Centre Number

Student Number

QATs
Quality Assessment Tasks

2014
HSC TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-14

Total marks – 70

Section I Pages 3-6
10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 7-11
60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0, \ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x>0$

Section I (10 marks) **Marks**

(1) Given that $(x-3)$ is a factor of $P(x)=x^3+kx^2-4x-6$, which of the following is the value of k? **1**

(A) $k=1$.

(B) $k=-1$.

(C) $k=\frac{21}{9}$.

(D) $k=3$.

(2) Which of the following equates to $\lim_{x\to 0}\left(\frac{\sin x\cos x}{2x}\right)$? **1**

(A) 2.

(B) $\frac{1}{2}$.

(C) $\frac{\cos x}{2}$.

(D) ∞.

(3) A group of six friends are seated randomly around a circular table. Which of the following is the probability that two particular people, Abasi and Isra sit opposite each other? **1**

(A) $\frac{2!5!}{6!}$.

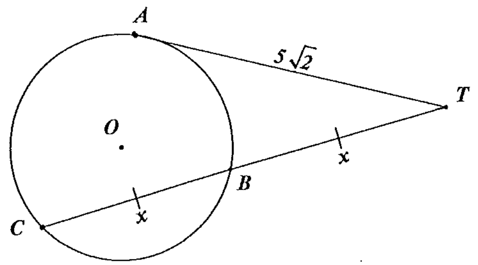
(B) $\frac{2!4!}{6!}$.

(C) $\frac{5!}{6!}$.

(D) $\frac{4!}{5!}$.

Marks

(4) **1**



AT is a tangent to a circle centre O. CT cuts the circle at B. If $BC=BT$ and $AT=5\sqrt{2}$, which of the following is the value of x?

(A) $5\sqrt{2}$.

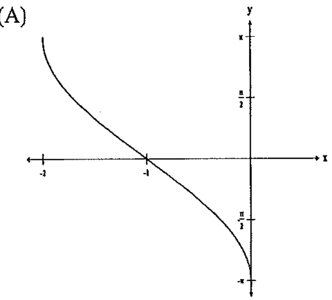
(B) $4\sqrt{2}$.

(C) 5.

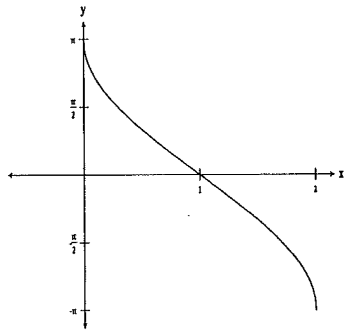
(D) $\sqrt{\frac{5\sqrt{2}}{2}}$.

(5) Which of the following represents the graph of $y=-2\sin^{-1}(x+1)$? **1**

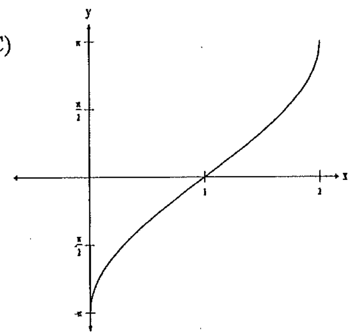
(A)



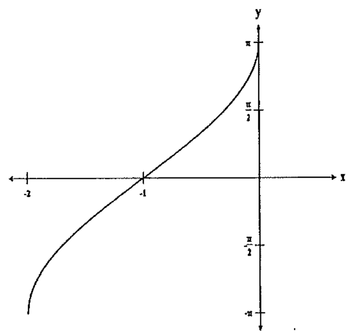
(B)



(C)



(D)



Marks

(6) A curve is defined by the parameters $x = 2p + \frac{2}{p}$, $y = p^2 + \frac{1}{p^2}$. Which of the following represents this curve in Cartesian form? **1**

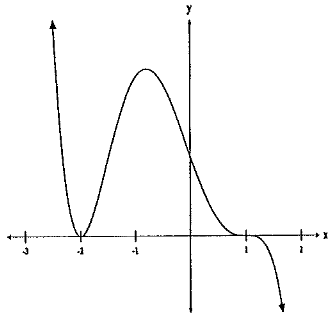
(A) $y = \frac{x^2}{4} - 2$.

