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Centre Number

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Student Number

2016 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION
MATHEMATICS EXTENSION 1

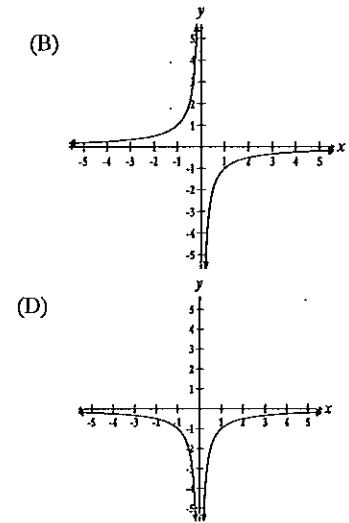
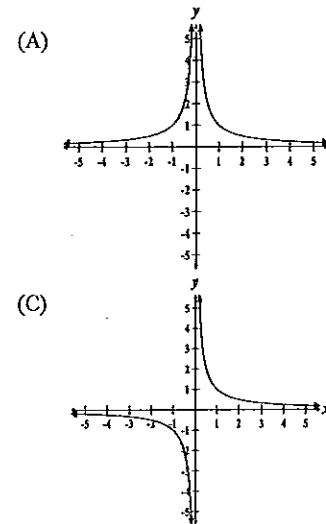
Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1-10

- 1 When the polynomial $P(x) = x^3 - 5x^2 + 2x + k$ is divided by $(x-1)$ the remainder is 4. Which of the following is the value of k ?
- (A) 2
(B) -2
(C) 12
(D) 6

- 2 Given that $f(x) = \frac{1}{x}$. Which of the following represents the graph of $y = f^{-1}(x)$.



2016 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Write your student number on this page and on the multiple-choice answer sheet
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Answer each question in a SEPARATE writing booklet. Extra writing booklets are available

Total marks – 70

Section I Pages 2-5

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 6-9

60 marks

- Attempt Question 11-14
- Allow about 1 hour and 45 minutes for this section

Disclaimer

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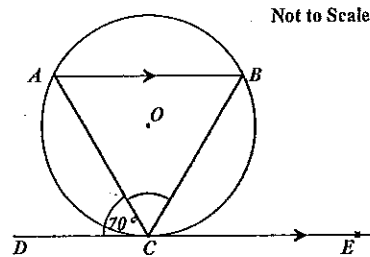
3 The acute angle between $4x+2y=7$ and $y=3x+1$ is θ . What is the value of $\tan \theta$?

- (A) $\frac{5}{7}$
- (B) 1
- (C) $\frac{1}{7}$
- (D) $\frac{1}{5}$

4 Which of the following is the exact value of $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$?

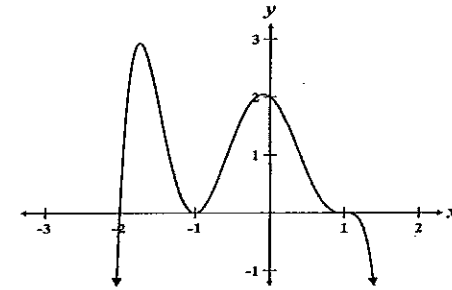
- (A) $\frac{24}{25}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{4}{5}$
- (D) $\frac{3}{4}$

5 The points A, B and C lie on a circle centre O . Points C, D and E lie on a tangent to the circle and AB is parallel to DE . $\angle ACD = 70^\circ$. Which of the following is the size of $\angle ACB$?



- (A) 70°
- (B) 50°
- (C) 60°
- (D) 40°

6 Which of the following best represents the graph below?



- (A) $y = (x+2)(x+1)^2(x-1)^3$
- (B) $y = (x+2)(x+1)^2(1-x)^3$
- (C) $y = (x+2)(1-x)^2(x-1)^3$
- (D) $y = (2-x)(x+1)^2(1-x)^3$

7 Given that $y = t^3 - 3t$ and $x = (t+1)^2$, which of the following equates to $\frac{dy}{dx}$?

- (A) $\frac{3(t-1)}{2}$
- (B) $2(t-1)(t+1)^2$
- (C) $\frac{3}{2}$
- (D) $\frac{2(t-1)}{3}$

8 Code words are created using all of the letters from the word CABRAMATTA. How many different words can be created?

- (A) $\frac{10!}{4!2!}$
- (B) $10 \times 4 \times 3!$
- (C) 10!
- (D) $\frac{9!}{3!2!}$

9 What is the constant term in the expansion of $\left\{ \left(2x + \frac{3}{x^2} \right)^{12} \left(2x - \frac{3}{x^2} \right)^{12} \right\}$?

