QATs
Quality Assessment Tasks

Centre Number

Student Number

2105 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 100

Section I Pages 2-5

10 marks

- Attempt questions 1-10
- Allow about 15 minutes for this section

Section II Pages 6-12

90 marks

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

STANDARD INTEGRALS

$\int x^n dx = \frac{1}{n+1}x^{n+1}, \; n \neq -1; \; x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x} dx = \ln x, \; x > 0$

$\int e^{ax} dx = \frac{1}{a}e^{ax}, \; a \neq 0$

$\int \cos ax \, dx = \frac{1}{a}\sin ax, \; a \neq 0$

$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \; a \neq 0$

$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \; a \neq 0$

$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \; a \neq 0$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \; a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \; a > 0, \; -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right), \; x > a > 0$

$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x, \; x > 0$

2015 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION
MATHEMATICS

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1-10

1 Which of the following is a solution to $5p^2-3p-5=0$?

(A) $\dfrac{-3\pm\sqrt{109}}{10}$

(B) $\dfrac{3\pm\sqrt{109}}{10}$

(C) $\dfrac{3\pm\sqrt{103}}{5}$

(D) $\dfrac{-3\pm\sqrt{103}}{5}$

2 Which expression is a factorisation of $27x^3-1$?

(A) $(3x-1)(9x^2-3x+1)$

(B) $(3x+1)(9x^2+6x+1)$

(C) $(3x+1)(9x^2-6x+1)$

(D) $(3x-1)(9x^2+3x+1)$

3 Which of the following is the evaluation of $2\sin\left(\dfrac{\pi}{7}\right)$?

(A) 0.868

(B) 0.016

(C) 0.0157

(D) 0.782

4 What is the derivative of $\ln(3x^2-1)$?

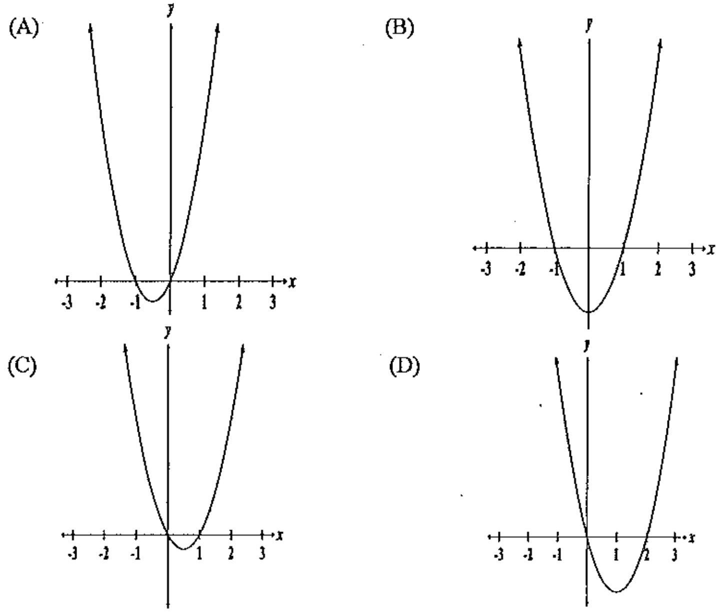
(A) $\dfrac{6}{3x-1}$

(B) $\dfrac{2}{x-1}$

(C) $\dfrac{6x}{3x^2-1}$

(D) $\dfrac{2x}{x^2-1}$

5 Which of the following graphs could be the graph of $y=x^2-x$?



6 Which of the following is a solution to the equation $|x-6|=2x$?

(A) $x=-6,2$

(B) $x=2,6$

(C) $x=-6$

(D) $x=2$

7 In a lottery there are 3 prizes and 50 tickets are sold. Xena buys 5 tickets. What is the probability that Xena wins all three prizes? *(note: once a ticket is chosen, it is not replaced).*

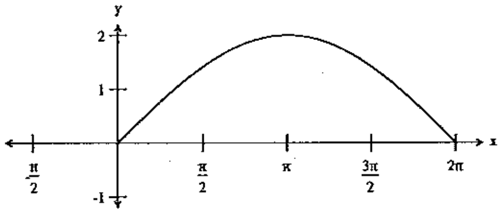
(A) $\frac{957}{19600}$

(B) $\frac{1}{19600}$

(C) $\frac{5}{19600}$

(D) $\frac{1}{1000}$

8 Which of the following could be the equation the graph below for $0 \le x \le 2\pi$?



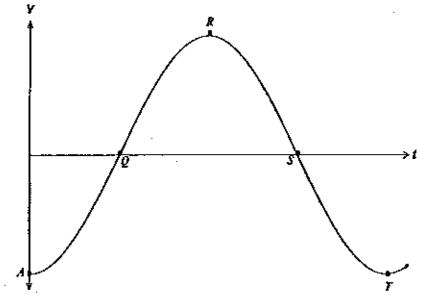
(A) $y=2\sin\left(\frac{x}{2}\right)$

(B) $y=2\sin(2x)$

(C) $y=\sin\left(\frac{x}{2}\right)+1$

(D) $y=2\cos\left(\frac{x}{2}\right)$

9 The graph of the **velocity** v of a particle moving along a straight line as a function of time t.



At which point does the maximum acceleration of the particle occur?

