QATs
Quality Assessment Tasks

Centre Number

Student Number

2015 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Answer each question in a SEPARATE writing booklet. Extra writing booklets are available

Total marks – 84

Section I Pages 2-5

10 marks
- Attempt Question 1-10
- Allow about 15 minutes for this section

Section II Pages 6-9

60 marks
- Attempt Question 11-14
- Allow about 1 hour and 45 minutes for this section

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

2015 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION
MATHEMATICS EXTENSION 1

Section I

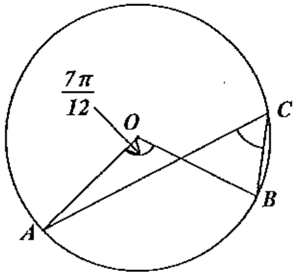
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1-10

1 When the polynomial $P(x)=x^3+2x^2+k$ is divided by $(x+2)$, the remainder is 3. What is the value of k?

(A) 3
(B) -13
(C) 19
(D) 0

2 Points A, B and C lie on a circle centre O. Given that $\angle AOB=\frac{7\pi}{12}$ *radians*, which of the following is the size of $\angle ACB$?



(A) $\frac{7\pi}{6}$

(B) $\frac{7\pi}{12}$

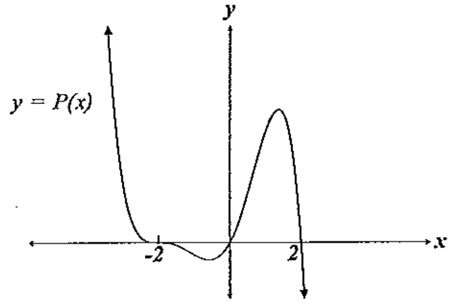
(C) $\frac{14\pi}{3}$

(D) $\frac{7\pi}{24}$

3 Given that $A=(4,-1)$ and $B=(-3,5)$, which of the following divides the interval AB **externally** in the ratio of $3:2$?

(A) $(18,-13)$
(B) $(-1,13)$
(C) $(-17,17)$
(D) $(6,2)$

4 Which of the following could be the equation of the graph $y=P(x)$?



(A) $y=-2x(x-2)(x+2)^3$

(B) $y=ax(2-x)^2(x+2)^2$

(C) $y=-3x^2(x^2-4)$

(D) $y=3x(x+2)^2(x-2)^3$

5 Which of the following is the general solution of $\sin\frac{x}{2}=\frac{\sqrt{3}}{2}$?

(A) $x=4\pi n\pm\frac{\pi}{3}$, where n is an integer

(B) $x=2\pi n+\frac{\pi}{3}$, where n is an integer

(C) $x=2\pi n+(-1)^n\times\frac{2\pi}{3}$, where n is an integer

(D) $x=4\pi n+(-1)^n\times\frac{2\pi}{3}$, where n is an integer

6 A curve is defined by the parameters $x=2\sin\theta,\ y=\cos 2\theta$. Which of the following represents this curve in Cartesian form?

(A) $x^2+y^2=4$

(B) $y=1-\dfrac{x^2}{2}$

(C) $y=2x+1$

(D) $y^2=\dfrac{x}{2}-1$

7 Which term is independent of x in the binomial expansion of $\left(\dfrac{2}{x^9}-3x^3\right)^8$?

(A) ${}^8C_2(2)^6(3)^2$

(B) $28\times4\times729$

(C) ${}^8C_5(2)^3(-3)^5$

(D) ${}^8C_3(2)^3(-3)^5$

8 Which of the following equates to $\sin\left[\cos^{-1}\left(-\dfrac{1}{2}\right)\right]$?

(A) $-\dfrac{\sqrt{3}}{2}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $-\dfrac{1}{2}$

9 Which of the following is the inverse function of $y=x^2-6x+8$?

(A) $y=3\pm\sqrt{x-1}$

(B) $y=3(x+1)$

(C) $y=\sqrt{x+1}$

(D) $y=3+\sqrt{x+1}$

10 A particle moves with velocity $v=\sqrt{9-x^2}$. If initially the particle has displacement $x=3$, which of the following is the displacement equation with respect to t ?