(B) $y = \frac{x^2}{2}$.

(C) $x + y^2 = 2$.

(D) $y = x^2 - 2$.

(7) Which of the following could be the equation of the graph $y = P(x)$ shown below. **1**



(A) $y = -3(x-1)^2(x+2)^2$.

(B) $y = -x(1-x)^3(x+2)$.

(C) $y = a(1-x)(x+2)^3 \quad , a > 0$.

(D) $y = 2(1-x)^3(x+2)^2$.

Marks

(8) Which of the following equates to $\cos\left[\sin^{-1}\left(-\frac{2}{5}\right)\right]$? **1**

(A) $\frac{\sqrt{21}}{5} - \pi$.

(B) $\frac{3}{5}$.

(C) $\frac{\sqrt{21}}{5}$.

(D) $\pi - \frac{\sqrt{21}}{5}$.

(9) A particle moves with constant acceleration of $4\, m/s$. If initially the particle is stationary at $x = \frac{1}{4}$, which of the following represents the velocity of the particle when $x = \frac{19}{4} m$? **1**

(A) $\pm 6\ m/s$.

(B) $0\ m/s$.

(C) $19\ m/s$.

(D) $6\ m/s$.

(10) Which of the following is the inverse function of $y = \frac{x+1}{x-1}$? **1**

(A) $y = \frac{x-1}{x+1}$.

(B) $y = \frac{x+1}{x-1}$.

(C) $y = \frac{1+x}{1-x}$.

(D) $y = \frac{1+x^2}{-x}$.

Section II

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the coordinates of the point P which divides the interval AB externally in the ratio 2:3; where $A=(4,-1)$ and $B=(-6,3)$. **2**

(b) Find the acute angle formed at the intersection of $y=3x-7$ and $3x+y-7=0$. **2**

(c) Differentiate $\cos^{-1}(\sin x)$; leaving your answer in simplest form. **2**

(d) Find $\int \frac{dx}{\sqrt{36-25x^2}}$. **2**

(e) Solve the following inequality for x: **3**

$$\frac{1}{x} \leq \frac{1}{x-2}$$

(f) The equation $y=x^3+x^2-4x-k^2$ has only positive roots. If one of the roots is the product of the other two roots:

(i) Show that $x=k$ is a root of the equation. **2**

(ii) Hence or otherwise find the value of k. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Express $\sqrt{3}\sin 2x+\cos 2x$ in the form $R\sin(2x+\alpha)$, where α is in radians. **2**

(ii) Hence or otherwise, find the general solution of the equation $\sqrt{3}\sin 2x+\cos 2x=1$. **2**

(b) Use the substitution $u=1-x$ to evaluate $\int_{-3}^{0}\left(\frac{x}{\sqrt{1-x}}\right)dx$. **3**

(c) Consider the function $f(x)=\frac{x^2}{x^2-9}$.

(i) Find the vertical and horizontal asymptotes. **2**

(ii) Show that $(0,0)$ is the only stationary point on the curve. **2**

(iii) Given that $y=f(x)$ is an EVEN function, sketch the graph of $y=f(x)$, clearly labelling all essential features. **2**

(d) Find the value of the term independent of x in the expression $\left(x^2-\frac{2}{x^4}\right)^6$. **2**

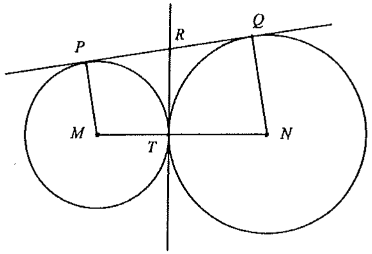
Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Prove by mathematical induction that **3**

$1+(1+2)+\dots\dots\dots(1+2+\dots\dots 2^{n-1})=2^{n+1}-n-2$ for all integers $n\geq 1$.

(b)

NOT TO SCALE



Two circles touch externally at T. The circles which have centres M and N are touched by a common tangent PQ. The common tangent at T meets PQ at R.