- (A) ${}^{24}C_4 2^{16} \times 3^8$
- (B) ${}^{12}C_8 4^4 \times 9^8$
- (C) ${}^{12}C_4 4^8 \times 9^4$
- (D) ${}^{24}C_8 4^8 \times 9^4$

10 A geometric series is given by $1 - \tan^2 x + \tan^4 x - \tan^6 x + \dots$ for $0 < x < \frac{\pi}{4}$.
Which of the following are possible values of the limiting sum (S) of this series?

- (A) $0 \leq S_\infty \leq 1$
- (B) $\frac{1}{2} < S_\infty < 1$
- (C) $-1 \leq S_\infty \leq 1$
- (D) $S_\infty = \text{all real numbers}$

Section II

60 marks

Attempt Question 11-14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, you responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE Writing Booklet.

- (a) Find the co-ordinates of the point P which divides the interval AB internally in the ratio of 3:2 where $A = (-3, 2)$ and $B = (4, -1)$. 2
- (b) Evaluate $\lim_{x \rightarrow 5} \frac{2 \sin(x-5)}{(x-5)}$. 1
- (c) Write down the Range of $y = \pi + 2 \sin^{-1} x$. 2
- (d) Find $\int \frac{2}{9+4x^2} dx$. 2
- (e) Solve $\frac{x^2 - 12}{x} \leq 4$. 3
- (f) Use the substitution $u = \cos x$ to evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x dx$. 3
- (g) Express $6 \sin x + 8 \cos x$ in the form $R \sin(x + \alpha)$; where $0 \leq \alpha \leq \frac{\pi}{2}$. 2

End of Question 11

Question 12 (15 marks) Use a SEPARATE Writing Booklet.

- (a) Find the exact value of $\sin 75^\circ$. 2
- (b) The probability that any one of the 30 days in June is raining is 0.3. Write an expression for the probability that June will have exactly 10 rainy days. 2

- (c) Newton's Law of cooling states that when an object at temperature $T^\circ\text{C}$ is placed in an environment at temperature $A^\circ\text{C}$, the rate of temperature loss is given by the equation

$$\frac{dT}{dt} = k(T - A)$$

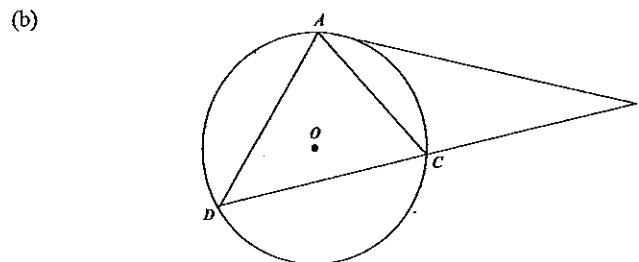
Where t is the time in minutes and k is a constant.

- (i) Verify that $T = A + Be^{kt}$ is a solution to the above equation, where A is a constant. 1
- (ii) A train is travelling through the mountains where the outside temperature is 2°C . The heating inside the carriage breakdown causing the inside temperature to drop from 22°C to 15°C in 30 minutes. Find the exact value of k . 2
- (iii) Find the temperature after a further 30 minutes. Leave your answer to the nearest minute. 1
- (d) Solve the following equation, giving your answer in general form, where x is in radians. 3
- $$\cos 2x - 5 \cos x - 2 = 0$$
- (e) (i) Show graphically that $\sin^{-1}(x) = (x-1)^2$ has a root between $x=0$ and $x=1$. 2
- (ii) By taking $x_1 = 0$ as the first approximation to this root, use one step of Newton's method to find a better approximation. 2

End of Question 12

Question 13 (15 marks) Use a SEPARATE Writing Booklet.

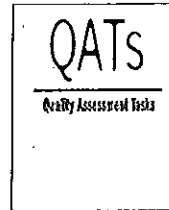
- (a) Prove by mathematical induction that $\sum_{r=1}^n (r^2 + 1)r! = n(n+1)!$ for all integral $n \geq 1$. 3



AB is a tangent to the circle centre O . The line BD meets the circle in C .