(A) S

(B) R

(C) T

(D) Q

10 In a geometric progression, the sum of the first two terms is 20 and the limiting sum is 36. Which of the following is the common ratio?

(A) $\pm\frac{1}{3}$

(B) $\pm\frac{\sqrt{5}}{3}$

(C) $\pm\frac{2}{3}$

(D) $\pm\frac{3}{\sqrt{5}}$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate Writing Booklet.

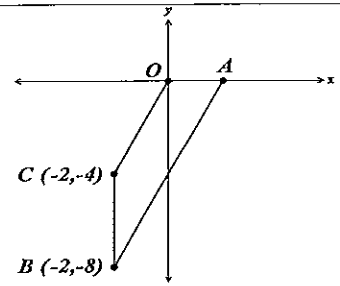
(a)	Rationalise the denominator of $\frac{1}{\sqrt{11}-3}$.	2
(b)	Solve the following equation $\frac{1}{3}(x+8)=\frac{1}{2}(2-x)$.	2
(c)	Evaluate $\lim_{x\to 3}\left(\frac{x^2+2x-15}{x^2-9}\right)$.	2
(d)	Differentiate $(3x^2-2)^4$.	2
(e)	Find $\int \frac{4x}{4x^2+3}\, dx$.	2
(f)	Evaluate $\int_{\frac{\pi}{2}}^{\pi} \sin\left(\frac{x}{3}\right)\, dx$.	3
(g)	Graph the region defined by $\frac{x}{2}+\frac{y}{3}\le 1$.	2

End of Question 11

Question 12 (15 marks) Use a separate Writing Booklet.

(a)	A bag contains four green marbles and five yellow marbles. Two marbles are chosen at random without replacement.		
	(i)	What is the probability that the 1st marble chosen is green.	1
	(ii)	What is the probability that the 2 marbles chosen are of the same colour.	2

(b)



In the diagram above $B=(-2,-8), C=(-2,-4)$ and O is the origin. OC is parallel to AB and A lies on the x-*axis*.

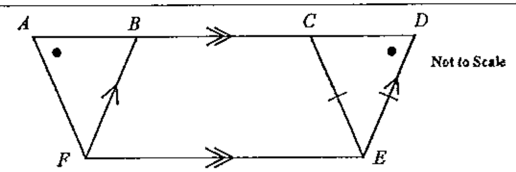
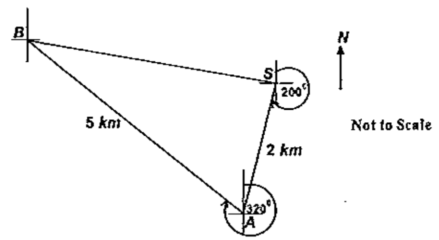
(i)	Find the gradient of OC	1
(ii)	Show that the equation of AB is $2x-y-4=0$.	2
(iii)	Hence find the co-ordinates of A.	1
(iv)	Find the length of AB.	1
(v)	Find the perpendicular distance from O to the line AB.	1
(vi)	Hence find the area of $OABC$.	2

(c) Zainab opened a popular ice cream shop in a busy arcade. On opening day, she sold 10 scoops of ice cream. The number of scoops sold each subsequent day increased so that each subsequent day's total sales formed an arithmetic progression. Zainab sold 100 scoops of ice cream on the 31st day of operation.

(i)	Show that the common difference is 3.	1
(ii)	Find the total number of scoops sold in the first 31 days of operation.	1
(iii)	Zainab makes a profit of \$1 per scoop sold. On which day will she reach an overall total profit of \$2500 from the first day of operation.	2

End of Question 12

Question 13 (15 marks) Use a separate Writing Booklet.

(a)	Find the domain of the function $f(x)=\frac{1}{\sqrt{4-x}}$		1
(b)	 In the above diagram $BDEF$ is a parallelogram. A and C lie on the line $ABCD$. $EC=ED$ and $\angle FAB=\angle EDC$.		
	(i)	Prove that $BF=EC$.	1
	(ii)	Prove that $\angle ABF=\angle CDE$.	2
	(iii)	Hence prove that $\Delta ABF\equiv\Delta CDE$.	3
(c)	A point $P(x,y)$ moves so that it is always equidistant from $A(-3,1)$ and $B(2,-4)$.		
	(i)	By finding the distances of AP and PB, find the equation of the locus of P.	2
	(ii)	Show that the locus of P is the perpendicular bisector of AB.	2
(d)	 Matais is competing in an orienteering competition. He commences at the start S and travels for 2 *km* on a bearing of $200°T$ to point A. He then travels for 5 *km* at a bearing of $320°T$ to Point B.		
	(i)	Show that $\angle BAS=60°$.	1
	(ii)	Hence find the exact length of SB.	2
	(iii)	Matais now needs to return to S from his current position at B. Find the bearing (*to the nearest degree*) he must travel on.	1

End of Question 13

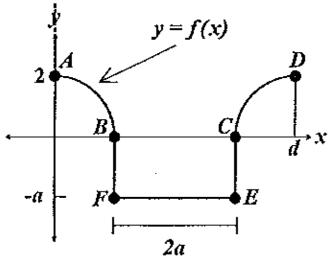
Question 14 (15 marks) Use a separate Writing Booklet.