(A) $x=\cos\left(t-\dfrac{\pi}{2}\right)+3$

(B) $x=3\sin\left(t+\dfrac{\pi}{2}\right)$

(C) $x=2-\sin\left(t-\dfrac{\pi}{2}\right)$

(D) $x=3-\cos\left(t+\dfrac{\pi}{2}\right)$

Section II

60 marks
Attempt Question 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, you responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE Writing Booklet.

(a) Find $\lim_{x\to 0}\left(\dfrac{\sin\frac{x}{3}\cos\frac{x}{3}}{\frac{2}{3}}\right)$. **1**

(b) State the domain of $y = 2\cos^{-1}(x+2)$ **2**

(c) (i) Find $\dfrac{d}{dx}(x\sin x)$. **1**

(ii) Hence evaluate $\int_0^{\pi} x\cos x\, dx$. **2**

(d) The acute angle between the line $mx + y - 1 = 0$ and the tangent to the curve $y = \sin^{-1}(3x)$ at $x = 0$ is $45°$. Find the value of m. **3**

(e) Solve $\dfrac{x^2-3}{x} \le -\dfrac{1}{2}$. **3**

(f) At an airport terminal, research has shown that two in every five travel bags are black in colour. If 20 bags were checked in at this airport terminal, write an expression for the probability that at least 18 bags were black in colour. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE Writing Booklet.

(a) Find $\int \dfrac{dx}{4+x^2}$. **2**

(b) (i) Show that the equation $x^3 = \dfrac{1}{x} + 1$ has a root between $x = 1$ and $x = 3$. **1**

(ii) Taking $x_1 = 2$ as a first approximation to this root, use one step of Newton's method to find a better approximation *(correct to 2 decimal places)*. **2**

(c) Evaluate $\int_e^{e^2} \dfrac{dx}{x\ln x}$, Using the substitution $u = \ln x^2$. **3**

(d) The region bounded by $y = \sin\left(x - \dfrac{\pi}{2}\right)$ and the x-axis between $x = 0$ and $x = \dfrac{\pi}{2}$ is rotated about the x-axis to form a solid. Find the volume of this solid in exact form. **3**

(e) The volume V, of a sphere of radius r metres is increasing at a constant rate of $160\,mm^3$ per second. Noting that $v = \dfrac{4}{3}\pi r^3$ and Surface Area $= 4\pi r^2$

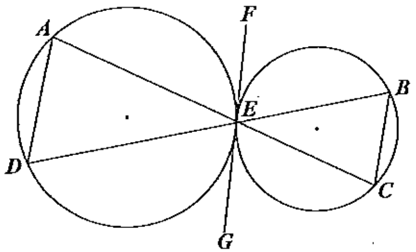
(i) Find $\dfrac{dr}{dt}$ in terms of r. **2**

(ii) Find the rate of change of the surface area A of the sphere when the radius is $40\,mm$. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE Writing Booklet.

(a)



In the diagram above, two circles meet externally at E. FG is a common tangent.

(i) Prove that AD is parallel to BC. **2**

(ii) If $EB=EC$, prove that $ABCD$ is cyclic. **2**

(b) Prove by mathematical induction that $3^{3n}+2^{n+2}$ is divisible by 5 for all integers $n\geq 1$. **3**

(c) Newton's Law of cooling states that when an object at temperature $T°$C is placed in an environment at temperature $T_0°$C, the rate of temperature loss is given by the equation

$$\frac{dT}{dt}k(T-T_0)$$

Where t is the time in seconds and k is a constant.

(i) Show that $T=T_0+Ae^{kt}$. **1**

(ii) A packet of peas, initially at $24°$C is placed in a snap-freeze refrigerator in which the internal temperature is maintained at $-40°$C. Find T_0 and A. **1**

(iii) After 5 seconds, the temperature of the packet is $19°$C. How long will it take for the packet's temperature to reduce to $0°$C. Leave your answer to the nearest second. **2**

(d) From the digits 1-9 (inclusive), four digit numbers are randomly selected without repetition. What is that probability that:

(i) The number selected is greater than 8000 and divisible by 5. **2**

(ii) The number selected is in ascending order. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE Writing Booklet.