- (i) Prove that $\triangle ABD$ is similar to $\triangle ABC$. 3
- (ii) Hence prove that $AB^2 = BC \times BD$. 1
- (c) The velocity of a particle moving on the x -axis is given by $v = \frac{e^{-x}}{x}$. Initially the particle is at $x = \frac{1}{4}$.
- (i) Find an expression for acceleration and hence find where the particle reaches maximum speed. 3
- (ii) Give a brief explanation why the particle will always travel in a negative direction. 1
- (d) At a dinner party, 4 couples sit at a round table. Find the probability that
- (i) each couple sit next to each other. 2
- (ii) Lena does not sit between Theo and Enzo. 2

End of Question 13



2016 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

MATHEMATICS EXTENSION 1
MARKING GUIDELINES

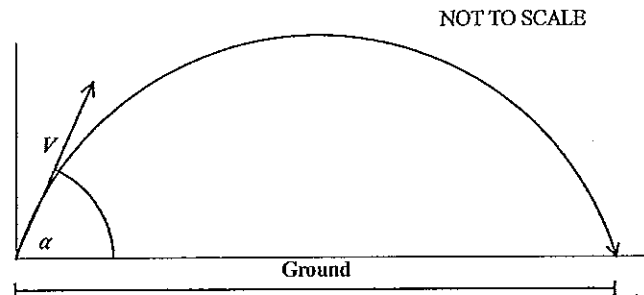
Section I

Multiple-choice Answer Key

| Question | Answer |
|----------|--------|
| 1 | D |
| 2 | C |
| 3 | B |
| 4 | A |
| 5 | D |
| 6 | B |
| 7 | A |
| 8 | A |
| 9 | C |
| 10 | B |

Question 14 (15 marks) Use a SEPARATE Writing Booklet.

- (a) The motion of a particle is given by $x = 3 + 2 \sin\left(4t + \frac{\pi}{3}\right)$.
- (i) Find an expression for acceleration of the particle in terms of x , and hence explain why the motion of this particle is simple harmonic. 3
- (ii) Find the speed of the particle when it is at the centre of motion. 2
- (b) (i) Given $(3 + 4x)^{15}$, use the Binomial Theorem to write an expression for T_k , $0 \leq k \leq 15$. 1
- (ii) Show that $\frac{T_{k+1}}{T_k} = \frac{64 - 4k}{3k}$. 3
- (iii) Hence find the greatest co-efficient in the expansion of $(3 + 4x)^{15}$. 2
- (c) At a golf course, a ball is hit off the ground with initial velocity v m/s, at an angle α° to the horizontal. Let force due to gravity $= -gm/s^2$ and noting $\alpha > 45^\circ$.



- (i) Write down the six equations of motion. (do not derive equations). 1
- (ii) Prove that during the motion when angle of elevation of the ball is 45° , this will occur when $t = \frac{v(\sin \alpha - \cos \alpha)}{g}$ seconds. 1
- (iii) If the time found in part (ii) is $\frac{1}{4}$ of the total time of flight, find the value of α . 2

End of Paper

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Section II

Question 11 (a)

| Criteria | Marks |
|---|-------|
| • Correct solution | 2 |
| • Achieves correct answer for either x or y | 1 |

Sample answer

$$P = \left(\frac{12-6}{5}, \frac{-3+4}{5} \right)$$

$$= \left(\frac{6}{5}, \frac{1}{5} \right)$$

Question 11 (b)

| Criteria | Mark |
|------------------|------|
| • Correct answer | 1 |

Sample answer

$$\lim_{x \rightarrow 5} \frac{2 \sin(x-5)}{(x-5)} = 2 \lim_{x \rightarrow 5} \frac{\sin(x-5)}{(x-5)}$$

$$= 2$$

Question 11 (c)

| Criteria | Marks |
|---|-------|
| • Correct answer | 2 |
| • Notes that $-\pi \leq 2 \sin^{-1} x \leq \pi$ | 1 |

Sample answer

$$\text{Range} = \{y : 0 \leq y \leq 2\pi\}$$

Question 11 (d)

| Criteria | Marks |
|---|-------|
| • Correct solution | 2 |
| • Achieves $\tan^{-1}\left(\frac{2x}{3}\right)$ | 1 |

Sample answer

$$\int \frac{2}{9+4x^2} dx = \frac{2}{3} \int \frac{3}{9+4x^2} dx$$

$$= \frac{2}{3 \times 2} \tan^{-1}\left(\frac{2x}{3}\right) + c$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{2x}{3}\right) + c$$

