QATs

Quality Assessed Tasks

Centre Number

Student Number

2016 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- Write you student number on this page and on the multiple-choice answer sheet
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 100

Section I Pages 2-5

10 marks

- Attempt questions 1-10
- Allow about 15 minutes for this section

Section II Pages 6-12

90 marks

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1-10

1 Which of the following is the value of $3e^3$, correct to 3 significant figures?

(A) 60.2

(B) 60.3

(C) 60.256

(D) 60.257

2 $\dfrac{1}{5\sqrt{3}-2}$ is equal to which of the following?

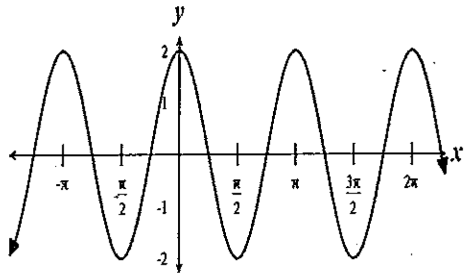
(A) $\dfrac{5\sqrt{3}+2}{73}$

(B) $\dfrac{5\sqrt{3}-2}{221}$

(C) $\dfrac{5\sqrt{3}-2}{77}$

(D) $\dfrac{5\sqrt{3}+2}{71}$

3



Which of the following is the equation of the above diagram?

(A) $y = 2\cos 2x$

(B) $y = 2\sin 2x$

(C) $y = 2\cos\frac{x}{2}$

(D) $y = -2\sin\frac{x}{2}$

4 The perpendicular distance from the point $(2,-2)$ to the line $2y = 3x + 4$ is equal to?

(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{6}{\sqrt{15}}$

(C) $\frac{14}{\sqrt{13}}$

(D) $\frac{14}{\sqrt{8}}$

5 The derivative of $2x\sin\frac{x}{2}$ is?

(A) $x\cos\frac{x}{2}$

(B) $2\sin\frac{x}{2} + 2x\cos\frac{x}{2}$

(C) $2\sin\frac{x}{2} + x\cos\frac{x}{2}$

(D) $\cos\frac{x}{2} - x\sin\frac{x}{2}$

6 A bag contains 6 green marbles and 4 yellow marbles. Arjun takes out one marble at a time from the bag without replacing it.
What is the probability that the second marble he takes out is yellow?

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{9}{25}$

(D) $\frac{6}{25}$

7 The sum of the first 10 terms of the sequence $(12+3x)$, $(6+4x)$, $5x$, ... is equal to?

(A) $66-6x$

(B) $390-15x$

(C) $100-32x$

(D) $75x-150$

8 Which of the following is the equation of the directrix to the parabola $(y-3)^2 = -12(x+1)$?

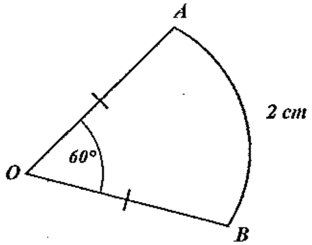
(A) $x = 2$

(B) $x = -4$

(C) $x = 6$

(D) $x = 0$

9



The diagram above represents a sector of a circle with an angle of 60° and an arc length $AB = 2\,cm$. Which of the following is the area of the sector?

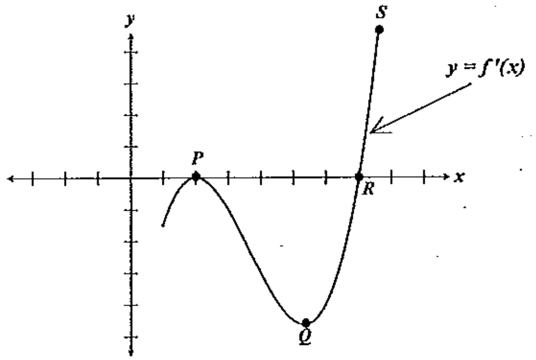
(A) $\frac{1}{30} cm^2$

(B) $\frac{6}{\pi} cm^2$

(C) $\pi^2\, cm^2$

(D) $\frac{12}{\pi} cm^2$

10



The diagram above shows points *P, Q, R and S* on the **GRADIENT** graph of $y = f(x)$. At which point does a local minimum exist?

(A) Q

(B) S

(C) P

(D) R

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate Writing Booklet.

