Centre Number

Student Number

QATs
Quality Assessment Tasks

2014
HSC TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks – 100

Section I Pages 3-6
10 marks
- Attempt Questions 1-10
- Allow 15 minutes for this section

Section II Pages 7-13
60 marks
- Attempt Questions 11-16
- Allow 2 hours and 45 minutes for this section

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt all questions
Allow about 15 minutes for this section

Use the multiple choice answer sheet.

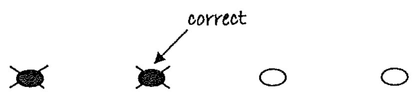
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Section 1 (10 marks) **Marks**

(1) Which of the following is a solution to the equation $x^3 = 2x^2$? **1**

(A) $x = 0, 2$

(B) $x = 2$

(C) $x = -2$

(D) $x = 0, \frac{1}{2}$

(2) The probabilities of the sex of a new born child being female or male are equal. If a family plan to have three children, the probability of having three girls is? **1**

(A) $\frac{3}{8}$

(B) $\frac{1}{4}$

(C) $\left[\frac{50}{100} \times \frac{49}{99} \times \frac{48}{98}\right]$

(D) $\frac{1}{8}$

(3) Which of the following equations describes the locus of all points with vertex $(3,3)$ and directrix $x = 1$? **1**

(A) $(x-3)^2 = 4(y-3)$

(B) $(y-3)^2 = 8(x-3)$

(C) $(y-3)^2 = -8(x-3)$

(D) $(x+3)^2 = -8(y-3)$

Marks

(4) If $\tan\theta = \dfrac{12}{5}$ and $\cos\theta < 0$, which of the following would $\sin\theta$ equate to? **1**

(A) $67°23'$

(B) $-\dfrac{12}{13}$

(C) $-\dfrac{3}{5}$

(D) $\dfrac{5}{13}$

(5) Given that $A = e^{2\ln x} + e^{3\ln y}$, which of the following is a correct statement? **1**

(A) $A = x^2 + y^3$

(B) $A = 2x + 3y$

(C) $A = x^2 y^3$

(D) $A = 6xy$

(6) In a geometric progression, the second term is 12 and the third term is -18. Which of the following is the value of the first term? **1**

(A) -18

(B) 6

(C) 8

(D) -8

(7) Which of the following represents the domain of $y = \sqrt{\dfrac{x+2}{2-x}}$? **1**

(A) $\{x : -2 \le x \le 2\}$

(B) $\{x : x \ne 2\}$

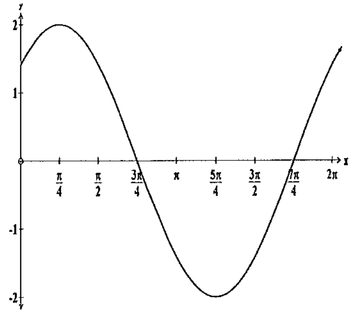
(C) $\{x : -2 \le x < 2\}$

(D) $\{x : x < -2 \ or \ x > 2\}$

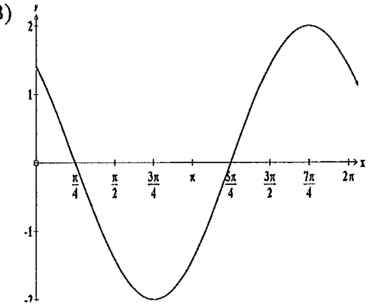
Marks

(8) Which of the following graphs represents $y = 2\sin\left(x - \dfrac{\pi}{4}\right)$? **1**

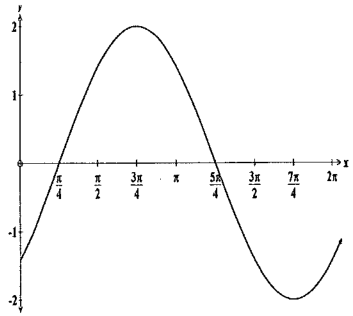
(A)



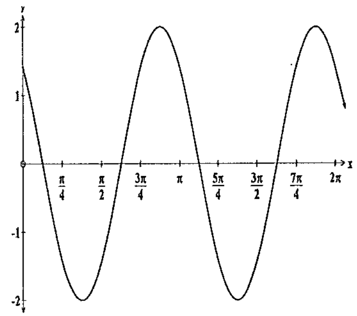
(B)