Question 11 (e)

| Criteria | Marks |
|----------------------------------|-------|
| • Correct solutions | 3 |
| • Achieves $x(x-6)(x+2) \leq 0$ | 2 |
| • Multiplies both sides by x^2 | 1 |

Sample answer

$$\frac{x^2-12}{x} \leq 4 \quad x \neq 0$$

$$x(x^2-12) \leq 4x^2$$

$$x^3-12x-4x^2 \leq 0$$

$$x^3-4x^2-12x \leq 0$$

$$x(x^2-4x-12) \leq 0$$

$$x(x-6)(x+2) \leq 0$$

$$x \leq -2 \text{ or } 0 < x \leq 6$$

Question 11 (f)

| Criteria | Marks |
|-------------------------------------|-------|
| • Correct solutions | 3 |
| • Achieves $I = -\int_1^0 1-u^2 du$ | 2 |
| • Achieves new limits | 1 |

Sample answer

$$I = \int_0^{\frac{\pi}{2}} \sin^3 x dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$x = \frac{\pi}{2} \rightarrow u = 0$$

$$x = 0 \rightarrow u = 1$$

$$\begin{aligned} I &= -\int_1^0 1-u^2 du \\ &= \int_1^0 u^2 - 1 du \\ &= \left[\frac{u^3}{3} - u \right]_1^0 \\ &= \left[0 - \left(\frac{1}{3} - 1 \right) \right] \\ &= \frac{2}{3} \end{aligned}$$

Question 11 (g)

| Criteria | Marks |
|---|-------|
| • Correct answer | 2 |
| • Correct value of either R or α | 1 |

Sample answer

$$\begin{aligned} 6 \sin x + 8 \cos x &= 10 \left\{ \frac{6}{10} \sin x + \frac{8}{10} \cos x \right\} \\ &= 10 \sin(x + \alpha) \quad \text{where } \tan \alpha = \frac{8}{6} \\ &= 10 \sin(x + 0.927) \quad \alpha = \tan^{-1} \left(\frac{8}{6} \right) = 0.927 \end{aligned}$$

Question 12 (a)

| Criteria | Marks |
|-----------------------------------|-------|
| • Correct solution | 2 |
| • Works positively towards answer | 1 |

Sample answer

$$\begin{aligned} \sin 75^\circ &= \sin(45^\circ + 30^\circ) \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \end{aligned}$$

Question 12 (b)

| Criteria | Marks |
|------------------------------------|-------|
| • Correct solution | 2 |
| • Correct working with one mistake | 1 |

Sample answer

$$\begin{aligned} P(R) &= \frac{3}{10} \quad P(\bar{R}) = \frac{3}{10} \\ \therefore P(R) &= {}^{30}C_{10} \left(\frac{7}{10} \right)^{20} \left(\frac{3}{10} \right)^{10} \end{aligned}$$

Question 12 (c) (i)

| Criteria | Mark |
|--------------------|------|
| • Correct solution | 1 |

Sample answer

$$T = B + Ae^{kt}$$

$$\frac{dT}{dt} = k[Ae^{kt}]$$

$$\therefore T = B + Ae^{kt}$$

$$\therefore T - B = Ae^{kt}$$

$$\therefore \frac{dT}{dt} = k[T - B]$$

Question 12 (c) (ii)

| Criteria | Marks |
|--------------------------------|-------|
| • Correct solution | 2 |
| • Achieves correct B and A | 1 |

Sample answer

$$T = 2 + Ae^{kt}$$

$$22 = 2 + A$$

$$A = 20$$

$$T = 2 + 20e^{kt}$$

$$15 = 2 + 20e^{30k}$$

$$\frac{13}{20} = e^{30k}$$

$$\frac{\ln\left(\frac{13}{20}\right)}{30} = k$$

Question 12 (c) (iii)

| Criteria | Mark |
|------------------|------|
| • Correct answer | 1 |

Sample answer

$$T = 2 + 20e^{\left[\frac{\ln\left(\frac{13}{20}\right)}{30} \times 60\right]}$$

$$T = 10.45 \text{ minutes}$$

$$T = 10^\circ$$

Question 12 (d)

| Criteria | Marks |
|---|-------|
| • Correct solution | 3 |
| • Achieves $\cos x = -\frac{1}{2}$ $\cos x = 3$ | 2 |
| • Achieves $2\cos^2 x - 1 - 5\cos x - 2 = 0$ | 1 |