(i) Prove that $PR=RQ$. **1**

(ii) Hence prove $\angle PTQ=90^{o}$. **2**

(iii) Hence, by using $\angle PMT$ and $\angle QNT$, prove that PM is parallel to NQ. **2**

(c) (i) Show that $\dfrac{d}{dx}\left(\sqrt{1-x^2}\,\sin^{-1}x\right)=1-\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$. **2**

(ii) Hence or otherwise evaluate $\displaystyle\int_{-\frac{1}{2}}^{0}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)dx$. **2**

(d) The area bounded by the curve $y=\cos(x-\pi)$, the *x-axis*, $x=\dfrac{\pi}{2}$ and $x=0$ is rotated through a complete revolution about the *x-axis*.

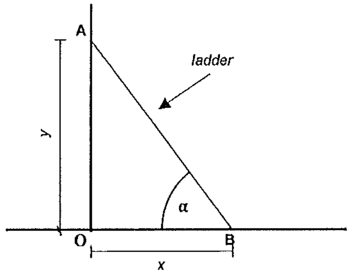
(i) Show that the volume generated is given by $\dfrac{\pi}{2}\displaystyle\int_{0}^{\frac{\pi}{2}}\cos(2x-2\pi)+1\,dx$ **2**

(ii) Hence find this volume leaving your answer in exact form. **1**

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) In a particular suburb in Sydney, during any day in the week, the probability that a car accident occurs is $\dfrac{1}{30}$. Find the probability that at least 5 accidents occur (each one on a separate day) in one week. **3**
(leave your answer to scientific notation).

(b)



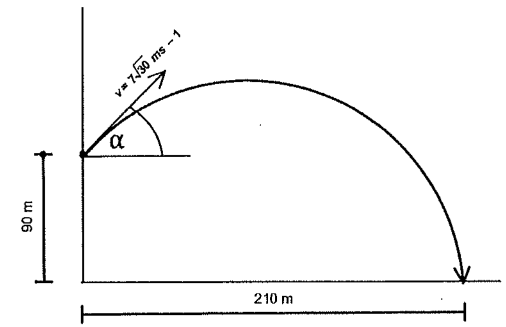
A ladder AB, 10 metres long, is leaning against a vertical wall OA (y metres), with its foot B, on horizontal ground OB (x metres). The top of the ladder begins to slide down to the ground at a constant speed of 1 metre per second. The angle the ladder makes with the ground is α radians.

(i) Prove that $\dfrac{dy}{d\alpha}=10\cos\alpha$. **1**

(ii) Hence find the rate at which α is changing when $x=6$. **2**

(c) A particle executes SHM so that $x=1+4\cos 2t$, where t is measured in seconds. Find the time it takes for the particle to travel from the centre of motion to half the amplitude. **2**

(d)



Question 14(d) is continued overleaf

Question 14(d) (continued) **Marks**

A stone thrown from the top of a cliff $90\,m$ high with an initial velocity of $7\sqrt{30}\,ms^{-1}$ and at an angle of α^{o} lands on a beach at a distance $210m$ horizontally from the foot of the cliff. (Take $g = 10\,ms^{-2}$)

(i) Write down the 6 equations of motion and show that $x = 7\sqrt{30}\,t\cos\alpha$ and $y = -5t^2 + 7\sqrt{30}t\sin\alpha + 90$ are the equations of motion for the stone. **2**

(ii) Hence derive the Cartesian equation of the path of the stone in simplest form. **2**

(iii) Hence find two possible angles for which the stone can been thrown to land $210\,m$ horizontally from the foot of the cliff. **3**

END OF PAPER

QATs
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MATHEMATICS EXTENSION 1
MARKING GUIDELINES

The sample answers indicate features that should be found in a response that receives full marks.