(a) Factorise $3x^2 - 5x + 2$. **1**

(b) Evaluate $\lim_{x \to 3} \left(\frac{x^3 - 27}{2x - 6} \right)$. **2**

(c) Find the equation of the tangent to the curve $y = x^3 + 1$ at the point $x = 1$. **2**

(d) Differentiate $(\ln x + 4)^5$. **2**

(e) Solve and graph the solution of $|3x - 5| \le 4$. **3**

(f) Find $\int \frac{1}{(x-4)^3}\, dx$. **2**

(g) Sketch the region defined as: **3**

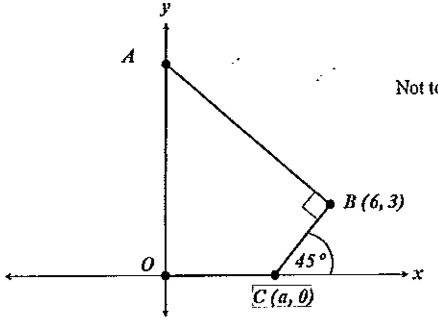
$$x^2 + y^2 \le 4 \quad \text{and} \quad y \ge 2 - x^2.$$

End of Question 11

Question 12 (15 marks) Use a separate Writing Booklet.

(a) Given that $\cos\theta = -\frac{12}{13}$ and $\tan\theta < 0$, find the exact value of $\sin\theta$. **2**

(b)



The points $OABC$ form a quadrilateral, where, $B = (6,3)$, $C = (a,0)$, O is the origin and A lies on the y-axis. The angle of inclination of BC is $45°$ and AB is perpendicular to BC.

(i) For $C = (a,0)$, show that $a = 3$. **1**

(ii) Show that the equation of line AB is $x + y - 9 = 0$. **2**

(iii) Find the coordinates of A. **1**

(iv) Hence find the area of $OABC$. **2**

(c) The sales team at Frontier phone company sell 12 000 phones in the first month of operation. They increase their sales by 800 phones each month on the preceding month's sales.

(i) Find the number of phones sold in the last month of the second year of operation. **2**

(ii) Find the number of phones sold over the entire two year period. **2**

(iii) Sampson, another phone company commenced business at exactly the same time as Frontier. Their sales team sell 5 000 phones in their first month of operation and increase their sales by 1 500 each month on the preceding month's sales.
After how many months will both companies total sales become equal? **3**

End of Question 12

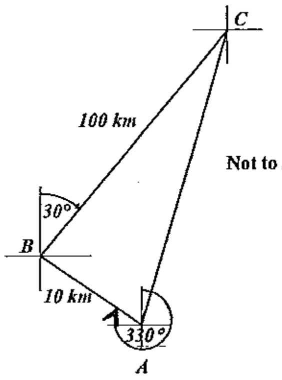
Question 13 (15 marks) Use a separate Writing Booklet.

(a) Find the value(s) of m for which the equation $x^2 + mx + (m+3) = 0$ has:

(i) One root equal to 3. **1**

(ii) Real roots. **2**

(b)



On a sailing trip Amelia sails from port A to Port B on a bearing of 330° for 10 *km*. She then changes direction and sails to port C on a bearing of 030° for 100 *km*.

(i) Copy the diagram in your solution booklet and show $\angle ABC = 120°$. **1**

(ii) Use the cosine rule to find the length of AC to 2 decimal places. **2**

(iii) Find the bearing of A from C to the nearest degree. **2**

(c) Consider the function $y = x^2(3-x)$.

(i) Find where the curve cuts the x-*axis*. **1**

(ii) Find the coordinates of the two stationary points and determine their nature. **3**

(iii) Find the coordinates of any points of inflexion. **1**

(iv) Sketch the curve $y = x^2(3-x)$. **2**

End of Question 13

Question 14 (15 marks) Use a separate Writing Booklet.

(a) Find the domain of $y=\sqrt{x^2-1}$. **1**

(b) (i) Show that $\frac{x}{x^2-1}+\frac{2}{x-1}=\frac{3x+2}{x^2-1}$. **2**

(ii) Hence or otherwise $\int \frac{3x+2}{x^2-1}\,dx$. **2**

(c) The finals of the Wagga Wagga Basketball competition is between the Mavericks and the Rams. The finals are the best out of three games. If a team wins both the 1st and 2nd games, the third game is not played. In any game, the probability of the Mavericks winning is $\frac{3}{5}$, while the Rams have a probability of $\frac{3}{10}$ of winning any game.