(a) (i) Write down the chord of contact from the external point $P(x_0, y_0)$ to the parabola $x^2=12y$. **1**

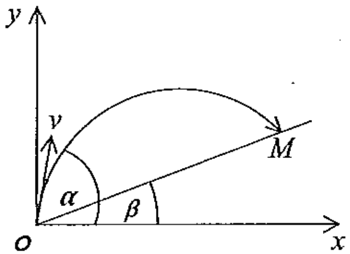
(ii) If this chord meets the parabola at points A and B, show that the x co-ordinate of the midpoint M of AB in the same as that of P. **2**

(b) A particle moves along a straight line with velocity v ms^{-1} such that $v^2=15+10x-5x^2$ where x is the displacement from the origin in *metres*.

(i) Show that the particle travels in Simple Harmonic Motion. **1**

(ii) Find the maximum acceleration of the particle. **2**

(c)



A ball is thrown from O with initial velocity v and angle $\alpha°$ to the horizontal. A fixed straight road passes through O and is on an incline to the horizontal at angle $\beta°$, where $\alpha>\beta$. The ball hits the road at M.
Let the length $OM=d$ *metres* and the acceleration due to gravity is $g\,ms^{-2}$

(i) Given that $x=vt\cos\alpha$ and $y=-\frac{1}{2}gt^2+vt\sin\alpha$, show that the equation of path of the ball is $y=x\tan\alpha-\frac{gx^2\sec^2\alpha}{2v^2}$ **2**

(ii) Write down the co-ordinates of M, in terms of d and β. **1**

(iii) Hence show that $d=\frac{2v^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$. **2**

(iv) Show that the maximum value of d occurs when $\alpha=\frac{1}{2}\left(\beta+\frac{\pi}{2}\right)$ **2**

(v) Show that the maximum value of d is $\frac{v^2}{g(1+\sin\beta)}$. **2**

End of Question 14

QATs

Quality Assessment Tasks

2015 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

MATHEMATICS EXTENSION 1 MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	D
3	C
4	A
5	C
6	B
7	B
8	C
9	D
10	B

Section II

Question 11 (a)

Criteria	Mark
• Correct answer	1

Sample answer

$$\lim_{x\to 0}\left(\frac{\sin\frac{x}{3}\cos\frac{x}{3}}{\frac{2}{3}}\right)=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sin\frac{2x}{3}}{\frac{2}{3}}\right)=\frac{1}{2}$$

Question 11 (b)

Criteria	Marks
• Correct solution	2
• Notes $-1\le x+2\le 1$ or similar	1

Sample answer

$-1\le x+2\le 1$

$-3\le x\le -1$

$\therefore \text{Domain}=\{x:-3\le x\le -1\}$

Question 11 (c) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$$\frac{d}{dx}(x\sin x)=\sin x+x\cos x$$

Question 11 (c) (ii)

Criteria	Marks
• Correct solution	2
• Noting $\int_0^{\pi} x\cos x\,dx = \int_0^{\pi} \sin x + x\cos x - \sin x\,dx$	1

Sample answer

$$\int_0^{\pi} x\cos x\,dx = \int_0^{\pi} \sin x + x\cos x - \sin x\,dx$$
$$= \left[x\sin x + \cos x\right]_0^{\pi}$$
$$= \left[0 - 1 - (0+1)\right]$$
$$= -2$$

Question 11 (d)

Criteria	Marks
• Correct solution	3
• Notes either $\frac{3+m}{1-3m} = 1$ *or* $\frac{3+m}{1-3m} = -1$	2
• Obtains either $m_1 = 3$ *or* $m_2 = -m$	1

Sample answer

$$y = \sin^{-1}(3x)$$
$$y' = \frac{3}{\sqrt{1-9x^2}}$$
$$f'(0) = 3$$

$\therefore m_1 = 3$ and $m_2 = -m$

$$Tan45^{\circ} = \left|\frac{3+m}{1-3m}\right|$$

$\frac{3+m}{1-3m} = 1$ *or* $\frac{3+m}{1-3m} = -1$

$3 + m = 1 - 3m$ $3 + m = 3m - 1$

$4m = -2$ $4 = 2m$

$m = -\frac{1}{2}$ $m = 2$

Question 11 (e)

Criteria	Marks
• Correct solution	3
• Achieves $x(2x+3)(x-2) \le 0$	2
• Notes $2x(x^2-3) \le x^2$	1

