QATs
Quality Assessment Tools

Centre Number

Student Number

2016 HIGHER SCHOOL CERTIFICATE
MID-YEAR EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 8-11
- Write your Centre Number and Student Number at the top of this page and the Multiple Choice Answer Sheet.

Total marks – 67

Section I Pages 2-5
7 marks
- Attempt Questions 1-7
- Allow 10 minutes for this section

Section II Pages 6-10
60 marks
- Attempt Questions 8-11
- Allow 1 hour and 50 minutes for this section

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MMID16_EXAM

Section 1 (7 marks)

(1) Which of the following is a simplification of the expression $\frac{a^3-8}{3a-6}$? 1

(A) $\frac{a^2-2a+4}{3}$

(B) $\frac{a^2+2a+4}{3}$

(C) $\frac{a+2}{a-2}$

(D) $\frac{a^2-4}{3}$

(2) If $\frac{7}{3\sqrt{2}-2}=p+q\sqrt{2}$, which of the following statements is true? 1

(A) $p=1$

(B) $q=\frac{21}{32}$

(C) $p=\frac{7}{8}$

(D) $q=-\frac{9}{14}$

(3) Which of the following is the derivative of $\frac{1}{2x-3}$? 1

(A) $\frac{1}{2}\ln(2x-3)$

(B) $\frac{-1}{2(2x-3)^2}$

(C) $\frac{1}{2}(2x-3)^{-2}$

(D) $\frac{-2}{(2x-3)^2}$

(4) Which of the following equations describes the locus of all points with vertex $=(-2,-2)$ and directrix $y=2$. 1

(A) $(x+2)^2=-16(y+2)$

(B) $(x-2)^2=-8(y-2)$

(C) $(y+2)^2=-8(x+2)$

(D) $(y-2)^2=4(x-2)$

(5) If $\sin x=\frac{5}{13}$ and $\cos x<0$, which of the following is a correct statement? 1

(A) $\cos x=\frac{12}{13}$

(B) $\operatorname{cosec} x=-\frac{13}{12}$

(C) $\tan x=-\frac{5}{12}$

(D) $\sec x=-\frac{5}{13}$

(6) Which of the following is the domain of the function $y=\sqrt{x^2-4}+\frac{1}{x-2}$? 1

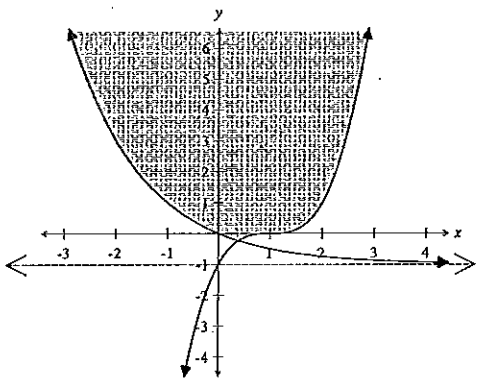
(A) Domain $=\{x:-2\le x<2\}$

(B) Domain $=\{x:x\le -2 \text{ or } x>2\}$

(C) Domain $=\{x: \textit{all real } x,\ x\ne 2\}$

(D) Domain $=\{x:x\ne \pm 2\}$

(7) 1



Which of the following pairs of inequations could describe the shaded region in the diagram above?

(A) $y\ge 2^{-x}-1$ and $y\le x^3-1$

(B) $y\le 1-2^x$ and $y\ge x^3-1$

(C) $y\le 2^x-2$ and $y\le (x+1)^3$

(D) $y\ge 2^{-x}-1$ and $y\ge (x-1)^3$

Section II

Question 8 (15 marks) Use a SEPARATE writing booklet.

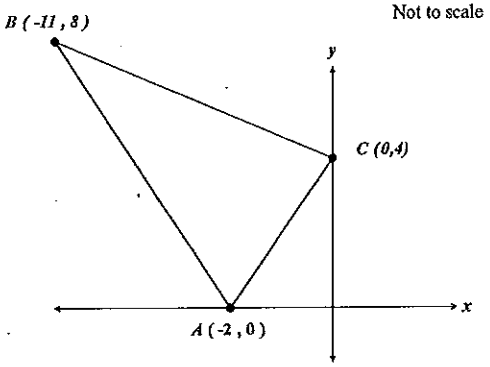
(a) Solve $\frac{a}{3}-\frac{a-2}{4}=2$. 2

(b) Evaluate $\lim_{a\to 2}\left(\frac{2a^2+a-10}{a^2-4}\right)$. 2

(c) The gradient of the curve $y=f(x)$ is given by $f'(x)=x^2+1$. The curve passes through $\left(1,\frac{2}{3}\right)$. Find the equation of the curve. 2

(d) Find the sum of the following series $3,9,27,-----,6561$. 2

(e)



Not to scale

In the diagram above, $A=(-2,0)$, $B=(-11,8)$ and $C=(0,4)$.