(a)		Given that $\sin 2\theta = -\frac{\sqrt{3}}{2}$, find the exact value of θ, for $0 \le \theta \le 2\pi$	2
(b)		Find the value of k so that the quadratic equation $2x^2 + (k+3)x + (k+1) = 0$ has equal roots.	3
(c)		On a Saturday at a local municipal library, the number (N) of visitors to the library at any given time over a $2\frac{1}{2}$ hour period (*t hours*) is given by: $N = 2t^3 - 9t^2 + 12t + 25;\ \ 0 \le t \le 2.5$	
	(i)	Find the initial number of visitors to the library.	1
	(ii)	Find the local maximum and local minimum number of visitors to the library during this time.	3
	(iii)	Find the time when the number of visitors to the library was decreasing most rapidly.	2
	(iv)	Hence, neatly sketch $N = 2t^3 - 9t^2 + 12t + 25;\ \ 0 \le t \le 2.5$, showing all essential features.	2
(d)		Solve the following equation $e^{2x} - 5e^x + 4 = 0$	2

End of Question 14

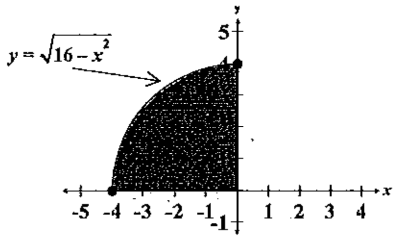
Question 15 (15 marks) Use a separate Writing Booklet.

(a) **2**



$ABFECD$ is a continuous curve and is denoted as $y = f(x)$. AB and CD are arcs of a circle with the same radius. $BCEF$ is a rectangle. If $\int_0^d f(x)\ dx = -6\pi$, find the value of a.

(b)



(i) Copy and complete the table below *(leaving your answers to three decimal places)*. **1**

x	-4	-3	-2	-1	0
$\sqrt{16-x^2}$					

(ii) Use the above table (five ordinates) and Simpson's rule to evaluate approximately $\int_{-4}^{0} \sqrt{16 - x^2}\ dx$. **2**

(iii) If the Trapezoidal Rule was used in part (ii), the answer would have been smaller. Give a brief reason for this in relation to the graph above. **1**

(c) The price ($$P$) of a particular share on the Australian stock market was severely negatively affected due to the mismanagement of the company. From the 1st September, 2015 the price of a share was given $P = M_0 e^{-kt}$, where M and k are constants and t is measured in days. You may assume that shares are traded every day of the year.

(i) If the share price lost half its value in the first 30 days from 1 September 2015, find the value of k *(answer to 3 decimal places)*. **2**

Question 15 continues over the page

Question 15 (continued)

(ii) A person who bought her shares on 1 September, 2105 decides to sell her shares when 35% of the value of her investment has been lost. On what date will this occur. **2**

(d) A particle moves along a straight line so that its displacement x *metres* from a fixed point O is given by $x=\frac{\ln(t+1)}{(t+1)}$, where t is measured in seconds.

(i) Find an expression for v. **2**

(ii) Show that the particle is stationary at $t=(e-1)$ *seconds*, and this is the maximum velocity. **2**

(iii) Describe the motion of the particle after $t=(e-1)$ *seconds*, as $t\to\infty$. **1**

End of Question 15

Question 16 (15 marks) Use a separate Writing Booklet.

(a) Arjun is playing a game on his mobile phone called "*Silly bird*". If he scores 50 or more points in one round he wins the game and if he scores 2 or less points in a round he loses, is eliminated and must stop playing. If he scores more than 2 and less that 50, he is allowed to play another round.

In any one round, the probabilities that that Arjun scores 50 or more points is $\frac{1}{10}$ and 2 or less points is $\frac{2}{5}$ respectively.

(i) Show that the probability of Arjun playing a second round is $\frac{1}{2}$. **1**

(ii) Find the probability that Arjun will win in his second round. **1**

(iii) Find the probability that Arjun will win the game before being eliminated. **3**

(b) (i) On the same number plane graph the following equations: **2**

$y=-\sin 2x$ *and* $y=\cos 2x;\ 0\le x\le\frac{\pi}{2}$

(ii) When the two graphs in (i) are added, they generate a new curve with the following equation $y=\cos 2x-\sin 2x$; $0\le x\le\frac{\pi}{2}$. Find where this new curve cuts the $x-axis$. **1**

(ii) Noting that $\sin 4x=2\cos 2x\sin 2x$; find the exact volume when the area bounded by $y=\cos 2x-\sin 2x$; $0\le x\le\frac{\pi}{2}$ and the coordinate axes is rotated about the $x-axis$. **3**

(c) Emma and Ben buy an apartment and borrow $400,000. Bank A and Bank B have similar offers. They are asked to repay the loan plus interest over 30 years at r % pa. The agreement is that they will pay P regular payments annually each of $\$M$. Interest is calculated and charged immediately before each repayment.