Question	Marks	Outcomes Assessed	Targeted Performance Bands
1	1	PE3	E2-E3
2	1	PE2	E2-E3
3	1	PE3	E2-E3
4	1	PE3	E2-E3
5	1	HE4	E2-E3
6	1	PE4	E2-E3
7	1	PE3	E2-E3
8	1	HE4	E2-E4
9	1	HE5	E2-E4
10	1	HE4	E2-E3
11a	2	PE2	E2-E3
11b	2	PE2	E2-E3
11c	2	HE4	E2-E3
11d	2	HE4	E2-E3
11e	3	PE3	E2-E3
11f(i)	2	PE3	E2-E3
11f(ii)	2	PE3	E2-E4
12a(i)	2	HE2	E2-E3
12a(ii)	2	PE2	E2-E3
12b	3	HE6	E2-E4
12c(i)	2	H5	E2-E3
12c(ii)	2	H6	E2-E3
12c(iii)	2	PE6	E3-E4
12d	2	HE3	E2-E3
13a	3	HE2	E3-E4
13b(i)	1	PE3	E2-E3
13b(ii)	2	PE3	E2-E4
13b(iii)	2	PE3	E2-E4
13c(i)	2	HE4	E2-E4
13c(ii)	2	HE4	E2-E4
13d(i)	2	PE6	E2-E3
13d(ii)	1	PE2	E2-E3
14a	3	HE3	E3-E4
14b(i)	1	HE5	E2-E3
14b(ii)	2	HE5	E2-E3
14c	2	HE3	E3-E4
14d (i)	2	HE3	E2-E3
14d (ii)	2	HE3	E3-E4
14d (iii)	3	HE3	E3-E4

Section I

Question	Marks	Answer	Outcomes Assessed	Targeted Performance Bands
1	1	B	PE3	E2-E3
2	1	B	PE2	E2-E3
3	1	D	PE3	E2-E3
4	1	C	PE3	E2-E3
5	1	A	HE4	E2-E3
6	1	A	PE4	E2-E3
7	1	D	PE3	E2-E3
8	1	C	HE4	E2-E4
9	1	D	HE5	E2-E4
10	1	B	HE4	E2-E3

Question 11 (15 marks)
11(a) (2 marks)
Outcomes Assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Uses correct formulae noting the ratio is -2:3	1
• Correct answer	1

Answer

$$P=\left[\frac{12+12}{1},\frac{-6-3}{1}\right]$$
$$=(24,-9)$$

11(b) (2 marks)
Outcomes Assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Uses correct formulae with correct m_1 and m_2	1
• Correct answer	1

Answer

$m_1=3 \quad m_2=-3$

$$\tan\theta=\left|\frac{3+3}{1-9}\right|$$
$$\tan\theta=\frac{6}{8}$$
$$\theta=36°52'$$

11(c) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E2-E3

Criteria	Marks
• Obtains $\frac{-1}{\sqrt{1-\sin^2 x}}\times\cos x$	1
• Correct answer	1

Answer

$$\frac{d}{dx}\left[\cos^{-1}(\sin x)\right]=\frac{-1}{\sqrt{1-\sin^2 x}}\times\cos x$$
$$=-\frac{\cos x}{\cos x}$$
$$=-1$$

11(d) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E2-E3

Criteria	Marks
• Obtains correct answer with one error	1
• Correct answer	1

Answer

$$\int\frac{dx}{\sqrt{36-25x^2}}=\frac{1}{5}\sin^{-1}\frac{5x}{6}+c$$

11(e) (3 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• Multiplies throughout by $x^2(x-2)^2$	1
• Obtain $-2x(x-2)\le 0$	1
• Correct answer	1

Answer

$$\frac{1}{x}\le\frac{1}{x-2}, \quad x\ne 0,2$$
$$x(x-2)^2\le(x-2)x^2$$
$$x(x-2)^2-x^2(x-2)\le 0$$
$$x(x-2)[x-2-x]\le 0$$
$$-2x(x-2)\le 0$$
$$x(x-2)\ge 0$$
$$\therefore x<0 \quad or \quad x>2$$

11(f) (i) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• Identifies correct roots and uses product of roots	1
• Correct answer	1