(i) Find the gradient of AC. 1

(ii) Hence find the angle of inclination of AC *(answer to nearest minute)*. 1

(iii) Find the equation of line AC. 2

(iv) Find point M, the midpoint of AC and hence show that MB is perpendicular to AC. 2

(v) Hence deduce what type of triangle is ABC, providing clear reasons. 1

Question 9 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\sum_{r=0}^{3}(r+1)^r$. 1

(b) Ursula started a new job at a starting salary of \$42,000. Each subsequent year her annual salary increased by \$1600.

(i) Find Ursula's salary in the 15th year. 1

(ii) Find the total amount Ursula will earn in the first 15 years. 2

(iii) If Ursula saves 25% each year of her annual salary, how many years does she need to work to save \$327600. 2

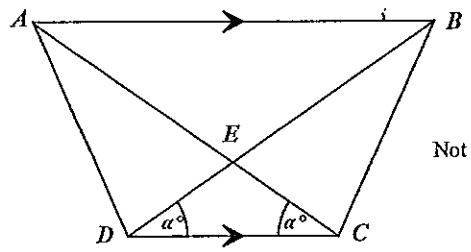
(c) If $y=\frac{2x-1}{2x+1}$

(i) Show that $\frac{dy}{dx}=\frac{4}{(2x+1)^2}$. 1

(ii) Find the equation of the normal to this curve at the point $P\left(\frac{1}{2},0\right)$. 2

(iii) Find the co-ordinates of Q, the other point on this curve where the tangent is parallel to the tangent at P. 2

(d) In the diagram below, AB is parallel to CD and $\angle BDC=\angle ACD=\alpha^o$.



Not to scale

(i) Show that $AE=EB$. 1

(ii) Prove that $\Delta ACD\cong\Delta BCD$. 2

(iii) Hence prove that $\angle ADE=\angle BCE$. 1

Question 10 (15 marks) Use a SEPARATE writing booklet.

(a) During an orienteering race, Abdul and Ben set off from the same starting point O. Abdul (A) ran at a bearing of $60^\circ T$, while Ben (B) ran at a bearing of $100^\circ T$. After 3 hours the distance OA was equal to the distance AB.

(i) Draw a clear diagram representing this information and explain why $\angle AOB = 40^\circ$. 1

(ii) If $OB = 14\,km$, find the length OA (*to the nearest km*). 2

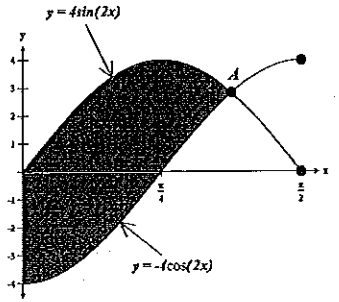
(iii) Find the bearing of A from B. 1

(b) Differentiate the following with respect to x.

(i) $y = 2x\tan\left(\frac{x}{2}\right)$. 2

(ii) $y = \cos^3(3x-2)$. 2

(c) In the diagram below, the curves $y = 4\sin 2x$ and $y = -4\cos 2x$, $0 \le x \le \frac{\pi}{2}$ meet at A.



(i) By equating the two equations $y = 4\sin 2x$ and $y = -4\cos 2x$; and solving simultaneously, show that the *x-coordinate* of A is $x = \frac{3\pi}{8}$. 2

(ii) Hence, find the exact area of the shaded region. 2

(d) A curve is defined as $y = 8\sin \pi x - 4\pi x - 3$, $0 < x \le 2$

(i) Find the *x-values* of all stationary points in the above domain. (***do not determine their nature***) 2

(ii) Find the *x-value* of the point where the curve is increasing at the greatest rate. 1

Question 11 (15 marks) Use a SEPARATE writing booklet.