(C)



(D)



(9) Given that $y = \log_{10} x$, which of the following statements is correct? **1**

(A) $10^x = y$

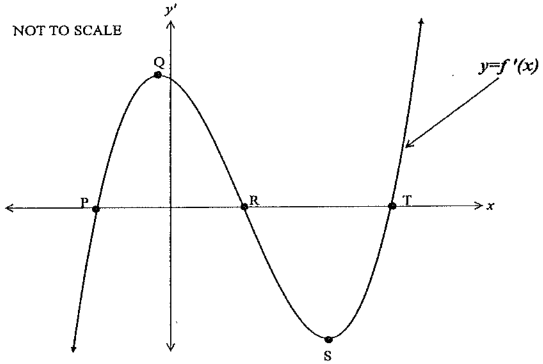
(B) $\dfrac{dy}{dx} = \dfrac{\ln 10}{x}$

(C) $\dfrac{dy}{dx} = \dfrac{1}{x\ln 10}$

(D) $\dfrac{dy}{dx} = \dfrac{1}{x}$

Marks

(10) 1



The diagram above represents a sketch of the **gradient function** of the curve $y = f(x)$.
Which of the following points have $f''(x)=0$ and $f'(x)<0$?

(A) R

(B) Q

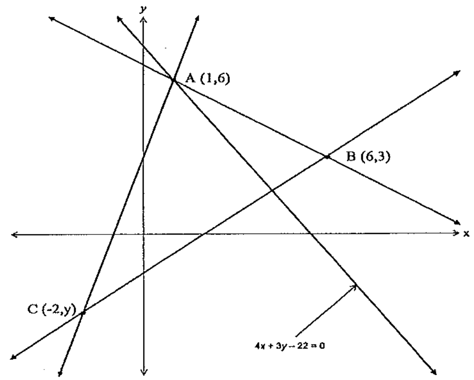
(C) T

(D) S

Section II
Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $|3x-2|\le 1$. 2

(b)



In the diagram above $A=(1,6)$, $B=(6,3)$ and $C=(-2,y)$. A line that passes through A and is **perpendicular** to line CB has the equation $4x+3y-22=0$.

(i) Find the equation of the line BC. 2

(ii) Hence show that $C=(-2,-3)$. 1

(iii) Find the length of BC. 1

(iv) Show that the distance from A to the line BC is $\frac{27}{5}$ units. 2

(v) Hence or otherwise, find the area of ΔABC. 1

(c) Graph the region bounded by $x^2+y^2<4$ and $y\le x^2+1$. 3

(d) Find the equation of the tangent to the curve $y=\ln(2x+1)$, at $x=0$. 3

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Convert 0.9 radians into degrees and minutes *(leave your answer to the nearest minute).* **1**

(b) Differentiate the following with respect to x.

(i) $\left(e^x+1\right)^3$. **2**

(ii) $\dfrac{\cos 3x}{x}$. **2**

(c) Find $\int \sec^2 \frac{x}{2}\, dx$. **2**

(d) Evaluate $\int_0^2 \frac{x^3}{2+2x^4}\, dx$, leaving your answer in simplified exact form. **3**

(e) A miner is mining for a precious metal in the deserts of Western Australia. The amount of precious metal mined in each of the first three months of operation were 4000g, 3920g, 3840g respectively and this pattern continues throughout the operation. The mine runs out of the precious metal after 50 months.

(i) How many grams were mined in the 12^{th} month? **1**

(ii) How many grams were mined over the first year? **2**

(iii) If the miner only sells 75% of the amount of precious metal mined each month, how many months does he need to mine to sell a total of 73.2kg? **2**

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) For the equation $4x^2-3x-2=0$, find the values of the following.