Answer

Let the roots be $\alpha,\beta,\alpha\beta$

$$\therefore\sum\alpha\beta\gamma=(\alpha\beta)^2=k^2$$
$$\therefore\alpha\beta=k$$

since $\alpha\beta$ is a root

$\therefore x=k$ is a root

11(f) (ii) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• Notes $k^3+k^2-4k-k^2=0$	1
• Correct answer	1

Answer

Solve
$f(k)=0$
$\therefore k^3+k^2-4k-k^2=0$
$k^3-4k=0$
$k(k^2-4)=0$
$k=0,\pm 2$
$k=2$ as $k>0$

Question 12 (15 marks)
12(a) (i) (2 marks)
Outcomes Assessed: HE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Achieves correct R	1
• Achieves correct α	1

Answer

$$\sqrt{3}\sin 2x+\cos 2x=2\left[\frac{\sqrt{3}}{2}\sin 2x+\frac{1}{2}\cos 2x\right]$$
$$=2\sin(2x+\alpha)$$
$$=2\sin\left(2x+\frac{\pi}{6}\right)$$

note: $\cos\alpha=\dfrac{\sqrt{3}}{2}$

$\therefore \alpha=\dfrac{\pi}{6}$

12(a) (ii) (2 marks)
Outcomes Assessed: PE2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Achieves $2x+\frac{\pi}{6}=\pi n+(-1)^n\frac{\pi}{6}$	1
• Correct answer	1

Answer

$$\sqrt{3}\sin 2x+\cos 2x=1$$
$$2\sin\left(2x+\frac{\pi}{6}\right)=1$$
$$\sin\left(2x+\frac{\pi}{6}\right)=\frac{1}{2}$$
$$2x+\frac{\pi}{6}=\pi n+(-1)^n\frac{\pi}{6} \qquad (for\ n\geq 0)$$
$$2x=\pi n+(-1)^n\frac{\pi}{6}-\frac{\pi}{6}$$
$$x=\frac{\pi}{2}n+(-1)^n\frac{\pi}{12}-\frac{\pi}{12}$$

12(b) (3 marks)
Outcomes Assessed: HE6
Targeted Performance Bands: E2-E4

Criteria	Marks
• Achieves $I = -\int_4^1 \left(\frac{1-u}{\sqrt{u}}\right) du$	1
• Achieves $\int_1^4 u^{-\frac{1}{2}} - u^{\frac{1}{2}}\ du$	1
• Correct answer	1

Answer

$u = 1 - x$
$du = -x\,dx$
$x = 0 \quad \rightarrow u = 1$
$x = -3 \quad \rightarrow u = 4$

$$\text{Let } I = \int_{-3}^{0} \left(\frac{x}{\sqrt{1-x}}\right) dx$$

$$\therefore I = -\int_4^1 \left(\frac{1-u}{\sqrt{u}}\right) du$$

$$= \int_1^4 u^{-\frac{1}{2}} - u^{\frac{1}{2}}\ du$$

$$= \left[2u^{\frac{1}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_1^4$$

$$= \left(4 - \frac{2}{3}(8)\right) - \left(2 - \frac{2}{3}\right)$$

$$= 4 - \frac{16}{3} - \frac{4}{3}$$

$$= 4 - \frac{20}{3}$$

$$= -2\frac{2}{3}$$

12(c) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct Vertical asymptote	1
• Correct Horizontal asymptote	1

Answer

Vertical Asymptotes occur at $x = \pm 3$

$$\text{Horizontal Asymptote} = \lim_{x\to\infty}\left(\frac{x^2}{x^2-9}\right)$$