(i) Show that the probability of a draw in any game in the finals is $\frac{1}{10}$. **2**

(ii) Find the probability that the Mavericks win the first two games. **1**

(iii) Find the probability of the Mavericks winning the championships in three games, where one game is a draw. **2**

(d) Traces of a particular drug used to remedy colds by athletes are permitted in a particular sport. The blood-medicine content (M) in the blood after a certain athlete has taken the medicine is:

$$M=0.32(0.8)^t$$

where t is in hours and M is *mg/ml*

(i) If the allowable blood-medicine limit to compete is 0.09 *mg/ml*, by what factor is the athlete initially over the limit. **2**

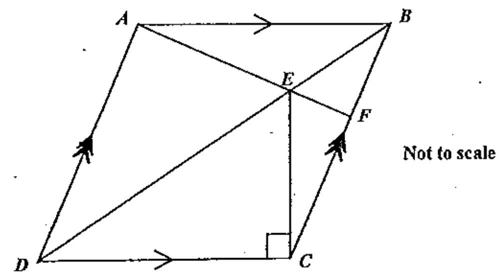
(ii) Show that M can be expressed as $M=0.32e^{-0.2231t}$. **1**

(iii) After how many hours and minutes will the athlete be legally able to compete. **2**

End of Question 14

Question 15 (15 marks) Use a separate Writing Booklet.

(a)



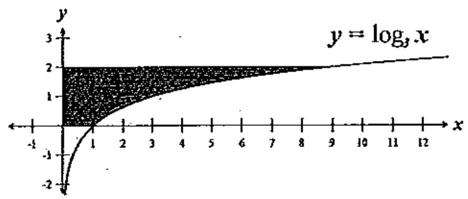
ABCD is a rhombus and *BD* is a diagonal. *DC* is perpendicular to *EC* and *AF* meets *BD* at *E*.

(i) Copy the diagram in your writing book and give reason(s) why $\angle ADE=\angle EDC$. **1**

(ii) Prove that $\Delta ADE\equiv\Delta CDE$. **3**

(iii) Hence prove that ΔEFC is a right angle triangle. **2**

(b)



The diagram above shows the graph of $y=\log_3 x$. The area bounded by the curve, the line $y=2$ and the x and y axes is rotated about the *y-axis*.

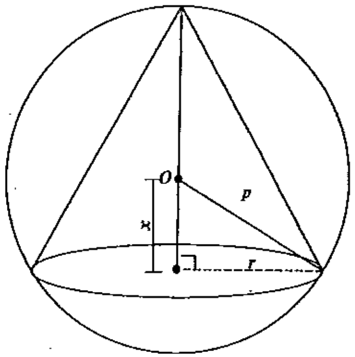
(i) Show that the volume of the solid formed is given by $V=\pi\int_0^2 e^{y\ln 9}\,dy$. **2**

(ii) Hence find the exact volume of the solid. **2**

Question 15 continues over the page

Question 15 (continued)

(c)



A cone is inscribed in a sphere of fixed radius p cm and centre O. The base of cone has a radius r cm and the length from the base of the cone to the centre of the sphere is x cm.

(i) Show that the volume V of the cone is given by $V = \frac{\pi}{3}\left[p^3 + p^2x - px^2 - x^3\right]$. **2**

(ii) Hence find the value of x for which the volume of the cone is a maximum. **3**

End of Question 15

Question 16 (15 marks) Use a separate Writing Booklet.

(a) Use Simpson's rule with three function values to show that **2**

$$\int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}} \operatorname{cosec} x \, dx = -\frac{\pi}{6}\left[2+\sqrt{2}\right].$$

(b) A particle is moving in a straight line. Its displacement from the origin (x cm) as a function of time (t minutes) is given by $x = e^{-t}\cos t$.

(i) Show that $v = -e^{-t}(\cos t + \sin t)$ and $a = 2e^{-t}\sin t$. **3**

(ii) Find the first two times when the particle is at rest. **2**

(iii) Find the first time when the particle has the greatest velocity. **1**

(iv) Give a brief description of the movement of the particle from $t = 0$ to $t \to \infty$ **1**

(c) Lola borrows \$300 000 to buy a house. She has agreed to pay back a total of \$22 500 each year until the loan is paid out. The loan agreement is for 6% p.a. reducible interest. Let p be the number of regular payments each year of \$$M$. Hence $P \times M = \$22500$ at the end of each year. Interest is calculated and charged just before each repayment.