Sample answer

$$\cos 2x - 5\cos x - 2 = 0$$

$$2\cos^2 x - 1 - 5\cos x - 2 = 0$$

$$2\cos^2 x - 5\cos x - 3 = 0$$

$$(2\cos x + 1)(\cos x - 3) = 0$$

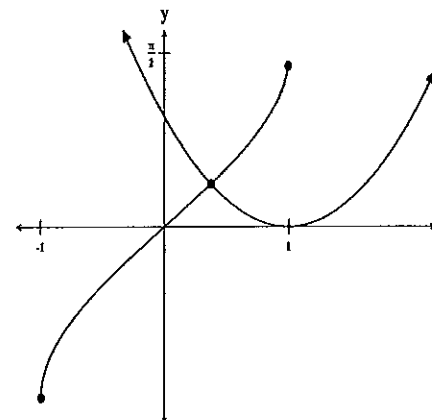
$$\cos x = -\frac{1}{2} \quad \cos x = 3 \text{ (no solution)}$$

$$x = 2\pi n \pm \frac{2\pi}{3} \quad \text{for } n = 0, 1, 2, \dots$$

Question 12 (e) (i)

| Criteria | Marks |
|--|-------|
| • Correct solution | 2 |
| • Correct graph of $y = \sin^{-1}(x)$ or $y = (x-1)^2$ | 1 |

Sample answer



Question 12 (e) (i)

| Criteria | Marks |
|--|-------|
| • Correct solution | 2 |
| • Uses correct formulae with one or two errors | 1 |

Sample answer

$$\sin^{-1}(x) = (x-1)^2$$

$$\sin^{-1}(x) - (x-1)^2 = 0$$

$$\text{Let } P(x) = \sin^{-1}(x) - (x-1)^2$$

$$P'(x) = \frac{1}{\sqrt{1-x^2}} - 2(x-1)$$

$$P(0) = -1$$

$$P'(0) = 3$$

$$x_2 = x_1 - \frac{P(x_1)}{P'(x_1)}$$

$$= 0 - \left[\frac{-1}{3} \right]$$

$$= \frac{1}{3}$$

Question 13 (a)

| Criteria | Marks |
|--|-------|
| • Correct proof | 1 |
| • Makes a correct substitution in correct step 3 (from step 2) | 1 |
| • Provides clear steps similar to the first three steps below | 1 |

Sample answer

Step 1: Prove the expression is true for $n=1$

$$(2)!! = 1(2)!$$

$$2 = 2 \quad (\text{true})$$

Step 2: Assume the expression is true for $n=k$ (where $k \geq 1$)

$$(1^2 + 1)!! + (2^2 + 1)2!! + \dots + (k^2 + 1)k! = k(k+1)!$$

Step 3: Prove the expression is true for $n=k+1$

$$(1^2 + 1)!! + (2^2 + 1)2!! + \dots + (k^2 + 1)k! + ((k+1)^2 + 1)(k+1)! = (k+1)(k+2)!$$

$$k(k+1)!! + ((k+1)^2 + 1)(k+1)! = (k+1)(k+2)! \quad \text{from assumption}$$

$$\text{LHS} = k(k+1)!! + (k^2 + 2k + 2)(k+1)!$$

$$= (k+1)!! [k + (k^2 + 2k + 2)]$$

$$= (k+1)!! (k^2 + 3k + 2)$$

$$= (k+1)!! (k+2)(k+1)$$

$$= (k+1)(k+2)!$$

$$= \text{RHS}$$

Hence the result is proven by mathematical induction.

Question 13 (b) (i)

| Criteria | Marks |
|--|-------|
| • Correct proof | 3 |
| • Correct proof with one mistake or no reason. | 2 |
| • One line correct in proof | 1 |

Sample answer

Prove $\triangle ABD$ is similar to $\triangle ABC$

$$\angle ABD = \angle ABC \quad (\text{common})$$

$$\angle ADB = \angle CAB \quad (\text{Alternate angle theorem})$$

$$\angle DAB = \angle ACB \quad (\text{angle sum of a triangle})$$

$\therefore \triangle ABD$ is similar to $\triangle ABC$ (equiangular)