$$= \lim_{x\to\infty}\left(\frac{1}{1-\frac{9}{x^2}}\right)$$

$$= 1$$

12(c) (ii) (2 marks)
Outcomes Assessed: H6
Targeted Performance Bands: E2-E3

Criteria	Marks
• Obtains correct $f'(x)$	1
• Correct working to achieve answer	1

Answer

$$f(x) = \frac{x^2}{x^2-9}$$

$$f'(x) = \frac{(x^2-9)\times 2x - x^2 \times 2x}{(x^2-9)^2}$$

Stationary points occur when $f'(x) = 0$

$$\therefore \frac{(x^2-9)\times 2x - x^2\times 2x}{(x^2-9)^2} = 0$$

$$2x(x^2-9) - 2x^3 = 0$$

$$2x^3 - 18x - 2x^3 = 0$$

$$18x = 0$$

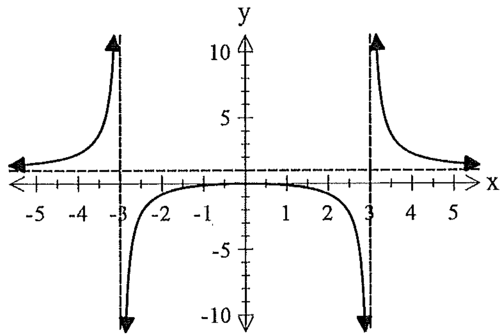
$$x = 0$$

$f(0) = 0 \quad \therefore (0,0)$ is the only stationary point

12(c) (iii) (2 marks)
Outcomes Assessed: PE6
Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct diagram with asymptotes and even function	1
• Correct answer	1

Answer



12(d) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• Achieves ${}^6C_2\left(x^2\right)^4\left(-\frac{2}{x^4}\right)^2$	1
• Correct answer	1

Answer

By inspection:

$$T_3 = {}^6C_2\left(x^2\right)^4\left(-\frac{2}{x^4}\right)^2$$
$$= {}^6C_2 \times 4$$
$$= \frac{6!}{2!4!} \times 4$$
$$= 60$$

Question 13 (15 marks)
13(a) (3 marks)
Outcomes Assessed: HE2
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides clear steps similar to the first three steps below	1
• Makes a substitution in step 3 (from step 2) and obtains required result	1
• Provides a clear explanation that 48 is a factor and provides a conclusion	1

Answer

Step 1: Prove the expression is true for n=1

$1 = 2^2 - 1 - 2$

$1 = 4 - 3$

$1 = 1 \quad (true)$

Step 2 : Assume the expression is true for n=k (where k is a positive integer)

$1 + (1+2) + \ldots\ldots\ldots\ldots(1+2+\ldots\ldots.2^{k-1}) = 2^{k+1} - k - 2$

Step 3 : Prove the expression is true for n=k+1

$1 + (1+2) + \ldots\ldots\ldots\ldots(1+2+\ldots\ldots.2^{k-1}) + (1+2+\ldots\ldots.2^k) = 2^{k+2} - (k+1) - 2$

$2^{k+1} - k - 2 + (1+2+\ldots\ldots.2^k) = 2^{k+2} - k - 3$ from assumption

$$\begin{aligned} \text{LHS} &= 2^{k+1} - k - 2 + (1+2+\ldots\ldots.2^k) \\ &= 2^{k+1} - k - 2 + \left[\frac{1(2^{k+1}-1)}{1}\right] \\ &= 2^{k+1} - k - 2 + 2^{k+1} - 1 \\ &= 2(2^{k+1}) - k - 3 \\ &= 2^{k+2} - k - 3 \\ &= RHS \end{aligned}$$

Hence if the expression is true when n=k, it is true when n=k+1
But the expression is true for n=1, ∴ it is true when n=2
If true for n=2, ∴ it is true when n=3
Therefore the expression is true for all n, $n \geq 1$.

13(b) (i) (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct explanation	1

Answer

$PR=RT$ (two tangent to a circle from an exterior point are equal)
$RT=RQ$ (two tangent to a circle from an exterior point are equal)
$\therefore PR = RQ$

13(b) (ii) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E4

Criteria	Marks
• Uses part (i) to work positively towards answer	1
• Correct proof	1

Answer

Let $\angle RPT = \angle RTP = a$ (base angles of an isosceles ΔPRT)
Let $\angle RQT = \angle RTQ = b$ (base angles of an isosceles ΔQRT)
In ΔPQT $\quad 2a+2b=180^o$ (angle sum of a triangle)
$\therefore a+b=90$
But $\angle RTP + \angle RTQ = a+b$
$\therefore \angle PTQ = 90^o$