(i) Show that the amount owing after n repayments is given as **2**

$$A_n=400,000\left(1+\frac{r}{100P}\right)^n-100MP\left\{\frac{\left(1+\frac{r}{100P}\right)^n-1}{r}\right\}$$

(ii) If Bank A is offering an interest rate of 6%p.a. to be repaid quarterly and Bank B is offering 6.5%p.a. to be repaid monthly, which is the cheaper offer. Justify your answer. **2**

End of paper

QATs
Quality Assessment Tasks

2015 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

MATHEMATICS – MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	D
3	A
4	C
5	C
6	D
7	B
8	A
9	D
10	C

Section II

Question 11 (a)

Criteria	Marks
• Multiplies by correct conjugate and achieves correct solution	2
• Multiplies by correct conjugate for numerator and denominator	1

Sample answer

$$\frac{1}{\sqrt{11}-3}=\frac{1}{\sqrt{11}-3}\times\frac{\sqrt{11}+3}{\sqrt{11}+3}$$
$$=\frac{\sqrt{11}+3}{2}$$

Question 11 (b)

Criteria	Marks
• Correct solution	2
• Correctly eliminates fractions from question	1

Sample answer

$$\frac{1}{3}(x+8)=\frac{1}{2}(2-x)$$
$$2(x+8)=3(2-x)$$
$$2x+16=6-3x$$
$$5x=-10$$
$$x=-2$$

Question 11 (c)

Criteria	Marks
• Correct solution	2
• Correctly factors both the numerator and denominator	1

Sample answer

$$\lim_{x\to 3}\left(\frac{x^2+2x-15}{x^2-9}\right)=\lim_{x\to 3}\left(\frac{(x+5)(x-3)}{(x-3)(x+3)}\right)$$
$$=\lim_{x\to 3}\left(\frac{(x+5)}{(x+3)}\right)$$
$$=\frac{4}{3}$$

Question 11 (d)

Criteria	Marks
• Correct solution	2
• Uses the Chain Rule with one error	1

Sample answer

$$\frac{d}{dx}\left(3x^2-2\right)^4 = 4\left(3x^2-2\right)^3 \times 6x$$

$$= 24x\left(3x^2-2\right)^3$$

Question 11 (e)

Criteria	Marks
• Correct answer	2
• Appreciates that the integral is $\ln\left(f(x)\right)$	1

Sample answer

$$\int \frac{4x}{4x^2+3}\ dx = \frac{1}{2}\ln\left(4x^2+3\right)+c$$

Question 11 (f)

Criteria	Marks
• Correct solution	3
• Correct integration or missing a multiple of 3 • Correct substitution into their integration	2
• Correct integration or missing a multiple of 3	1

Sample answer

$$\int_{\frac{\pi}{2}}^{\pi} \sin\left(\frac{x}{3}\right)\ dx = -3\left[\cos\frac{x}{3}\right]_{\frac{\pi}{2}}^{\pi}$$

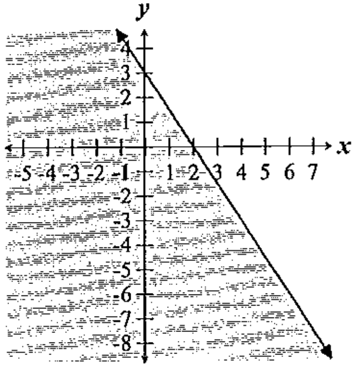
$$= -3\left[\cos\frac{\pi}{3} - \cos\frac{\pi}{6}\right]$$

$$= -3\left[\frac{1}{2} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{3\left(\sqrt{3}-1\right)}{2}$$

Question 11 (g)

Criteria	Marks
• Correct solution	2
• Correct drawing of line or correct shading	1



Question 12 (a) (i)

Criteria	Mark
• Correct answer	1

Sample answer

Answer $= \dfrac{4}{9}$

Question 12 (a) (ii)

Criteria	Marks
• Correct solution	2
• Achieves the correct answer for either $P(GG)$ *or* $P(YY)$	1

Sample answer

$$P(\text{both the same colour}) = P(GG)+P(YY)$$

$$= \left(\frac{4}{9}\times\frac{3}{8}\right)+\left(\frac{5}{9}\times\frac{4}{8}\right)$$

$$= \frac{1}{6}+\frac{5}{18}$$

$$= \frac{4}{9}$$

Question 12 (b) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$m_{OC}=\frac{-4}{-2}=2$

Question 12 (b) (ii)

Criteria	Marks
• Correct solution	2
• Uses the correct formulae with one error	1

Sample answer

$m_{AB}=2 \quad (m_{OC}=m_{AB}=2)$

$\therefore y+8=2(x+2)$

$y+8=2x+4$

$2x-y-4=0$

Question 12 (b) (iii)

Criteria	Mark
• Correct answer	1

Sample answer

Substitute $y=0$

$2x-4=0$

$x=2$

$\therefore A=(2,0)$

Question 12 (b) (iv)

Criteria	Mark
• Correct answer	1

Sample answer

$\text{Distance}_{AB}=\sqrt{(2+2)^2+(0+8)^2}$

$=\sqrt{16+64}$

$=\sqrt{80}$

$=4\sqrt{5}\ units$

Question 12 (b) (v)