(i) Show that the amount owing after the second repayment is: **1**

$$A_2 = \$300000\left(1+\frac{0.06}{P}\right)^2 - M\left[1+\left(1+\frac{0.06}{P}\right)\right].$$

(ii) Show that the amount owing after n repayments is: **3**

$$A_n = \$375000 - \$75000\left(1+\frac{0.06}{P}\right)^n.$$

(iii) Calculate how much quicker Lola's loan would be repaid if she paid the loan back in equal monthly instalments rather than equal quarterly instalments. **2**

End of Paper

QATs
Quality Assessment Tasks

2016 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

MATHEMATICS – MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	D
3	A
4	C
5	C
6	B
7	D
8	A
9	B
10	D

Section II

Question 11 (a)

Criteria	Mark
• Correct answer	1

Sample answer

$3x^2-5x+2=(3x-2)(x-1)$

Question 11 (b)

Criteria	Marks
• Correct solution	2
• Achieves $\lim_{x\to3}\frac{(x-3)(x^2+3x+9)}{2(x-3)}$	1

Sample answer

$$\lim_{x\to3}\left(\frac{x^3-27}{2x-6}\right)=\lim_{x\to3}\frac{(x-3)(x^2+3x+9)}{2(x-3)}$$
$$=\lim_{x\to3}\frac{(x^2+3x+9)}{2}$$
$$=\frac{27}{2}$$

Question 11 (c)

Criteria	Marks
• Correct solution	2
• Achieves correct gradient and y value of point	1

Sample answer

$y=x^3+1$

$y'=3x^2$

$f(1)=2 \quad (1,2)$

$f'(1)=3 \quad m=3$

$y-2=3(x-1)$

$y-2=3x-3$

$3x-y-1=0$

Question 11 (d)

Criteria	Marks
• Correct solution	2
• Achieves $5(\ln x+4)^4$	1

Sample answer

$$\frac{d}{dx}(\ln x+4)^5 = 5(\ln x+4)^4 \times \frac{1}{x}$$
$$= \frac{5(\ln x+4)^4}{x}$$

Question 11 (e)

Criteria	Marks
• Correct solution and graph •	3
• Achieves $\frac{1}{3} \le x \le 3$	2
• Achieves $x \le 3$ *or* $x \ge \frac{1}{3}$	1

Sample answer

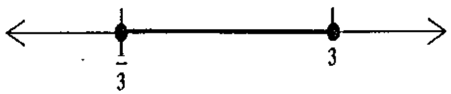
$$|3x-5| \le 4$$

$3x-5 \le 4$ *or* $3x-5 \ge -4$

$3x \le 9$ $\quad 3x \ge 1$

$x \le 3$ $\quad x \ge \frac{1}{3}$

$$\frac{1}{3} \le x \le 3$$



Question 11 (f)

Criteria	Marks
• Correct solution in any format, with or without c	2
• Notes $\int (x-4)^{-3}\, dx$ and attempts integration with one mistake	1

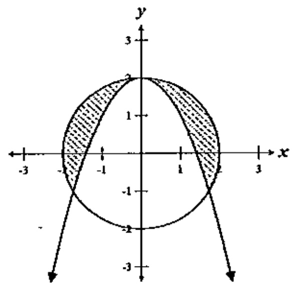
Sample answer

$$\int \frac{1}{(x-4)^3}\, dx = \int (x-4)^{-3}\, dx$$
$$= \frac{(x-4)^{-2}}{-2} + c$$
$$= \frac{-1}{2(x-4)^2} + c$$

Question 11 (g)

Criteria	Marks
• Correct solution	3
• Correct graph of $x^2+y^2 \le 4$ and $y \ge 2-x^2$	2
• Correct graph of $x^2+y^2 \le 4$ *or* $y \ge 2-x^2$	1

Sample answer



Question 12 (a)

Criteria	Marks
• Correct solution	2
• Appreciates that relevant angle lies in the second quadrant	1

Sample answer

Since $\cos\theta = -\frac{12}{13}$ and $\tan\theta < 0$,

θ lies in the second quadrant.

Hence $\sin\theta = \frac{5}{13}$

Question 12 (b) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$m_{BC} = \tan 45$

$m_{BC} = 1$

$m_{BC} = \frac{3-0}{6-a} \qquad \therefore \frac{3-0}{6-a} = 1 \qquad a = 3$

Question 12 (b) (ii)

Criteria	Marks
• Correct solution	2
• Achieves correct gradient from (i)	1

Sample answer

$m_{AB} = -1 \qquad (m_{BC} \times m_{AB} = -1)$

$y - 3 = -1(x - 6)$

$y - 3 = -x + 6$

$x + y - 9 = 0$

Question 12 (b) (iii)

Criteria	Mark
• Correct solution	1

Sample answer

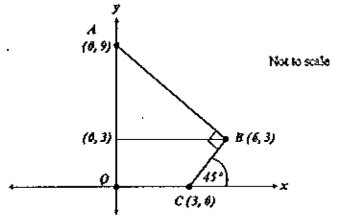
For $x + y - 9 = 0$, let $x = 0$

$\therefore y = 9 \quad A = (0, 9)$

Question 12 (b) (iv)