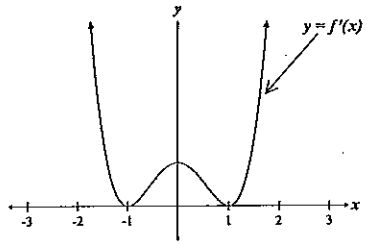
(a) Five values of the function $y = f(x)$ are shown in the table below. 2

x	0	2	4	8	16
$f(x)$	1	4	8	14	25

Using the table (5 ordinates) and Simpson's rule, evaluate $\int_0^{16} f(x)\,dx$.

(b) For what values of k does the quadratic $kx^2 + (2k-3)x + 2 = 0$ have real roots. 3

(c)

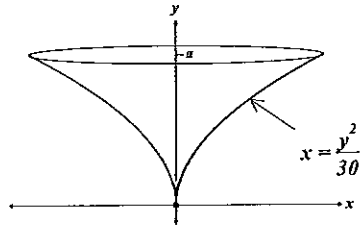


The above diagram is a sketch of the **gradient** function of the curve $y = f(x)$.

(i) Given that $f(0) = 0$, draw a sketch of the function $y = f(x)$. 2

(ii) Hence or otherwise, evaluate $\int_{-a}^{a} f(x)\,dx$, where a is any real value. 1

(d) 3

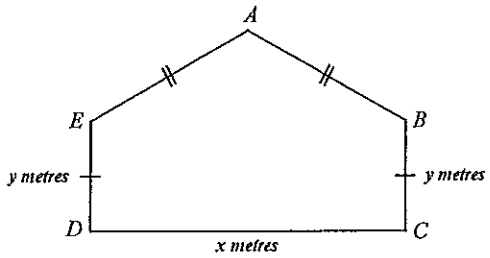


The glass ornament above has a shape obtained by rotating part of the parabola $x = \frac{y^2}{30}$, $0 \le y \le a$; about the *y-axis*. The glass has a height of a *cm*. If the volume of the glass is $\frac{200\pi}{9}\,cm^3$, find the value of a.

Question 11 continueS over the page

(e) The figure $ABCDE$ below represents a paddock of perimeter 48 metres. The sloping sides AB and AE are equal in length with each being 30% longer than the horizontal side CD, where $CD = x$ *metres*. The vertical sides $ED = BC = y$ *metres*.

Not to scale



(i) Prove that $y = \left(24 - \frac{9}{5}x\right)$ *metres*. **1**

(ii) Hence find the maximum possible area of the paddock. **3**

END OF PAPER

QATs
Quality Assessment Tasks

2016 HIGHER SCHOOL CERTIFICATE MID-YEAR EXAMINATION

MATHEMATICS MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	A
3	D
4	A
5	C
6	B
7	D

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Section II

Question 8 (a)

Criteria	Marks
• Correct solution	2
• Achieve $4a-3(a-2)=24$	1

Sample answer

$$\frac{a}{3}-\frac{a-2}{4}=2$$
$$4a-3(a-2)=24$$
$$4a-3a+6=24$$
$$a=18$$

Question 8 (b)

Criteria	Marks
• Correct solution.	2
• Achieves $\lim_{a\to 2}\frac{(2a+5)(a-2)}{(a-2)(a+2)}$	1

Sample answer

$$\lim_{a\to 2}\left(\frac{2a^2+a-10}{a^2-4}\right)=\lim_{a\to 2}\frac{(2a+5)(a-2)}{(a-2)(a+2)}$$
$$=\lim_{a\to 2}\frac{(2a+5)}{(a+2)}$$
$$=\frac{9}{4}$$

Question 8 (c)

Criteria	Marks
• Correct solution.	2
• Achieves $f(x)=\frac{x^3}{3}+x+c$	1

Sample answer

$$f'(x)=x^2+1$$
$$f(x)=\frac{x^3}{3}+x+c$$
$$\frac{2}{3}=\frac{1}{3}+1+c$$
$$c=-\frac{2}{3}$$
$$f(x)=\frac{x^3}{3}+x-\frac{2}{3}$$

Question 8 (d)

Criteria	Marks
• Correct solution	2
• Achieves $n=8$	1

Sample answer

$$T_n=ar^{n-1}$$
$$6561=3(3)^{n-1}$$
$$2187=(3)^{n-1}$$
$$3^7=(3)^{n-1}$$
$$n=8$$
$$S_n=\frac{a(r^n-1)}{r-1}$$
$$S_8=\frac{3(3^8-1)}{3-1}$$
$$=9840$$

Question 8 (e) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$$m_{AC} = \frac{4-0}{0+2} = 2$$

Question 8 (e) (ii)