Sample answer

$$\frac{x^2-3}{x} \le -\frac{1}{2} \qquad x \ne 0$$
$$2x(x^2-3) \le x^2$$
$$2x^3 - 6x \le x^2$$
$$2x^3 - x^2 - 6x \le 0$$
$$x(2x^2 - x - 6) \le 0$$
$$x(2x+3)(x-2) \le 0$$
$$x \le -\frac{3}{2} \quad or \quad 0 < x \le 2$$

Question 11 (f)

Criteria	Marks
• Correct solution	3
• Obtains at least one of the following: ${}^{20}C_{18}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^{18}$ or ${}^{20}C_{19}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^{19}$ or ${}^{20}C_{20}\left(\frac{2}{5}\right)^{20}$	2
• Notes $P(black) = \frac{2}{5}$ *and* $P\left(\overline{black}\right) = \frac{3}{5}$	1

Sample answer

Let x=0, 1, 2….20 be the number of black bags in the expansion $\left(\frac{3}{5}+\frac{2}{5}\right)^{20}$

$$P(x = 18,19,20) = {}^{20}C_{18}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^{18} + {}^{20}C_{19}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^{19} + {}^{20}C_{20}\left(\frac{2}{5}\right)^{20}$$

Question 12 (a)

Criteria	Marks
• Correct answer	2
• Achieves $a\tan^{-1}\left(\frac{x}{2}\right)$, or similar	1

Sample answer

$$\int \frac{dx}{4+x^2} = \frac{1}{2}\int\left(\frac{2}{4+x^2}\right)dx$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

Question 12 (b) (i)

Criteria	Mark
• Correct working and reasoning	1

Sample answer

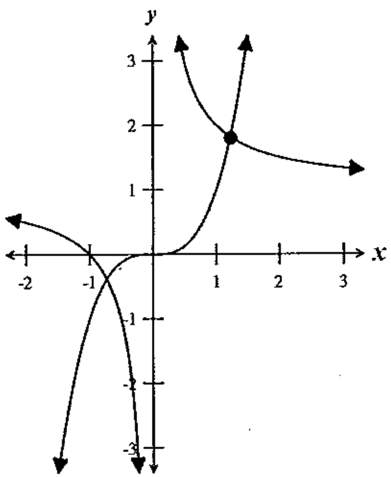
Let $P(x)=x^3-\frac{1}{x}-1$

$P(1)=1-1-1=-1$

$P(3)=27-\frac{1}{3}-1=\frac{77}{3}$

Since $P(1)<0$ and $P(3)>0$ and $P(x)$ is continuous for $1\le x\le 3$, a root exists.

OR graphical answer:



Question 12 (b) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $P(2)=8-\frac{1}{2}-1=6.5$ and $P'(2)=12+\frac{1}{4}=12.25$	1

Sample answer

$P(x)=x^3-\frac{1}{x}-1$

$P'(x)=3x^2+\frac{1}{x^2}$

$P(2)=8-\frac{1}{2}-1=6.5$

$P'(2)=12+\frac{1}{4}=12.25$

$$x_2=x_1-\frac{P(x_1)}{P(x_1)}$$
$$=2-\frac{6.5}{12.25}$$
$$=1.469$$
$$=1.47$$

Question 12 (c)

Criteria	Marks
• Correct solution	3
• Achieves $\int_a^b \frac{du}{u}$	2
• Achieves $\frac{1}{2}du=\frac{dx}{x}$ or $\left[x=e^2\to u=4 \text{ and } x=e\to u=2\right]$	1

Sample answer

$u=\ln x^2$

$u=2\ln x$

$\frac{1}{2}du=\frac{dx}{x}$

$x=e^2 \quad \to u=4$

$x=e \quad \to u=2$

$$Let\ I=\int_e^{e^2}\frac{dx}{x\ln x}$$
$$=\int_2^4\frac{du}{u}$$
$$=[\ln u]_2^4$$
$$=\ln 4-\ln 2$$
$$=\ln 2$$

Question 12 (d)

Criteria	Marks
• Correct solution	3
• Achieves $V=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}1-\cos(2x-\pi)\ dx$	2
• Achieves $V=\pi\int_0^{\frac{\pi}{2}}\sin^2\left(x-\frac{\pi}{2}\right)\ dx$	1