Criteria	Marks
• Correct solution	2
• Works positively towards answer	1

Sample answer



$\text{Area} = \left[\frac{1}{2} \times 6 \times 6\right] + \frac{3}{2}[3 + 6]$

$= 18 + \frac{27}{2}$

$= 31\frac{1}{2}\ units^2$

Question 12 (c) (i)

Criteria	Marks
• Correct solution	2
• Substitutes both a and d in correct formulae	1

Sample answer

$a = 12\,000 \qquad d = 800$

$T_n = a + (n-1)d$

$T_{24} = 12\,000 + 23(800)$

$= 30\,400$ phones

Question 12 (c) (ii)

Criteria	Marks
• Correct solution	2
• Correct working with one mistake	1

Sample answer

$S_n = \frac{n}{2}\{a+l\}$

$S_{24} = 12\{12\,000 + 30\,400\}$

$= 508\,800$ phones

Question 12 (c) (iii)

Criteria	Marks
• Correct solution	3
• Achieves correct quadratic from their working	2
• Uses S_n for each company in solution	1

Sample answer

$S_{sampson} = S_{Frontier}$

$\frac{n}{2}\{24\,000 + (n-1)800\} = \frac{n}{2}\{10\,000 + (n-1)1500\}$

$n\{800n + 23200\} = n\{1500n + 8500\}$

$800n^2 + 23200n = 1500n^2 + 8500n$

$700n^2 - 14\,700n = 0$

$n^2 - 21n = 0$

$n = 0, 21$

Total sales become equal after 21 months.

Question 13 (a) (i)

Criteria	Mark
• Correct solution	1

Sample answer

For $x^2 + mx + (m+3) = 0$, $x = 3$ is a root

$\therefore 9 + 3m + (m+3) = 0$

$4m = -12$

$m = -3$

Question 13 (a) (ii)

Criteria	Marks
• Correct solution	2
• Obtains discriminant and notes $\Delta \geq 0$	1

Sample answer

$\Delta = b^2 - 4ac$

$= m^2 - 4(m+3)$

$= m^2 - 4m - 12$

For real roots $\Delta \geq 0$

$m^2 - 4m - 12 \geq 0$

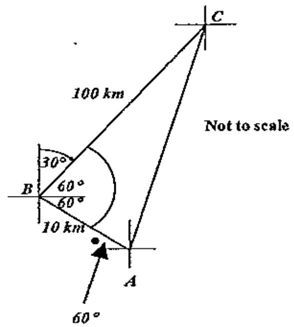
$(m-6)(m+2) \geq 0$

$m \leq -2$ or $m \geq 6$

Question 13 (b) (i)

Criteria	Mark
• Clearly notes answer in diagram or indicate a combination of alternate angle and complement of 30^0	1

Sample answer



Hence $\angle ABC = 60^\circ + 60^\circ = 120^\circ$

Question 13 (b) (ii)

Criteria	Marks
• Correct solution	2
• Substitutes correctly in the cosine rule formulae	1

Sample answer

$AC^2 = 100^2 + 10^2 - (2 \times 100 \times 10 \times \cos 120°)$

$AC = 105.3565$

$= 105.36$

Question 13 (b) (iii)

Criteria	Marks
• Correct solution	2
• Substitutes correctly in the cosine rule formulae	1

Sample answer

$\frac{\sin\theta}{10} = \frac{\sin 120°}{105.36}$

$\sin\theta = \frac{10\sin 120°}{105.36}$

$\theta = 4° 43'$

$\theta = 5^0$

Bearing from A to $C = 270° - (60° + 5°) = 205°$

Question 13 (c) (i)

Criteria	Mark
• Correct answer	1

Sample answer

Roots of the equation occur when $y = 0$

$x^2(3 - x) = 0$

$\therefore x = 0, 3$

Question 13 (c) (ii)

Criteria	Marks
• Correct solution	3
• Achieves both stationary points	2
• Differentiates correctly and achieves one stationary point	1

Sample answer

$y = x^2(3 - x)$

$y = 3x^2 - x^3$

$y' = 6x - 3x^2$

$y'' = 6 - 6x$

Stationary points occur when $y' = 0$

$6x - 3x^2 = 0$

$2x - x^2 = 0$

$x(2 - x) = 0$

$x = 0, 2$

Stationary points occur at $(0,0)$ and $(2,4)$

Check concavity

$f''(0) = 6 \quad \therefore (0,0)$ is a minimum turning point

$f''(2) = -6 \quad \therefore (2,4)$ is a maximum turning point

Question 13 (c) (iii)

Criteria	Mark
• Correct solution	1

Sample answer

Points of inflexion occur when $y'' = 0$ and the curve changes concavity.