Criteria	Mark
• Correct answer	1

Sample answer

$\text{Distance}=\left|\frac{-4}{\sqrt{4+1}}\right|$

$=\frac{4}{\sqrt{5}}\ units$

Question 12 (b) (vi)

Criteria	Marks
• Correct solution (correct numerical expression)	2
• Obtains the length of OC or substitutes in correct formulae	1

Sample answer

$\text{Distance}_{OC}=\sqrt{4+16}$

$=\sqrt{20}$

$=2\sqrt{5}$

$A=\frac{1}{2}\times\frac{4}{\sqrt{5}}\left[4\sqrt{5}+2\sqrt{5}\right]$

$=\frac{1}{2}(16+8)$

$=12 units^2$

Question 12 (c) (i)

Criteria	Mark
• Correct solution	1

Sample answer

$T_n=a+(n-1)d$

$100=10+30d$

$90=30d$

$d=3$

Question 12 (c) (ii)

Criteria	Mark
• Correct solution	1

Sample answer

$S_n = \frac{n}{2}\{a+l\}$

$S_{31} = \frac{31}{2}\{10+100\}$

$= 1705$

Question 12 (c) (iii)

Criteria	Marks
• Correct solution	2
• Achieves $2500 = \frac{n}{2}\{20+(n-1)3\}$	1

Sample answer

$2500 = \frac{n}{2}\{20+(n-1)3\}$

$5000 = n\{3n+17\}$

$3n^2 + 17n - 5000 = 0$

$n = \frac{-17 \pm \sqrt{289+60000}}{6}$

$= \frac{-17 \pm \sqrt{60289}}{6}$

$n = 38.089 \quad (n > 0)$

Overall total reached on 39th day of operation.

Question 13 (a)

Criteria	Mark
• Correct answer	1

Sample answer

$4 - x > 0$

$x < 4$

$Domain = \{x : x < 4\}$

Question 13 (b) (i)

Criteria	Mark
• Correct solution	1

Sample answer

$BF = DE$ (opposite sides of a parallelogram are equal)

$Since\ CE = DE$ $(given)$

$\therefore BF = CE$

Question 13 (b) (ii)

Criteria	Marks
• Correct solution	2
• Noting $\angle BFE = \angle CDE$ or similar	1

Sample answer

$\angle BFE = \angle CDE$ (opposite angles of a parallelogram are equal)

$\angle ABF = \angle BFE$ (alternate angles in parallel lines, AD parallel to FE)

$\therefore \angle ABF = \angle CDE$

Question 13 (b) (iii)

Criteria	Marks
• Correct solution	3
• Correct proof without reasons	2
• Works positively towards solution	1

Sample answer

Note

$\angle DCE = \angle CDE$ (base angles of an isosceles triangle are equal)

Hence from (ii)

$\angle BAF = \angle ABF = \angle DCE = \angle CDE$

To prove:

$\Delta ABF \equiv \Delta CDE$

$\angle FAB = \angle ECD$ $(proven\ above)$

$\angle ABF = \angle CDE$ $(proven\ above)$

$BF = EC$ $(proven\ above)$

$\therefore \Delta ABF \equiv \Delta CDE\ (AAS)$

Question 13 (c) (i)

Criteria	Marks
• Correct solution	2
• Obtains Distance_{AP} and Distance_{BP}	1

Sample answer

$\text{Distance}_{AP} = \sqrt{(x+3)^2 + (y-1)^2}$

$\text{Distance}_{BP} = \sqrt{(x-2)^2 + (y+4)^2}$

Now $\text{Distance}_{AP} = \text{Distance}_{BP}$

$\therefore \sqrt{(x+3)^2 + (y-1)^2} = \sqrt{(x-2)^2 + (y+4)^2}$

$(x+3)^2 + (y-1)^2 = (x-2)^2 + (y+4)^2$

$x^2 + 6x + 9 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 + 8y + 16$

$10x - 10y - 10 = 0$

$x - y - 1 = 0$

Question 13 (c) (ii)

Criteria	Marks
• Correct answer	2
• Proves either perpendicular or bisector	1

Sample answer

For $x - y - 1 = 0 \quad m = 1$

$m_{AB} = \dfrac{1+4}{-3-2} = -1$

$\therefore m_{AB} \times m = -1$

$-1 \times 1 = -1$

Also $\text{midpoint}_{AB} = \left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$

Substitute $\left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$ in $x - y - 1 = 0$

$-\dfrac{1}{2} + \dfrac{3}{2} - 1 = 0$

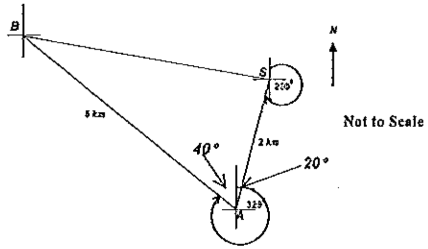
$0 = 0$

Therefore the locus of P is the perpendicular bisector of AB.