Criteria	Mark
• Correct answer	1

Sample answer

$\tan\theta = 2$

$\theta = 63°26'$

Question 8 (e) (iii)

Criteria	Marks
• Correct solution	2
• Uses correct formulae and either point A or C	1

Sample answer

$m = 2 \quad C = (0,4)$

$\therefore y = 2x + 4$

$2x - y + 4 = 0$

Question 8 (e) (iv)

Criteria	Marks
• Correct working and solution	2
• Achieves $M_{AC} = (-1,2)$	1

Sample answer

$M_{AC} = (-1,2)$

$$m_{MB} = \frac{8-2}{-11-1} = -\frac{1}{2}$$

For perpindicular

$m_{MB} \times m_{AC} = -1$

$$2 \times -\frac{1}{2} = -1$$

$-1 = -1$

Question 8 (e) (v)

Criteria	Mark
• Correct answer and explanation	1

Sample answer

ABC is an isosceles triangle *AB=BC (perpendicular bisector of AC passes through B.)*

Question 9 (a)

Criteria	Mark
• Correct answer	1

Sample answer

$$\sum_{r=0}^{3}(r+1)^r = 1+2+9+64 = 76$$

Question 9 (b) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$a = \$42,000 \quad d = \$1,600$

$T_n = a + (n-1)d$

$T_{15} = 42,000 + (14 \times 1600)$

$= \$64,400$

Question 9 (b) (ii)

Criteria	Marks
• Correct solution	2
• Uses the answer obtained in (i) into correct formulae or uses other formulae correctly	1

Sample answer

$$S_n = \frac{n}{2}\{a+l\}$$

$$S_{15} = \frac{15}{2}\{42000 + 64400\}$$

$= \$798000$

Question 9 (b) (iii)

Criteria	Marks
• Correct solution	2
• Achieves $2n^2+105n-3278=0$ or similar	1

Sample answer

$a=10500 \quad d=400$

$$S_n=\frac{n}{2}\{2a+(n-1)d\}$$

$$327600=\frac{n}{2}\{21000+(n-1)400\}$$

$$327600=10500n+200n^2-200n$$

$$2n^2+103n-3276=0$$

$$n=\frac{-103\pm\sqrt{10609+26208}}{4}$$

$$=\frac{-103\pm\sqrt{36817}}{4}$$

$$=\frac{-103\pm191.9}{4}$$

$$=22.225,-73.7$$

$$=23 \quad (n>0)$$

It will take 23 years.

Question 9 (c) (i)

Criteria	Mark
• Correct working	1

Sample answer

$$y=\frac{2x-1}{2x+1}$$

$$y'=\frac{(2x+1)\times2-(2x-1)\times2}{(2x+1)^2}$$

$$=\frac{4x+2-4x+2}{(2x+1)^2}$$

$$=\frac{4}{(2x+1)^2}$$

Question 9 (c) (ii)

Criteria	Marks
• Correct solution	2
• Uses m=-1	1

Sample answer

$$f'\left(\frac{1}{2}\right)=\frac{4}{4}=1$$

For normal $m=-1 \quad (m_1\times m_2=-1) \quad pt(\frac{1}{2},0)$

$$y-0=-1\left(x-\frac{1}{2}\right)$$

$$y=-x+\frac{1}{2}$$

$$2y=-2x+1$$

$$2x+2y-1=0$$

Question 9 (c) (iii)

Criteria	Marks
• Correct solution	2
• Achieves $4=(2x+1)^2$	1

Sample answer

$$\frac{4}{(2x+1)^2}=1$$

$$4=(2x+1)^2$$

$$2x+1=\pm2$$

$$x=\frac{1}{2},-\frac{3}{2}$$

$$Answer=\left(-\frac{3}{2},2\right)$$

Question 9 (d) (i)

Criteria	Mark
• Correct proof	1

Sample answer

$\angle EAB=\angle ACD=\alpha$ *(alternate angles in prallel lines AB parallel to CD)*

$\angle ABE=\angle BDC=\alpha$ *(alternate angles in prallel lines AB parallel to CD)*

Since $\angle EAB=\angle ABC$

$\therefore AE=EB$ *(equal sides of an isosceles triangle ABE)*

Question 9 (d) (ii)

Criteria	Marks
• Correct proof	2
• Uses correct test and working with one mistake or no reasoning	1