$6 - 6x = 0$

$x = 1 \quad (1,2)$

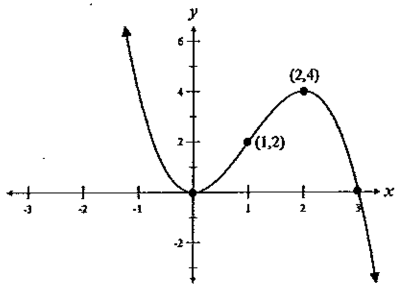
x	0	1	2
y''	6	0	-6

$\therefore (1,2)$ is a vertical point of inflexion

Question 13 (c) (iv)

Criteria	Marks
• Correct diagram	2
• Uses their solution to represent graph with one error	1

Sample answer



Question 14 (a)

Criteria	Mark
• Correct answer	1

Sample answer

$$x^2-1\geq 0$$
$$(x-1)(x+1)\geq 0$$
$$x\leq -1 \quad or \quad x\geq 1$$
$$\text{Domain}=\{x: x\leq -1 \ \ or \ \ x\geq 1\}$$

Question 14 (b) (i)

Criteria	Marks
• Corrects solution	2
• Works positively towards answer	1

Sample answer

$$LHS=\frac{x}{x^2-1}+\frac{2}{x-1}$$
$$=\frac{x}{(x-1)(x+1)}+\frac{2}{(x-1)}$$
$$=\frac{x+2(x+1)}{(x-1)(x+1)}$$
$$=\frac{3x+2}{x^2-1}$$

Question 14 (b) (ii)

Criteria	Marks
• Correct solutions (disregard exclusion of *c*)	2
• Uses part (i) to rewrite and achieves one part of solution	1

Sample answer

$$\int\frac{3x+2}{x^2-1}\,dx=\int\frac{x}{x^2-1}+\frac{2}{x-1}\,dx$$
$$=\frac{1}{2}\ln\left|x^2-1\right|+2\ln\left|x-1\right|+c$$

Question 14 (c) (i)

Criteria	Marks
• Correct solution	2
• Demonstrates an understanding of complement of events	1

Sample answer

$$P(Draw)=1-\left[P(W_{mavericks})+P(W_{Rams})\right]$$
$$=1-\left[\frac{3}{5}+\frac{3}{10}\right]$$
$$=\frac{1}{10}$$

Question 14 (c) (ii)

Criteria	Mark
• Correct solution	1

Sample answer

$$P(WW_{mavericks}) = \frac{3}{5}\times\frac{3}{5} = \frac{9}{25}$$

Question 14 (c) (iii)

Criteria	Marks
• Correct solution	2
• Achieves one of the possibilities or presents both options without solution	1

Sample answer

$$P(\textit{Mavericks winning in three games}) = P(DWW) + P(WDW)$$
$$= \left(\frac{1}{10}\times\frac{3}{5}\times\frac{3}{5}\right)+\left(\frac{3}{5}\times\frac{1}{10}\times\frac{3}{5}\right)$$
$$= \frac{18}{250}$$
$$= \frac{9}{125}$$

Question 14 (d) (i)

Criteria	Marks
• Correct solution	2
• Achieves initial M	1

Sample answer

$M = 0.32(0.8)^t$

$t = 0 \quad M = 0.32$

Factor over the limit $= \dfrac{0.32}{0.09} = 3.6$

Question 14 (d) (ii)

Criteria	Mark
• Correct solution	1

Sample answer

Show $(0.8)^t = e^{-0.2231t}$

$$(0.8)^t = \left(e^{-0.2231}\right)^t$$

By calculator:

$e^{-0.2231} = 0.80003$

$\therefore e^{-0.2231} = 0.8$

$\therefore (0.8)^t = e^{-0.2231t}$

Hence $M = 0.32e^{-0.2231t}$

Question 14 (d) (iii)

Criteria	Marks
• Correct solution	2
• Solves $0.09 = 0.32e^{-0.2231t}$ using natural logs	1

Sample answer

$$0.09 = 0.32e^{-0.2231t}$$
$$0.28125 = e^{-0.2231t}$$
$$\ln(0.28125) = -0.2231t$$
$$\frac{\ln(0.28125)}{-0.2231} = t$$
$$t = 5.686$$
$$t = 5 \text{ hours } 41 \text{ minutes}$$

Question 15 (a) (i)

Criteria	Mark
• Correct reasons	1

Sample answer

$\angle ADE = \angle EDC$ *(Diagonals of a rhombus bisect the vertices of a rhombus).*

Question 15 (a) (ii)

Criteria	Marks
• Correct solution	3
• Correct solution with one mistake in reasoning	2
• Positively working towards solution	1