Question 13 (d) (i)

Criteria	Mark
• Correct working	1

Sample answer



$\angle BAS = 40° + 20° = 60°$ (see diagram)

Question 13 (d) (ii)

Criteria	Marks
• Correct solution	2
• Substitutes correctly into the cosine rule	1

Sample answer

$SB^2 = 5^2 + 2^2 - (2 \times 5 \times 2 \times \cos 60^0)$

$= 29 - 10$

$= 19$

$SB = \sqrt{19}\ km$

Question 13 (d) (iii)

Criteria	Mark
• Correct solution	1

Sample answer

$$\frac{\sin\theta}{2}=\frac{\sin 60}{\sqrt{19}}$$

$$\sin\theta=\frac{2\sin 60}{\sqrt{19}}$$

$$\sin\theta=\sqrt{\frac{3}{19}}$$

$$\theta=23^0 25'$$

$$\cong 23^0$$

$$Bearing=180-(40+23)$$

$$=117^\circ T$$

Question 14 (a)

Criteria	Marks
• Correct solution	2
• Achieves two correct angles	1

Sample answer

$$\sin 2\theta=-\frac{\sqrt{3}}{2}$$

$$2\theta=240^\circ,300^\circ,600^0,660^\circ$$

$$\theta=120^\circ,150^\circ,300^0,330^\circ$$

Question 14 (b)

Criteria	Marks
• Correct solution	3
• Achieves $k^2-2k+1=0$	2
• Obtains the discriminant or notes that that equal roots occur when $\Delta=0$	1

Sample answer

$$\Delta=(k+3)^2-8(k+1)$$

$$=k^2+6k+9-8k-8$$

$$=k^2-2k+1$$

Equal roots occur when $\Delta=0$

$$k^2-2k+1=0$$

$$(k-1)^2=0$$

$$k=1$$

Question 14 (c) (i)

Criteria	Mark
• Correct answer	1

Sample answer

$$N=2t^3-9t^2+12t+25$$

When $t=0$ $N=25$

Question 14 (c) (ii)

Criteria	Marks
• Correct solution	3
• Obtains stationary points	2
• Achieves both N' and N''	1

Sample answer

$N = 2t^3 - 9t^2 + 12t + 25$

$N' = 6t^2 - 18t + 12$

$N'' = 12t - 18$

Stationary points occur when $N' = 0$

$6t^2 - 18t + 12 = 0$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

Check concavity

$N''(1) = -6$ maximum

$N''(2) = 6$ minimum

Local Maximum $N(1) = 2 - 9 + 12 + 25 = 30$

Local Maximum $N(2) = 16 - 36 + 24 + 25 = 29$

Question 14 (c) (iii)

Criteria	Marks
• Correct solution and noting that $N' < 0$	2
• Achieves both N' and N''	1

Sample answer

Occurs at point of inflection

$N'' = 0$

$12t - 18 = 0$

$t = \frac{3}{2}$

t	1	$\frac{3}{2}$	2
$N''(t)$	-6	0	6

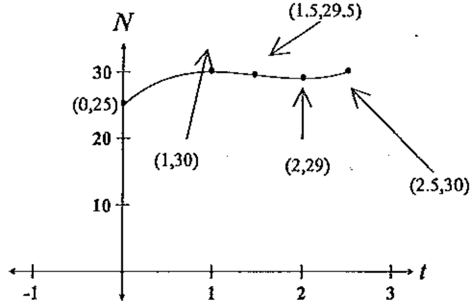
Note also $N'\left(\frac{3}{2}\right) = \frac{27}{2} - 27 + 12 = -\frac{3}{2}$ *(decreasing)*

Question 14 (c) (iv)

Criteria	Marks
• Correct solution	2
• Works positively towards answer	1

Sample answer

$N(2.5) = 30$



Question 14 (d)

Criteria	Marks
• Correct solution	2
• Achieves $e^x = 4$ *or* $e^x = 1$	1

Sample answer

$e^{2x} - 5e^x + 4 = 0$

$(e^x - 4)(e^x - 1) = 0$

$e^x = 4$ *or* $e^x = 1$

$x = \ln 4$ *or* $x = 0$

Question 15 (a)

Criteria	Marks
• Correct solution	2
• Notes the area of a semicircle and area of rectangle	1

Sample answer

$\int_0^d f(x)\, dx = \frac{1}{2}\pi(2)^2 - 2a^2 = -6\pi$

$\therefore 2\pi - 2a^2 = -6\pi$

$2a^2 = 8\pi$

$a^2 = 4\pi$

$a = 2\sqrt{\pi} \quad (a > 0)$

Question 15 (b) (i)

Criteria	Mark
• Correct answer (allow one error)	1

Sample answer

x	-4	-3	-2	-1	0
$\sqrt{16-x^2}$	0	2.646	3.464	3.873	4

Question 15 (b) (ii)

Criteria	Marks
• Correct answer	2
• Uses correct weightings and finds the relevant ordinates	1

Sample answer

x	-4	-3	-2	-1	0	
$\sqrt{16-x^2}$	0	2.646	3.464	3.873	4	
weight	1	4	2	4	1	
result	0	10.584	6.928	15.492	4	37.004

$$\therefore \int_{-4}^{0} \sqrt{16-x^2}\ dx = \frac{1}{3}(37.004)$$

$= 12.335$ (3 dec places)

Question 15 (b) (iii)

Criteria	Mark
• A reasonable graphical explanation	1

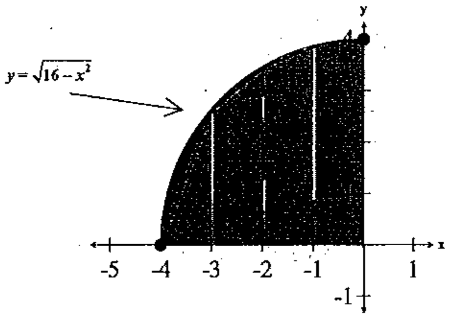
Sample answer

The function is concave down.