13(b) (iii) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2-E4

Criteria	Marks
• Uses part (i) to work positively towards answer	1
• Correct proof	1

Answer
$\angle PMT = 2a$ (angle in the alternate segment and angle in the centre is twice angle at circumference)
Also
$\angle QMT = 2b$ (angle in the alternate segment and angle in the centre is twice angle at circumference)
Now $a+b=90$
$\therefore 2a+2b=180$
$\therefore \angle PMT + \angle QNT = 180$
$\therefore \angle PMT$ is cointerior to $\angle QNT$
$\therefore PM$ is parallel to NQ

13(c) (i) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E2-E4

Criteria	Marks
• Uses the product rule correctly	1
• Correct working	1

Answer

$$\frac{d}{dx}\left(\sqrt{1-x^2}\sin^{-1}x\right) = \sin^{-1}x \times \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}} \times -2x + \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$
$$= 1 - \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$$

13(c) (ii) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E2-E4

Criteria	Marks
• Achieves $\left[\sqrt{1-x^2}\sin^{-1}x - x\right]_0^{\frac{1}{2}}$ or similar	1
• Correct answer	1

Answer

$$\int_{-\frac{1}{2}}^{0}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)dx = -\int_{-\frac{1}{2}}^{0} 1 - \left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right) - 1\,dx$$
$$= \int_{0}^{-\frac{1}{2}} 1 - \left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right) - 1\,dx$$
$$= \left[\sqrt{1-x^2}\sin^{-1}x - x\right]_0^{-\frac{1}{2}}$$
$$= \left[\left(\frac{\sqrt{3}}{2}\sin^{-1}\left(-\frac{1}{2}\right) + \frac{1}{2}\right) - (0-0)\right]$$
$$= \left(-\frac{\sqrt{3}}{2}\sin^{-1}\left(\frac{1}{2}\right) + \frac{1}{2}\right)$$
$$= -\frac{\sqrt{3}}{2} \times \frac{\pi}{6} + \frac{1}{2}$$
$$= \frac{1}{2} - \frac{\sqrt{3}\pi}{12}$$

13(d) (i) (2 marks)

Outcomes Assessed: PE6

Targeted Performance Bands: E2-E3

Criteria	Marks
• Uses the correct formulae and makes a good attempt at achieving answer	1
• Correct answer	1

Answer

$$V=\pi\int y^2dx$$

$$y=cos(x-\pi)$$

$$y^2=cos^2(x-\pi)$$

Now

$$cos(2x-2\pi)=2cos^2(x-\pi)-1$$

$$\therefore\frac{1}{2}[cos(2x-2\pi)+1]=cos^2(x-\pi)$$

$$\therefore V=\pi\int_0^{\frac{\pi}{2}}\frac{1}{2}[cos(2x-2\pi)+1]dx$$

$$=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}[cos(2x-2\pi)+1]dx$$

13(d) (ii) (1 mark)

Outcomes Assessed: P6

Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct answer	1

Answer

$$V=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}[cos(2x-2\pi)+1]dx$$

$$=\frac{\pi}{2}\left[\frac{1}{2}\sin(2x-2\pi)+x\right]_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{2}\left[\frac{1}{2}\sin(-\pi)+\frac{\pi}{2}-0\right]$$

$$=\frac{\pi}{2}\left[\frac{\pi}{2}\right]$$

$$=\frac{\pi^2}{4}\ units^3$$

Question 14 (15 marks)

14(a) (3 marks)

Outcomes Assessed: HE3

Targeted Performance Bands: E3-E4

Criteria	Marks
• Uses the concept of binomial expansion for answer	1
• Makes a very positive attempt towards answer	1
• Correct answer	1

Answer

$$P(E)={}^7C_5\left(\frac{29}{30}\right)^2\left(\frac{1}{30}\right)^5+{}^7C_6\left(\frac{29}{30}\right)^1\left(\frac{1}{30}\right)^6+{}^7C_7\left(\frac{1}{30}\right)^7$$

$$=8.08\times10^{-7}+9.28\times10^{-9}+4.57\times10^{-11}$$

$$=8.17\times10^{-7}$$

14(b) (i) (1 mark)