Question 13 (c) (ii)

| Criteria | Mark |
|-------------------|------|
| • Correct working | 1 |

Sample answer:

$$\frac{AB}{CB} = \frac{BD}{AB} \quad (\text{corresponding sides in similar triangles are in ratio})$$

$$\therefore AB^2 = BC \times BD$$

Question 13 (c) (i)

| Criteria | Marks |
|---|-------|
| • Correct solution | 3 |
| • Works positively at achieves $\ddot{x} = -\frac{e^{-2x}(x+1)}{x^3}$ | 2 |
| • Notes maximum speed occurs when $\ddot{x} = 0$ | 1 |

Sample answer:

$$v = \frac{e^{-x}}{x}$$

$$\frac{1}{2}v^2 = \frac{e^{-2x}}{2x^2}$$

$$\ddot{x} = \frac{2x^2 \cdot -2e^{-2x} - e^{-2x} \cdot 4x}{4x^4}$$

$$= \frac{-4x^2 e^{-2x} - 4x e^{-2x}}{4x^4}$$

$$= \frac{-4x e^{-2x}(x+1)}{4x^4}$$

$$= \frac{-e^{-2x}(x+1)}{x^3}$$

Maximum speed occurs when $\ddot{x} = 0$

$$\frac{-e^{-2x}(x+1)}{x^3} = 0$$

$$x = -1$$

Question 13 (c) (ii)

| Criteria | Mark |
|---------------------|------|
| • Correct reasoning | 1 |

Sample answer:

Particle starts at $x = -\frac{1}{4}$ moving to the left and particle will never stop as $v = \frac{e^{-x}}{x} \neq 0$.

Question 13 (d) (i)

| Criteria | Marks |
|--------------------------------|-------|
| • Correct solution | 2 |
| • Achieves correct denominator | 1 |

Sample answer:

$$P(E) = \frac{(2!)^4 3!}{7!}$$

$$= \frac{96}{5040}$$

$$= \frac{2}{105}$$

Question 13 (d) (ii)

| Criteria | Marks |
|--|-------|
| • Correct solution | 2 |
| • Provides a method of counting which has some merit | 1 |

Sample answer:

$$P(E) = 1 - P(\text{Lena sits between Theo and Enzo})$$

$$= 1 - \frac{2!5!}{7!}$$

$$= \frac{240}{5040}$$

$$= \frac{1}{21}$$

Question 14 (a) (i)

| Criteria | Marks |
|--|-------|
| • Correct solution | 3 |
| • Achieves $\ddot{x} = -16(x-3)$ | 2 |
| • Correctly explains why particle's movement is SHM or achieves $\ddot{x} = -32 \sin\left(4t + \frac{\pi}{3}\right)$ | 1 |

Sample answer

$$x = 3 + 2 \sin\left(4t + \frac{\pi}{3}\right)$$

$$\dot{x} = 8 \cos\left(4t + \frac{\pi}{3}\right)$$

$$\ddot{x} = -32 \sin\left(4t + \frac{\pi}{3}\right)$$

$$= -16 \left[3 + 2 \sin\left(4t + \frac{\pi}{3}\right) - 3 \right]$$

$$\ddot{x} = -16(x-3)$$

Hence since when $x > 3$ $\ddot{x} < 0$ and when $x < 3$ $\ddot{x} > 0$, particle will oscillate about $x = 3$.

OR quoting Since $\ddot{x} = -n^2 X$

Therefore the particle executes SHM.

Question 14 (a) (ii)

| Criteria | Marks |
|---------------------------------------|-------|
| • Correct solution | 2 |
| • Works positively towards the answer | 1 |

Sample answer

The speed of the motion is at a maximum at the centre of motion.

Hence

$$-8 \leq 8 \cos\left(4t + \frac{\pi}{3}\right) \leq 8$$

$$\therefore \text{Speed} = 8 \text{ m/s}$$

Question 14 (b) (i)

| Criteria | Mark |
|------------------|------|
| • Correct answer | 1 |

Sample answer

$$T_k = {}^{15}C_{k-1} (3)^{16-k} (4x)^{k-1}$$

Question 14 (b) (ii)

| Criteria | Marks |
|---|-------|
| • Correct solution | 3 |
| • Achieves $\frac{{}^{15}C_{k-1} (3)^{16-k} (4x)^{k-1}}{k!(15-k)!} \div \frac{{}^{15}C_k (3)^{15-k} (4x)^k}{(k-1)!(16-k)!}$ | 2 |
| • Notes correctly T_{k+1} | 1 |