Simpson's rule uses parabolas (concave down in this case) to approximate the integral. Hence the approximation would be greater than the exact area.

Trapezoidal rule uses trapeziums (see diagram below).

Hence the answer would be smaller.



Question 15 (c) (i)

Criteria	Marks
• Correct solution	2
• Notes $\frac{1}{2}M_o = M_0e^{-30k}$ or similar	1

Sample answer

$$\frac{1}{2}M_o = M_0e^{-30k}$$
$$0.5 = e^{-30k}$$
$$\ln(0.5) = -30k$$
$$k = 0.023$$

Question 15 (c) (ii)

Criteria	Marks
• Correct solution	2
• Notes $0.65M_o = M_0e^{-0.023t}$ or similar	1

Sample answer

$$0.65M_o = M_0e^{-0.023t}$$
$$0.65 = e^{-0.023t}$$
$$\ln(0.65) = -0.023t$$
$$t = 18.73$$

She will sell the shares on September 19, 2015.

Question 15 (d) (i)

Criteria	Marks
• Correct solution	2
• Uses the quotient rule with one mistake	1

Sample answer

$$x=\frac{\ln(t+1)}{(t+1)}$$

$$v=\frac{(t+1)\times\frac{1}{(t+1)}-\ln(t+1)\times 1}{(t+1)^2}$$

$$=\frac{1-\ln(t+1)}{(t+1)^2}$$

Question 15 (d) (ii)

Criteria	Marks
• Correct solution by proving $t=e-1$ is a maximum	2
• Obtains or proves stationary point at $t=e-1$	1

Sample answer

Particle is stationary when v=0.

$$\frac{1-\ln(t+1)}{(t+1)^2}=0$$

$$1-\ln(t+1)=0$$

$$\ln(t+1)=1$$

$$t+1=e$$

$$t=e-1$$

t	$e-2$	$e-1$	e
v	0.155	0	-0.022

Since the velocity changes from positive to negative, $t=e-1$ is a maximum.

Question 15 (d) (iii)

Criteria	Mark
• Clear explanations noting particle will not stop	1

Sample answer

$As\, t\to\infty \quad \frac{\ln(t+1)}{(t+1)}\to 0$

Therefore the particle continues to slow down but never stops.

Question 16 (a) (i)

Criteria	Mark
• Correct working	1

Sample answer

$$P(\text{Arjun playing a second round (draw)})=1-\left(\frac{1}{10}+\frac{2}{5}\right)$$

$$=\frac{1}{2}$$

Question 16 (a) (ii)

Criteria	Mark
• Correct solution	1

Sample answer

Let the $P(\text{Arjun playing another round})=P(Draw)=P(D)$

Let the $P(\text{Arjun winning at any draw})=P(Win)=P(W)$

$\therefore P(\text{Arjun to win in his second round})=P(DW)$

$$=\frac{1}{2}\times\frac{1}{10}$$

$$=\frac{1}{20}$$

Question 16 (a) (iii)

Criteria	Marks
• Correct solution	3
• Reduces working to a limiting sum	2
• Notes $P(\text{Arjun wins}) = P(W) + P(DW) + P(DDW) + P(DDDW) + \ldots\ldots$	1

Sample answer

Let the $P(\text{Arjun wins}) = P(W) + P(DW) + P(DDW) + P(DDDW) + \ldots\ldots$

$$= \frac{1}{10} + \left(\frac{1}{2} \times \frac{1}{10}\right) + \left(\left(\frac{1}{2}\right)^2 \times \frac{1}{10}\right) + \left(\left(\frac{1}{2}\right)^3 \times \frac{1}{10}\right) + \ldots\ldots$$

$$= \frac{1}{10}\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\ldots\right)$$

$$= \frac{1}{10}\left(\frac{1}{1-\frac{1}{2}}\right)$$

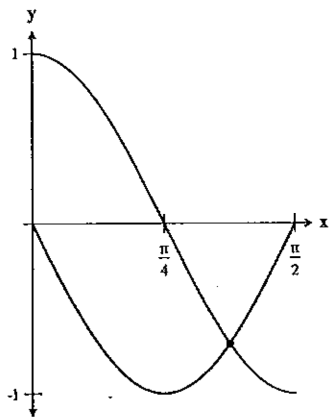
$$= \frac{1}{10} \times 2$$

$$= \frac{1}{5}$$

Question 16 (b) (i)

Criteria	Marks
• Correct graph for both functions	2
• Correct graph for one of the functions	1

Sample answer



Question 16 (b) (ii)