Sample answer

$$y^2=\sin^2\left(x-\frac{\pi}{2}\right)$$
$$V=\pi\int_0^{\frac{\pi}{2}}\sin^2\left(x-\frac{\pi}{2}\right)\ dx$$
$$=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}1-\cos(2x-\pi)\ dx$$
$$=\frac{\pi}{2}\left[x-\frac{1}{2}\sin(2x-\pi)\right]_0^{\frac{\pi}{2}}$$
$$=\frac{\pi}{2}\left[\left(\frac{\pi}{2}-0\right)-(0-0)\right]$$
$$=\frac{\pi^2}{4}\ units^3$$

Question 12 (e) (i)

Criteria	Marks
• Correct solution	2
• Obtains either $\frac{dV}{dt}=160$ or $\frac{dV}{dr}=4\pi r^2$	1

Sample answer

$$\frac{dV}{dt}=160$$
$$V=\frac{4}{3}\pi r^3$$
$$\frac{dV}{dr}=4\pi r^2$$
$$\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$$
$$=\frac{1}{4\pi r^2}\times 160$$
$$=\frac{40}{\pi r^2}$$

Question 12 (e) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $\frac{dA}{dt}=\frac{320}{r}$	1

Sample answer

$$A=4\pi r^2$$
$$\frac{dA}{dr}=8\pi r$$
$$\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}$$
$$=8\pi r\times\frac{40}{\pi r^2}$$
$$=\frac{320}{r}$$

When $r=40$

$$\frac{dA}{dt}=\frac{320}{40}=8\,mms^{-1}$$

Question 13 (a) (i)

Criteria	Marks
• Correct proof	2
• Proves either $\angle AEF = \angle ADE$ or $\angle GEC = \angle EBC$	1

Sample answer

$\angle AEF = \angle ADE$ *(Angle between a tangent and a chord is equal to the angle in the alternate segment)*

$\angle GEC = \angle EBC$ *(Angle between a tangent and a chord is equal to the angle in the alternate segment)*

but $\angle AEF = \angle GEC$ *(vertically opposite angles)*

$\therefore \angle ADE = \angle EBC$

$\therefore AD$ is parallel to BC (alternate angles in parallel lines)

Question 13 (a) (ii)

Criteria	Marks
• Correct proof	2
• Works positively towards proof	1

Sample answer

If $EB = EC$

$\therefore \angle EBC = \angle ECB$

$\therefore$ since $\angle ADE = \angle ECB$

$\therefore$ *ABCD is cyclic* (angles standing on the same arc are equal at the circumference- arc AB)

Question 13 (b)

Criteria	Marks
• Correct proof	3
• Makes a substitution in step 3 (from step 2) and proves LHS=RHS	2
• Provides clear steps similar to the first three steps below	1

Sample answer

Step 1: Prove the expression is true for n=1

$3^3 + 2^3 = 35$ which is divisible by 5

Step 2: Assume the expression is true for n=k

$3^{3k} + 2^{k+2} = 5M$ *where M is an integer*

$3^{3k} = 5M - 2^{k+2}$

Step 3: Prove the expression is true for n=k+1

$f(k+1) = 3^{3k+3} + 2^{k+3}$

$= 27 \times 3^{3k} + 2 \times 2^{k+2}$

$= 27\left[5M - 2^{k+2}\right] + 2 \times 2^{k+2}$ from step 2

$= 135M - 25 \times 2^{k+2}$

$= 5\left[27M - 5 \times 2^{k+2}\right]$

$f(k+1)$ is divisible by 5

Hence if the expression is true when n=k, it is true when n=k+1
But the expression is true for n=1, ∴ it is true when n=2
If true for n=2, ∴ it is true when n=3
Therefore the expression is true for all positive $n \geq 1$.