Sample answer

Prove $\Delta ACD \cong \Delta BCD$

DC *is common*

$\angle ACD = \angle BDC$ *(given)*

$AC = BD$ (since $AE = EB$ *(proven) and* $ED = EC$ *equal sides of an isosceles triangle ECD)*

$\therefore \Delta ACD \cong \Delta BCD$ *(S.A.S)*

Question 9 (d) (iii)

Criteria	Mark
• Correct proof	1

Sample answer

$\angle ADC = \angle BCD$ *(corresponding angles in congruent triangles are equal)*

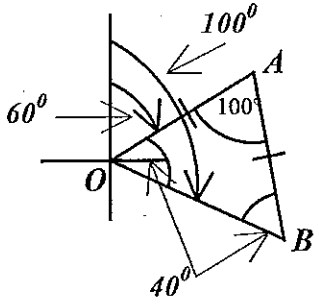
But $\angle BDC = \angle ACD$ *(given)*

$\therefore \angle ADE = \angle BCE$

Question 10 (a) (i)

Criteria	Marks
• Correct diagram and proof	2

Sample answer



From diagram

$\angle AOB = 100 - 60$

$= 40°$

Question 10 (a) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $\frac{OA}{\sin 40°} = \frac{14}{\sin 100°}$	1

Sample answer

$$\frac{OA}{\sin 40} = \frac{14}{\sin 100}$$

$$OA = \frac{14 \sin 40}{\sin 100}$$

$$= 9.13785\,km$$

$$= 9\,km \text{ (nearest } km)$$

Question 10 (a) (iii)

Criteria	Mark
• Correct answer	1

Sample answer

Bearing of A from $B = 270 + 10 + 40 = 320°T$

Question 10 (b) (i)

Criteria	Marks
• Correct answer	2
• Use the product rule with one error	1

Sample answer

$$y = 2x \tan\left(\frac{x}{2}\right)$$

$$y' = \tan\left(\frac{x}{2}\right) \times 2 + 2x \times \sec^2\left(\frac{x}{2}\right) \times \frac{1}{2}$$

$$= 2\tan\left(\frac{x}{2}\right) + x\sec^2\left(\frac{x}{2}\right)$$

Question 10 (b) (ii)

Criteria	Marks
• Correct answer	2
• Uses the chain rule with one error	1

Sample answer

$y=\cos^3(3x-2)$

$y'=3\cos^2(3x-2)\times-\sin(3x-2)\times 3$

$=-9\cos^2(3x-2)\sin(3x-2)$

Question 10 (c) (i)

Criteria	Marks
• Correct solution	2
• Achieves $\tan 2x=-1$	1

Sample answer

$4\sin 2x=-4\cos 2x$

$\tan 2x=-1$

$2x=\frac{3\pi}{4},\frac{7\pi}{4}$

$x=\frac{3\pi}{8},\frac{7\pi}{8}$

$x=\frac{3\pi}{8} \quad (0\le x\le\frac{\pi}{2})$

Question 10 (c) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $\left[-2\cos 2x+2\sin 2x\right]_0^{\frac{3\pi}{8}}$	1

Sample answer

$A=\int_0^{\frac{3\pi}{8}} 4\sin 2x+4\cos 2x \ dx$

$=\left[-2\cos 2x+2\sin 2x\right]_0^{\frac{3\pi}{8}}$

$=-2\left[\cos 2x-\sin 2x\right]_0^{\frac{3\pi}{8}}$

$=-2\left[-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-(1-0)\right]$

$=-2\left[-\frac{2}{\sqrt{2}}-1\right]$

$=\left(\frac{4}{\sqrt{2}}+2\right)units^2$

Question 10 (d) (i)

Criteria	Marks
• Correct solution	2
• Achieves $8\pi\cos\pi x-4\pi=0$	1

Sample answer

$y=8\sin\pi x-4\pi x-3$

$y'=8\pi\cos\pi x-4\pi$

Stationary points occur when $y'=0$

$8\pi\cos\pi x-4\pi=0$

$\cos\pi x=\frac{1}{2}$

$\pi x=\frac{\pi}{3},\frac{5\pi}{3}$

$x=\frac{1}{3},\frac{5}{3}$

Question 10 (d) (ii)

Criteria	Mark
• Correct solution	1

Sample answer

Point of inflexion occurs when $y'' = 0$ and concavity changes.