Criteria	Mark
• Correct answer	1

Sample answer

Find the root of $y = \cos 2x - \sin 2x$; $0 \le x \le \frac{\pi}{2}$

$$\cos 2x - \sin 2x = 0$$

$$\tan 2x = 1$$

$$2x = \frac{\pi}{4}$$

$$x = \frac{\pi}{8}$$

Question 16 (b) (iii)

Criteria	Marks
• Correct solution	3
• Achieves $\pi\int_0^{\frac{\pi}{8}}(1-\sin 4x)\,dx$	2
• Achieves $(\cos 2x-\sin 2x)^2=\cos^2 2x-2\cos 2x\sin 2x+\sin^2 2x$	1

Sample answer

$$V=\pi\int_0^{\frac{\pi}{8}}(\cos 2x-\sin 2x)^2\,dx$$

$$=\pi\int_0^{\frac{\pi}{8}}\left(\cos^2 2x-2\cos 2x\sin 2x+\sin^2 2x\right)dx$$

$$=\pi\int_0^{\frac{\pi}{8}}(1-2\cos 2x\sin 2x)\,dx$$

$$=\pi\int_0^{\frac{\pi}{8}}(1-\sin 4x)\,dx$$

$$v=\pi\left[x+\frac{1}{4}\cos 4x\right]_0^{\frac{\pi}{8}}$$

$$=\pi\left[\left(\frac{\pi}{8}+\frac{1}{4}\cos\frac{\pi}{2}\right)-\left(0+\frac{1}{4}\right)\right]$$

$$=\pi\left[\frac{\pi}{8}-\frac{1}{4}\right]$$

$$=\pi\left(\frac{\pi-2}{8}\right)\ units^3$$

Question 16 (c) (i)

Criteria	Marks
• Correct working	2
• $A_n=400,000\left(1+\frac{r}{100P}\right)^n-M\left\{1+\left(1+\frac{r}{100P}\right)+\left(1+\frac{r}{100P}\right)^2+\ldots+\left(1+\frac{r}{100P}\right)^{n-1}\right\}$	1

Sample answer

$$A_1=400,000\left(1+\frac{r}{100P}\right)-M$$

$$A_2=A_1\left(1+\frac{r}{100P}\right)-M$$

$$=\left\{400,000\left(1+\frac{r}{100P}\right)-M\right\}\left(1+\frac{r}{100P}\right)-M$$

$$=400,000\left(1+\frac{r}{100P}\right)^2-M\left(1+\frac{r}{100P}\right)-M$$

$$=400,000\left(1+\frac{r}{100P}\right)^2-M\left\{1+\left(1+\frac{r}{100P}\right)\right\}$$

$$\therefore A_3 = 400,000\left(1+\frac{r}{100P}\right)^3 - M\left\{1+\left(1+\frac{r}{100P}\right)+\left(1+\frac{r}{100P}\right)^2\right\}$$

$$A_n = 400,000\left(1+\frac{r}{100P}\right)^n - M\left\{1+\left(1+\frac{r}{100P}\right)+\left(1+\frac{r}{100P}\right)^2+\left(1+\frac{r}{100P}\right)^3+\dots+\left(1+\frac{r}{100P}\right)^{n-1}\right\}$$

$$= 400,000\left(1+\frac{r}{100P}\right)^n - M\left\{G.P\ a=1,\ \textit{common ratio}=\left(1+\frac{r}{100P}\right),\ \textit{number of terms}=n,\right\}$$

$$= 400,000\left(1+\frac{r}{100P}\right)^n - M\left[\frac{1\left\{\left(1+\frac{r}{100P}\right)^n-1\right\}}{\left(1+\frac{r}{100P}\right)-1}\right]$$

$$= 400,000\left(1+\frac{r}{100P}\right)^n - M\left[\frac{1\left\{\left(1+\frac{r}{100P}\right)^n-1\right\}}{\frac{r}{100P}}\right]$$

$$= 400,000\left(1+\frac{r}{100P}\right)^n - 100MP\left[\frac{\left\{\left(1+\frac{r}{100P}\right)^n-1\right\}}{r}\right]$$

Question 16 (c) (ii)

Criteria	Marks
• Correct solution	2
• Achieves one of the results	1

Sample answer
For Bank A
$r=6\quad P=4\quad A_n=0$

$$0 = 400,000\left[1+\frac{6}{400}\right]^{120} - 400M\left[\frac{\left(1+\frac{6}{400}\right)^{120}-1}{6}\right]$$

$0 = 2387729.15 - 331.2881915M$
$M = \$7,207.41$
Total interest charged $= (\$7207.41\times 120) - \$400,000$
$= \$464,889.96$

For Bank B
$r=6.5\quad P=12\quad A_n=0$

$$0 = 400,000\left[1+\frac{6.5}{1200}\right]^{360} - 1200M\left[\frac{\left(1+\frac{6.5}{1200}\right)^{360}-1}{6.5}\right]$$

$0 = 2796719.19 - 1106.178087M$
$M = \$2,528.27$
Total interest charged $= (\$2528.27\times 360) - \$400,000$
$= \$510,177.94$

Bank A is the cheaper offer.

End of solutions