Question 13 (c) (i)

Criteria	Mark
• Correct working	1

Sample answer

Now $T = T_0 + Ae^{kt}$

$\frac{dT}{dt} = k\left(Ae^{kt}\right)$

$\frac{dT}{dt} = k\left(T - T_0\right)$ since $Ae^{kt} = T - T_0$

Question 13 (c) (ii)

Criteria	Mark
• Correct T_0 & A	1

Sample answer

$T_0 = -40$

When $t = 0$, $24 = -40 + A$

$A = 64$

Question 13 (c) (iii)

Criteria	Marks
• Correct solution	2
• Correctly obtains the value of k	1

Sample answer

$T = -40 + 64e^{kt}$

$19 = -40 + 64e^{5k}$

$\frac{59}{64} = e^{5k}$

$k = -0.01629$

Hence $T = -40 + 64e^{-0.01629t}$

When $T = 0$

$0 = -40 + 64e^{-0.01629t}$

$\ln\left(\frac{40}{64}\right) = -0.01629t$

$t = 28.85\,\text{sec}$

$t = 29\,\text{sec}$

Question 13 (d) (i)

Criteria	Marks
• Correct answer	2
• Correct denominator or numerator	1

Sample answer

$P(E) = \frac{2 \times {}^7P_2}{{}^9P_4}$

$= \frac{84}{3024}$

$= \frac{1}{36}$

Question 13 (d) (ii)

Criteria	Marks
• Corrects answer	2
• Works positively towards answer	1

Sample answer

$P(E) = \frac{{}^9C_4}{{}^9P_4}$

$= \frac{126}{3024}$

$= \frac{1}{24}$

Question 14 (a) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$xx_0 = 2a(y + y_0)$, $a = 3$

$\therefore xx_0 = 6(y + y_0)$

Question 14 (a) (ii)

Criteria	Marks
• Correct solution	2
• Achieves $x^2 = 2xx_0 - 12y_0 = 0$	1

Sample answer

$Now\, xx_0 = 6(y + y_0)$

$\therefore \frac{xx_0}{6} - y_0 = y$ (1)

and $x^2 = 12y$ (2)

Substitute (1) in (2)

$\therefore x^2 = 12\left(\frac{xx_0}{6} - y_0\right)$

$x^2 = 2xx_0 - 12y_0 = 0$

Let the roots of the above equation $= \alpha$ and β

$\therefore \alpha + \beta = 2x_0$

Now the x coordinates of $M = \frac{\alpha + \beta}{2} = x_0$

Hence the x co-ordinate of the midpoint M of AB in the same as that of P.

Question 14 (b) (i)

Criteria	Mark
• Achieves $\ddot{x} = -5(x-1)$	1

Sample answer

$$v^2 = 15 + 10x - 5x^2$$

$$\frac{1}{2}v^2 = \frac{15}{2} + 5x - \frac{5x^2}{2}$$

$$\ddot{x} = 5 - 5x$$

$$\ddot{x} = -5(x-1)$$

Since $\ddot{x} = -n^2 X$.

Therefore the particle executes SHM.

Question 14 (b) (ii)

Criteria	Marks
• Correct answer	2
• Notes that $\ddot{x}_{max}$ occurs when v=0 or similar	1

Sample answer

Maximum acceleration occurs when $v = 0$

$$15 + 10x - 5x^2 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = -1, 3$$

When $x = -1$, $\ddot{x}_{max} = -5(-2) = 10ms^{-2}$

Question 14 (c) (i)

Criteria	Marks
• Correct solution	2
• Achieves $y = -\frac{1}{2}g\left[\frac{x}{vcos\alpha}\right]^2 + v\left[\frac{x}{vcos\alpha}\right]\sin\alpha$	1

Sample answer

$x = vt\cos\alpha$ (1)

$y = -\frac{1}{2}gt^2 + vt\sin\alpha$ (2)

Substitute (1) in (2)

$$\therefore y = -\frac{1}{2}g\left[\frac{x}{vcos\alpha}\right]^2 + v\left[\frac{x}{vcos\alpha}\right]\sin\alpha$$

$$= x\tan\alpha - \frac{gx^2}{2v^2\cos^2 x}$$

$$y = x\tan\alpha - \frac{gx^2\sec^2 x}{2v^2}$$

Question 14 (c) (ii)

Criteria	Mark
• Correct answer	1

Sample answer

$$\cos\beta = \frac{x}{d}$$

$$\therefore x = d\cos\beta$$

$$\sin\beta = \frac{y}{d}$$

$$\therefore y = d\sin\beta$$

$$\therefore M = (d\cos\beta, d\sin\beta)$$

Question 14 (c) (iii)