$$y'' = -8\pi^2 \sin \pi x$$

$$\therefore -8\pi^2 \sin \pi x = 0$$

$$\sin \pi x = 0$$

$$\pi x = \pi, 2\pi \quad 0 < x \leq 2$$

$$x = 1, 2$$

x	1.5	2	2.5
$f'(x)$	+	0	-

Answer: $x=2$

(Note: $f'(1) = -12$ hence at $x=1$, the curve is decreasing)

Question 11 (a)

Criteria	Marks
• Correct solution	2
• Uses correct weighting and Simpson's rule with a maximum of two mistakes	1

Sample answer

x	0	2	4	8	16
$f(x)$	1	4	8	14	25
Weight	1	4	2	4	1
Result	1	16	16	56	25

$$\int_0^{16} f(x)\,dx = \frac{2}{3}(114)$$

$$= 76 \text{ units}^2$$

Question 11 (b)

Criteria	Marks
• Correct solution	3
• Achieves $\Delta = 4k^2 - 20k + 9$	2
• Notes that $\Delta \geq 0$	1

Sample answer

Real roots occur when $\Delta \geq 0$

$$\Delta = (2k-3)^2 - 8k$$

$$= 4k^2 - 12k + 9 - 8k$$

$$= 4k^2 - 20k + 9$$

$$4k^2 - 20k + 9 \geq 0$$

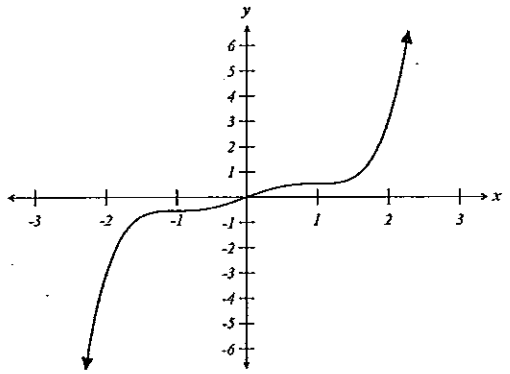
$$(2k-1)(2k-9) \geq 0$$

$$\therefore k \leq \frac{1}{2} \text{ or } k \geq \frac{9}{2}$$

Question 11 (c) (i)

Criteria	Marks
• Correct solution	2
• Makes a positive attempt towards answer	1

Sample answer



Question 11 (c) (ii)

Criteria	Mark
• Correct answer	1

Sample answer

$$\int_{-a}^{a} f(x)\,dx = 0 \text{ from diagram- odd function}$$

Question 11 (d)

Criteria	Marks
• Correct solution	3
• Achieves $\frac{200\pi}{9}=\frac{\pi}{5\times 900}\left[y^5\right]_0^a$	2
• Obtains $x^2=\frac{y^4}{900}$	1

Sample answer

$$\frac{200\pi}{9}=\pi\int_0^a\frac{y^4}{900}\,dy$$

$$\frac{200\pi}{9}=\frac{\pi}{5\times 900}\left[y^5\right]_0^a$$

$$100000=\left[a^5-0\right]$$

$$a=10$$

Question 11 (e) (i)

Criteria	Mark
• Correct proof	1

Sample answer

$$48=x+2y+2\left(\frac{13}{10}x\right)$$

$$48=x+2y+\frac{13}{5}x$$

$$2y=48-\frac{18}{5}x$$

$$y=24-\frac{9}{5}x$$

Question 11 (e) (ii)

Criteria	Marks
• Correct solutions	3
• Achieves $A=24x-\frac{6}{5}x^2$	2
• Achieves $h=\frac{6}{5}x$	1

Sample answer

Let the height of the triangle AEB = h

$$h=\sqrt{\frac{169x^2}{100}-\frac{x^2}{4}}$$

$$=\sqrt{\frac{36x^2}{25}}$$

$$h=\frac{6}{5}x$$

$$A=xy+\frac{1}{2}\times x\times\frac{6}{5}x$$

$$=xy+\frac{3}{5}x^2$$

$$=x\left(24-\frac{9}{5}x\right)+\frac{3}{5}x^2$$

$$=24x-\frac{9}{5}x^2+\frac{3}{5}x^2$$

$$A=24x-\frac{6}{5}x^2$$

$$A'=24-\frac{12}{5}x$$

$$A'=0$$

$$\therefore 24-\frac{12}{5}x=0$$

$$24=\frac{12}{5}x$$

$$x=10$$

$$A''(x)=-\frac{12}{5}$$

$$\therefore A''(10)=-\frac{12}{5}$$

$\therefore$ maximum

$$A=240-\frac{6}{5}(100)$$

$$=120\ m^2$$