle at time t is given by, **4**

$$x=\frac{Vt}{\sqrt{2}} \quad \text{and} \quad y=\frac{-gt^2}{2}+\frac{Vt}{\sqrt{2}}+1$$

(ii) Show that the path of the particle is given by $y=1+x-\frac{gx^2}{V^2}$ **2**

A volleyball player serves a ball with initial speed V metres per second at an angle of 45°. At that moment, the bottom of the ball is 1m above the ground and its horizontal distance from the net is 9.3 metres. The ball just clears the net, which is 2.3 metres high.

(iii) Show that the initial speed of the ball is approximately 10.3 metres per second (take $g=9.8ms^{-2}$) **2**

(iv) What is the horizontal distance from the net to the point where the ball lands? (answer to the nearest cm) **4**

End of Assessment

1) ~~$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$~~

C

2) D

3) C

4) ~~A~~ B

Q5 (i) Let $P(x) = 6x^3 - ax^2 + 2x - b$

$\therefore P(2) = 48 - 4a + 4 - b = 99$

$\therefore 4a + b = -47 \ldots (i)$

$P(-1) = -6 - a - 2 - b = 0$

$\therefore a + b = -8 \ldots (ii)$

$(i)-(ii) \quad 3a = -39 \qquad \therefore b = -8+13$

$\underline{a = -13} \qquad \underline{= 5}$

(ii) $P(x) = 6x^3 + 13x^2 + 2x - 5$

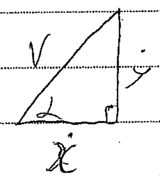
$P(\tfrac{1}{2}) = 6(\tfrac{1}{8}) + 13(\tfrac{1}{4}) + 2(\tfrac{1}{2}) - 5 = 0$

$\therefore$ $2x-1$ is also a factor of $P(x)$

$\therefore P(x) = (2x-1)(x+1)(3x+5)$ by inspection.

b)

(i) $\ddot{x} = 0$



$\dot{x} = V\cos\alpha$

$\int \ddot{x} = \int V\cos\alpha \, dt$

$x = Vt\cos\alpha + C$

when $t=0$ $x=0$ $\therefore C=0$

$\therefore x = Vt\cos\alpha$

now ~~...~~ $\alpha = 45°$

$\therefore x = Vt\cos 45°$

$x = \frac{Vt}{\sqrt{2}}$

$\ddot{y} = -g$

$\int \ddot{y} = \int -g \, dt$

$\therefore \dot{y} = -gt + C$

when $\dot{y} = V\sin\alpha$, $t=0$

$\therefore C = V\sin\alpha$

$\therefore \dot{y} = -gt + V\sin\alpha$

$\int \dot{y} = \int -gt + V\sin\alpha \, dt$

$y = \frac{-gt^2}{2} + Vt\sin\alpha + C$

when $t=0$, $y=1$ $\therefore C=1$

$\therefore y = \frac{-gt^2}{2} + Vt\sin\alpha + 1$

(4)

now. $\alpha = 45^{\circ}$

$\therefore y = \frac{-gt^2}{2} + vt\sin 45^{\circ} + 1$

$y = \frac{-gt^2}{2} + \frac{vt}{\sqrt{2}} + 1$

(ii) $x = \frac{vt}{\sqrt{2}}$

$\therefore \frac{x\sqrt{2}}{v} = t$ ①

$y = \frac{-gt^2}{2} + \frac{vt}{\sqrt{2}} + 1$ ②

① -> ②

$y = \frac{-g}{\cancel{2}}\left(\frac{\cancel{2}x^2}{v^2}\right) + \frac{\cancel{v}}{\cancel{\sqrt{2}}}\left(\frac{x\cancel{\sqrt{2}}}{\cancel{v}}\right) + 1$

$y = 1 + x - \frac{gx^2}{v^2}$

2

(iii) [illegible]

$\cancel{y = \frac{-gx^2}{2v^2}(1+\tan^2\alpha) + x\tan\alpha}$

$y = 1 + x - \frac{gx^2}{v^2}$

when $y = 2.3$, $x = 9.3$ and $g = 9.8$

$\therefore 2.3 = 1 + 9.3 - \frac{9.8(9.3^2)}{v^2}$

$-8v^2 = -847.602$

$v^2 = \frac{847.602}{8}$

$v = \sqrt{\frac{847.602}{8}}$ (taking positive as $v > 0$)

$= 10.3$ m/s [to nearest 1 d.p]

(iv) $y = 1 + x - \frac{gx^2}{v}$

Ball lands when $y = 0$ $\therefore 0 = 1 + x - \frac{9.8x^2}{v^2}$

$\frac{9.8x^2}{v^2} - x - 1 = 0$

(*) $x = \frac{1 \pm \sqrt{1 + 4\left(\frac{9.8}{v^2}\right)}}{2\left(\frac{9.8}{v^2}\right)}$ (taking positive as $x > 0$)

now, distance from net is (*) $- 9.3$

$\therefore$ distance from net $= 2.43$ m (to nearest m)

$= 243$ cm (to nearest cm)

Ask for another writing booklet if you need more space

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