Sample answer

Prove $\Delta ADE \equiv \Delta CDE$

$AD = CD$ *(sides of rhombus are equal)*

$\angle ADE = \angle EDC$ *(proven in part (i))*

ED is common

$\therefore \Delta ADE \equiv \Delta CDE$ *(SAS)*

Question 15 (a) (iii)

Criteria	Marks
• Correct solution with one mistake in reasoning	2
• Presents one line of solution with correct reasoning	1

Sample answer

$\angle DCE = \angle DAE = 90°$ (Corresponding angles in congruent triangles are equal)

$\angle DAE = \angle BFE$ (Alternate angles in parallel lines, AD parallel to CB)

$\angle EFB = \angle EFC = 90°$ (Straight line)

$\therefore \Delta EFC$ is a right angle triangle

Question 15 (b) (i)

Criteria	Marks
• Correct solution	2
• Achieves $x = e^{y\ln 3}$	1

Sample answer

$$y = \log_3 x$$
$$y = \frac{\ln x}{\ln 3}$$
$$y\ln 3 = \ln x$$
$$x = e^{y\ln 3}$$
$$x^2 = \left[e^{y\ln 3}\right]^2$$
$$x^2 = e^{2y\ln 3}$$
$$x^2 = e^{y\ln 9}$$

Question 15 (b) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $\frac{\pi}{\ln 9}\left[e^{y\ln 9}\right]_0^2$	1

Sample answer

$$V = \pi\int_0^2 e^{y\ln 9}\,dy$$
$$= \frac{\pi}{\ln 9}\left[e^{y\ln 9}\right]_0^2$$
$$= \frac{\pi}{\ln 9}\left[e^{2\ln 9} - 1\right]$$
$$= \frac{\pi}{2\ln 3}[81 - 1]$$
$$= \frac{40\pi}{\ln 3}\ units^3$$

Question 15 (c) (i)

Criteria	Marks
• Correct working	2
• Substitutes either $x^2 = h^2 - r^2$ or $h = x + p$ in correct volume formulae	1

Sample answer

Now

$x^2 + r^2 = p^2$ *also* $h = x + p$

$r^2 = p^2 - x^2$

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{\pi}{3}\left(p^2 - x^2\right)(x + p)$$
$$= \frac{\pi}{3}\left[xp^2 + p^3 - x^3 - px^2\right]$$
$$= \frac{\pi}{3}\left[p^3 + p^2x - px^2 - x^3\right]$$

Question 15 (c) (ii)

Criteria	Marks
• Correct solution	3
• Achieves $x=\frac{p}{3},-p$	2
• $V'=\frac{\pi}{3}\left[p^2-2px-3x^2\right]$	1

Sample answer

$$V=\frac{\pi}{3}\left[p^3+p^2x-px^2-x^3\right]$$

$$V'=\frac{\pi}{3}\left[p^2-2px-3x^2\right]$$

$$V''=\frac{\pi}{3}\left[-2p-6x\right]$$

Stationary points occur when $V'=0$

$$\frac{\pi}{3}\left[p^2-2px-3x^2\right]=0$$

$$3x^2+2px-p^2=0$$

$$(3x-p)(x+p)=0$$

$$x=\frac{p}{3},-p$$

$x=\frac{p}{3}\quad(x>0)$

$V''\left(\frac{p}{3}\right)=\frac{\pi}{3}\left[-4p\right]<0$ concave down maximum

Hence when $x=\frac{p}{3}$ the volume of the cone is a maximum.

Question 16 (a)

Criteria	Marks
• Correct solution	2
• Obtains correct function values	1

Sample answer

x	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	
cosec x	$-\sqrt{2}$	-1	$-\sqrt{2}$	
weight	1	4	1	
result	$-\sqrt{2}$	-4	$-\sqrt{2}$	Sum $=-4-2\sqrt{2}$

$$\int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}}\operatorname{cosec}x\,dx=\frac{\frac{\pi}{4}}{3}\left[-4-2\sqrt{2}\right]$$

$$=-\frac{\pi}{12}\left[4+2\sqrt{2}\right]$$

$$=-\frac{\pi}{6}\left[2+\sqrt{2}\right]$$

Question 16 (b) (i)

Criteria	Marks
• Insert standard referenced marking criterion here	3
• Obtains either v or a (from their v)	2
• Obtains either v or a (from their v) with one error	1