Criteria	Marks
• Correct solution	2
• Works positively towards answer	1

Sample answer

Substitute $M = (d\cos\beta, d\sin\beta)$ into $\quad y = x\tan\alpha - \dfrac{gx^2\sec^2 x}{2v^2}$

$$d\sin\beta = d\cos\beta\tan\alpha - \frac{g(d\cos\beta)^2}{2v^2\cos^2\alpha}$$

$$\sin\beta = \cos\beta\tan\alpha - \frac{gd\cos^2\beta}{2v^2\cos^2\alpha}$$

$$\frac{gd\cos^2\beta}{2v^2\cos^2\alpha} = \cos\beta\tan\alpha - \sin\beta$$

$$d = \frac{2v^2\cos^2\alpha(\cos\beta\tan\alpha - \sin\beta)}{g\cos^2\beta}$$

$$= \frac{2v^2\cos\alpha(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{g\cos^2\beta}$$

$$d = \frac{2v^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$$

Question 14 (c) (iv)

Criteria	Marks
• Correct solution (*no need to prove max*)	2
• Achieves $d' = \dfrac{2v^2}{g\cos^2\beta}\left[\sin(\alpha-\beta)\times -\sin\beta + \cos\alpha\cos(\alpha-\beta)\right]$	1

Sample answer

$$d' = \frac{2v^2}{g\cos^2\beta}\left[\sin(\alpha-\beta)\times -\sin\beta + \cos\alpha\cos(\alpha-\beta)\right]$$

$$= \frac{2v^2}{g\cos^2\beta}\cos\left[\alpha + (\alpha-\beta)\right]$$

$$= \frac{2v^2}{g\cos^2\beta}\cos(2\alpha-\beta)$$

Stationary points occurs when $d' = 0$

$$\frac{2v^2}{g\cos^2\beta}\cos(2\alpha-\beta) = 0$$

$$\cos(2\alpha-\beta) = 0$$

$$2\alpha - \beta = \frac{\pi}{2}$$

$$\alpha = \frac{1}{2}\left(\beta + \frac{\pi}{2}\right)$$

$$d'' = \frac{-4v^2}{\cos^2\beta}\sin(2\alpha-\beta)$$

$$d''\left[\frac{1}{2}\left(\beta+\frac{\pi}{2}\right)\right] = \frac{-4v^2}{\cos^2\beta}\sin\left(\frac{\pi}{2}\right)$$

$$= \frac{-4v^2}{\cos^2\beta} < 0 \qquad \therefore \text{max}$$

Question 14 (c) (v)

Criteria	Marks
• Correct solution	2
• Works positively towards solution.	1

Sample answer

Substitute $\alpha = \frac{1}{2}\left(\beta + \frac{\pi}{2}\right)$ into $d = \frac{2v^2 \cos\alpha \sin(\alpha - \beta)}{g\cos^2\beta}$

$$d = \frac{2v^2 \cos\left(\frac{\beta}{2} + \frac{\pi}{4}\right)\sin\left(\left(\frac{\beta}{2} + \frac{\pi}{4}\right) - \beta\right)}{g\cos^2\beta}$$

$$= \frac{2v^2 \cos\left(\frac{\beta}{2} + \frac{\pi}{4}\right)\sin\left(\frac{\pi}{4} - \frac{\beta}{2}\right)}{g\cos^2\beta}$$

$$= \frac{2v^2\left[\cos\frac{\beta}{2} \times \frac{1}{\sqrt{2}} - \sin\frac{\beta}{2} \times \frac{1}{\sqrt{2}}\right]\left[\frac{1}{\sqrt{2}}\cos\frac{\beta}{2} - \frac{1}{\sqrt{2}}\sin\frac{\beta}{2}\right]}{g\cos^2\beta}$$

$$= \frac{\frac{2}{2}v^2\left[\cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right]\left[\cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right]}{g\left(1 - \sin^2\beta\right)}$$

$$= \frac{v^2\left[\cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right]^2}{g\left(1 - \sin^2\beta\right)}$$

$$= \frac{v^2\left[\cos^2\frac{\beta}{2} - 2\sin\frac{\beta}{2}\cos\frac{\beta}{2} + \sin^2\frac{\beta}{2}\right]^2}{g\left(1 - \sin^2\beta\right)}$$

$$= \frac{v^2\left[1 - \sin\beta\right]}{g\left(1 - \sin\beta\right)\left(1 + \sin\beta\right)}$$

$$= \frac{v^2}{g\left(1 + \sin\beta\right)}$$