Sample answer

$$T_k = {}^{15}C_{k-1} (3)^{16-k} (4x)^{k-1}$$

$$T_{k+1} = {}^{15}C_k (3)^{15-k} (4x)^k$$

$$\frac{T_{k+1}}{T_k} = \frac{{}^{15}C_k (3)^{15-k} (4x)^k}{{}^{15}C_{k-1} (3)^{16-k} (4x)^{k-1}}$$

$$= \frac{{}^{15}C_k (3)^{15-k} (4x)^k}{k!(15-k)!} \times \frac{(k-1)!(16-k)!}{{}^{15}C_{k-1} (3)^{16-k} (4x)^{k-1}}$$

$$= \frac{4x(16-k)}{3k}$$

$$= \left[\frac{4(16-k)}{3k} \right] x$$

$$= \left[\frac{64-4k}{3k} \right] x$$

$$\text{Co-efficient} = \left[\frac{64-4k}{3k} \right]$$

Question 14 (b) (iii)

| Criteria | Marks |
|--|-------|
| • Correct answer (no need to present all cases for full marks) | 2 |
| • Achieves $k < 9.143$ or $k > 9.143$ | 1 |

Sample answer

Case 1

$$T_{k+1} > T_k$$

$$\therefore \frac{T_{k+1}}{T_k} > 1$$

$$\therefore \frac{64-4k}{3k} > 1$$

$$64-4k > 3k$$

$$64 > 7k$$

$$k < 9.143$$

$$k = 9, 8, 7, \dots, 1$$

$$\therefore T_{10} > T_9 > T_8, \dots$$

Case 2:

$$T_{k+1} \neq T_k \text{ as } k = 9.143 \text{ (not an integer)}$$

Case 3:

$$T_{k+1} < T_k$$

$$\frac{T_{k+1}}{T_k} < 1$$

$$k > 9.143$$

$$k = 10, 11, 12, \dots$$

$$\therefore T_{10} > T_{11} > T_{12}, \dots$$

$$T_{10} = {}^{15}C_9 (3)^6 (4)^9$$

Question 14 (c) (i)

| Criteria | Mark |
|--------------------------------------|------|
| • Correct answer (allow one mistake) | 1 |

Sample answer

$$\ddot{x} = 0 \quad \ddot{y} = -g$$

$$\dot{x} = v \cos \alpha \quad \dot{y} = -gt + v \sin \alpha$$

$$x = vt \cos \alpha \quad y = -\frac{1}{2}gt^2 + vt \sin \alpha$$

Question 14 (c) (ii)

| Criteria | Mark |
|--------------------|------|
| • Correct solution | 1 |

Sample answer

When the angle of elevation of the ball is $45^\circ \therefore \tan \theta = \frac{\dot{y}}{\dot{x}} = 1$.

$$\therefore \dot{x} = \dot{y}$$

$$v \cos \alpha = -gt + v \sin \alpha$$

$$gt = v \sin \alpha - v \cos \alpha$$

$$t = \frac{v(\sin \alpha - \cos \alpha)}{g}$$

Question 14 (c) (iii)

| Criteria | Marks |
|--|-------|
| • Correct solution | 2 |
| • Achieves $0 = -\frac{1}{2}g \left[\frac{4v(\sin \alpha - \cos \alpha)}{g} \right]^2 + v \left[\frac{4v(\sin \alpha - \cos \alpha)}{g} \right] \sin \alpha$ | 1 |

Sample answer

$$t = \frac{4v(\sin \alpha - \cos \alpha)}{g} \text{ when } y=0$$

$$0 = -\frac{1}{2}g \left[\frac{4v(\sin \alpha - \cos \alpha)}{g} \right]^2 + v \left[\frac{4v(\sin \alpha - \cos \alpha)}{g} \right] \sin \alpha$$

$$0 = \frac{-8v^2(\sin \alpha - \cos \alpha)^2}{g} + \frac{4v^2(\sin \alpha - \cos \alpha)\sin \alpha}{g}$$

$$8v^2(\sin \alpha - \cos \alpha)^2 = 4v^2(\sin \alpha - \cos \alpha)\sin \alpha$$

$$2(\sin \alpha - \cos \alpha) = \sin \alpha$$

$$\sin \alpha = 2 \cos \alpha$$

$$\tan \alpha = 2$$

$$\alpha = 63^\circ 26'$$