Sample answer

$$x=e^{-t}\cos t$$

$$v=\cos t\times\left(-e^{-t}\right)+e^{-t}\left(-\sin t\right)$$

$$v=-e^{-t}\left(\cos t+\sin t\right)$$

$$a=\left(\cos t+\sin t\right)\times\left(e^{-t}\right)+\left(-e^{-t}\right)\times\left(-\sin t+\cos t\right)$$

$$=e^{-t}\left[\cos t+\sin t-\left(-\sin t+\cos t\right)\right]$$

$$=2e^{-t}\sin t$$

Question 16 (b) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $\tan t = -1$	1

Sample answer
At rest when $v = 0$

$\therefore -e^{-t}(\cos t + \sin t) = 0$

$e^{-t} \neq 0 \qquad \cos t + \sin t = 0$

$\tan t = -1$

$t = \frac{3\pi}{4}, \frac{5\pi}{4}$

Question 16 (b) (iii)

Criteria	Mark
• Correct solution	1

Sample answer
Greatest/least velocity occurs when $a = 0$

$\therefore 2e^{-t}\sin t = 0$

$e^{-t} \neq 0 \qquad \sin t = 0$

$t = 0, \pi$

when $t = 0 \quad v = -1$

when $t = \pi \quad v = 1$

Greatest velocity occurs first when $t = \pi$

Question 16 (b) (iv)

Criteria	Mark
• Correct description	1

Sample answer
The particle is oscillating on either side of the origin, with distance on either side diminishing.
The particle is tending towards the origin.

Question 16 (c) (i)

Criteria	Mark
• Correct working	1

Sample answer

$$A_1 = \$300000\left(1+\frac{0.06}{P}\right)^1 - M$$

$$A_2 = \left[\$300000\left(1+\frac{0.06}{P}\right) - M\right]\left(1+\frac{0.06}{P}\right)^1 - M$$

$$= \$300000\left(1+\frac{0.06}{P}\right)^2 - M\left(1+\frac{0.06}{P}\right)^1 - M$$

$$A_2 = \$300000\left(1+\frac{0.06}{P}\right)^2 - M\left[1+\left(1+\frac{0.06}{P}\right)\right]$$

Question 16 (c) (ii)

Criteria	Marks
• Correct solution	3
• Achieves $A_n = \$300000\left(1+\frac{0.06}{P}\right)^n - \$22500\left[\frac{1\left(\left(1+\frac{0.06}{P}\right)^n-1\right)}{0.06}\right]$	2
• Achieves $A_n = \$300000\left(1+\frac{0.06}{P}\right)^n - M\left[1+\ldots\ldots\ldots+\left(1+\frac{0.06}{P}\right)^{n-1}\right]$	1

Sample answer

$$A_n = \$300000\left(1+\frac{0.06}{P}\right)^n - M\left[1+\ldots\ldots\ldots+\left(1+\frac{0.06}{P}\right)^{n-1}\right]$$

$$= \$300000\left(1+\frac{0.06}{P}\right)^n - M\left[\frac{1\left(\left(1+\frac{0.06}{P}\right)^n-1\right)}{\frac{0.06}{P}}\right]$$

$$= \$300000\left(1+\frac{0.06}{P}\right)^n - MP\left[\frac{1\left(\left(1+\frac{0.06}{P}\right)^n-1\right)}{0.06}\right]$$

$$= \$300000\left(1+\frac{0.06}{P}\right)^n - \$22500\left[\frac{1\left(\left(1+\frac{0.06}{P}\right)^n-1\right)}{0.06}\right] \qquad MP = \$22500$$

$$= \$300000\left(1+\frac{0.06}{P}\right)^n - \$375000\left(\left(1+\frac{0.06}{P}\right)^n-1\right)$$

$$= \$300000\left(1+\frac{0.06}{P}\right)^n - \$375000\left(1+\frac{0.06}{P}\right)^n + \$375000$$

$$= \$375000 - \$75000\left(1+\frac{0.06}{P}\right)^n$$

Question 16 (c) (iii)

Criteria	Marks
• Correct solution	2
• Obtains one of the results- monthly or quarterly	1

Sample answer

Monthly $p = 12$

$$\$375000 - \$75000\left(1+\frac{0.06}{12}\right)^n = 0$$

$$\$75000(1.005)^n = \$375000$$

$$n = \frac{\ln 5}{\ln(1.005)}$$

$$= 322.69\ \textit{months}$$

$$= 26.89\ \textit{years}$$

Quartley $p = 4$

$$\$375000 - \$75000\left(1+\frac{0.06}{4}\right)^n = 0$$

$$\$75000(1.015)^n = \$375000$$

$$n = \frac{\ln 5}{\ln(1.015)}$$

$$= 108.09\ \textit{quarters}$$

$$= 27.02\ \textit{years}$$

Monthly will be quicker by 0.13years.