Outcomes Assessed: HE5

Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct working	1

Answer

$$\sin\alpha=\frac{y}{10}$$

$$y=10\sin\alpha$$

$$\frac{dy}{d\alpha}=10\cos\alpha$$

14(b) (ii) (2 marks)
Outcomes Assessed: HE5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Uses $\frac{d\alpha}{dt}=\frac{d\alpha}{dy}\times\frac{dy}{dt}$ and noting that $\cos\alpha=\frac{3}{5}$	1
• Correct answer	1

Answer

$\cos\alpha=\frac{6}{10}=\frac{3}{5}$

$\frac{dy}{dt}=-1$

$\frac{d\alpha}{dt}=\frac{d\alpha}{dy}\times\frac{dy}{dt}$

$=\frac{1}{10\cos\alpha}\times(-1)$

$=-\frac{1}{10}\left(\frac{5}{3}\right)$

$=-\frac{1}{6}$ *radians*/seconds

14(c) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• Finds either the time at equilibrium or at half the amplitude	1
• Correct answer	1

Answer
Equilibrium position occurs at $x=1$
Amplitude=4
Time at centre of motion:

$1=1+4\cos 2t$

$4\cos 2t=0$

$\cos 2t=0$

$2t=\frac{\pi}{2},\frac{3\pi}{2}$

$t=\frac{\pi}{4},\frac{3\pi}{4},\ldots$

Time at half the amplitude

$3=1+4\cos 2t$

$4\cos 2t=2$

$\cos 2t=\frac{1}{2}$

$2t=\frac{\pi}{3},\frac{5\pi}{3},\ldots$

$t=\frac{\pi}{6},\frac{5\pi}{6}$

$Answer=\frac{5\pi}{6}-\frac{3\pi}{4}$

$=\frac{\pi}{12}$

14(d) (i) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
• Achieves at least four correct equations	1
• Correct working	1

Answer

$\ddot{x}=0$ $\qquad \ddot{y}=-10$

$\dot{x}=v\cos\alpha$ $\qquad \dot{y}=-10t+7\sqrt{30}\sin\alpha$

$\dot{x}=7\sqrt{30}\cos\alpha$ $\qquad \dot{y}=-10t+7\sqrt{30}\sin\alpha$

$x=7\sqrt{30}t\cos\alpha$ $\qquad y=-5t^2+7\sqrt{30}\,t\sin\alpha+90$

14(d) (ii) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• Substitutes $t=\frac{x}{7\sqrt{30}\cos\alpha}$ into y	1
• Correct answer	1

Answer

$x=7\sqrt{30}t\cos\alpha$ $\qquad y=-5t^2+7\sqrt{30}\,t\sin\alpha+90$

$t=\frac{x}{7\sqrt{30}\cos\alpha}$

$\therefore y=-5\left(\frac{x}{7\sqrt{30}\cos\alpha}\right)^2+7\sqrt{30}\left(\frac{x}{7\sqrt{30}\cos\alpha}\right)\sin\alpha+90$

$y=x\tan\alpha-\frac{x^2}{294}\sec^2\alpha+90$

14(d) (iii) (3 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
• Achieves $0=210\tan\alpha-\frac{(210)^2}{294}\sec^2\alpha+90$	1
• Achieves $5\tan^2\alpha-7\tan\alpha+2=0$	1
• Correct answer	1

Answer

$$0=210\tan\alpha-\frac{(210)^2}{294}\sec^2\alpha+90$$
$$0=210\tan\alpha-150\left(1+\tan^2\alpha\right)+90$$

$7\tan\alpha-5\left(1+\tan^2\alpha\right)+3=0$

$5\tan^2\alpha-7\tan\alpha+2=0$

$(5\tan\alpha-2)(\tan\alpha-1)=0$

$\tan\alpha=\frac{2}{5}$ *or* $\tan\alpha=1$

$\alpha=21°48',45°$