(i) $\alpha^2+\beta^2$. **2**

(ii) $\alpha^3+\beta^3$. **2**

(b) In a recent poll taken on whether Australia should become a republic, the results were as follows:

In favour of a republic=35% Against a republic=55% Undecided=10%

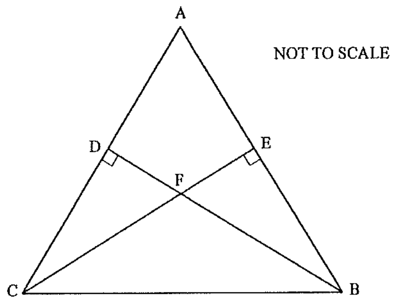
If two people were chosen at random from those who were surveyed, find the probability that:

(i) Both would be in favour of a republic. **1**

(ii) One would be in favour of a republic and one against a republic. **2**

(iii) At least one would be in favour of a republic. **3**

(c)



In diagram above, ΔABC is an **isosceles** triangle where $AB=AC$, $CE\perp AB$ and $BD\perp AC$.

(i) Prove that ΔCDB is congruent to ΔCEB. **3**

(ii) Explain why ΔCFB is an isosceles triangle **1**

(iii) Hence or otherwise prove that $DF=FE$ **1**

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Given that $3\sin\theta\tan^2\theta = \sin\theta$, find the exact values of θ, for $0 \le \theta \le 2\pi$. **3**

(b) The function $y = x(x-3)^2$ is defined in the domain $0 \le x \le 4$.

(i) Find the *x-intercepts*. **1**

(ii) Find the coordinates of any turning points and determine their nature. **3**

(iii) Show that when $x = 2$, the curve is decreasing most rapidly. **1**

(iv) Sketch the curve $y = x(x-3)^2$ for $0 \le x \le 4$, showing all essential features. **2**

(c) The blood-alcohol content (A) after a person has been drinking is given by $A = A_0 e^{-kt}$, where A_0 represents the blood-alcohol content level at the time a person stops drinking, t is measured in *hours* and A in *mg/ml*.

Melita stops drinking at 11pm on Saturday night (t=0) and her blood alcohol level was measured at 0.24 *mg/ml*. It took 28 hours for Melita's blood-alcohol level to be measured at 0.001 *mg/ml*.

(i) Explain why $A_0 = 0.24$ and find the value of k. *(4 decimal places)* **2**

(ii) The allowable blood-alcohol level limit for Melita to drive a car is 0.05 *mg/ml*. What is the earliest time on Sunday that Melita will be able to legally drive? *(leave your answer to the nearest hour)* **2**

(iii) What is the rate of decrease of the blood-alcohol level content in Melita's blood at 8.00am on Sunday? **1**

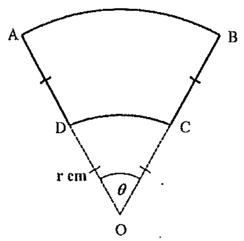
Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\log_2(x+4) = 4$ **1**

(b) (i) Graph the curve $y = e^x + 1$, showing all essential features. **1**

(ii) The area bounded by the curve $y = e^x + 1$, the *x-axis*, $x = 0$ and $x = \ln 2$ is rotated about the *x-axis*. Find the exact volume of the solid formed. **3**

(c)



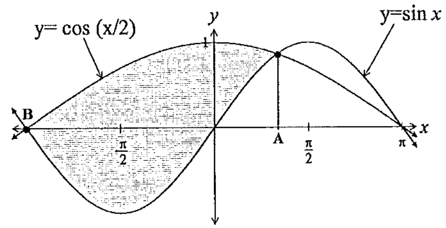
In the diagram above, $AD = OD = OC = CB = r\,cm$ and $\angle AOB = \theta$ radians. The perimeter of $ABCD = 12\,cm$. AB and CD are arcs of circles centre O.

(i) Find an expression for r in terms of θ. **1**

(ii) Show that A, the area of the $ABCD$ in cm^2 is given by $A = \dfrac{216\theta}{(2+3\theta)^2}$. **2**

(iii) Hence find the value of θ which produces the maximum area for $ABCD$. **2**

(d)



A section of the graphs of $y = \sin x$ and $y = \cos\frac{x}{2}$ for are represented above.

Question 15(d) continues over the page

Questions 15(d) (continued) **Marks**

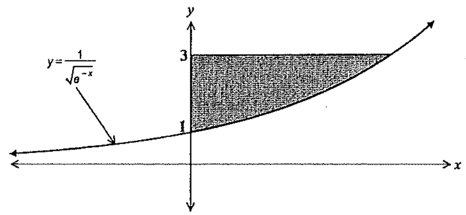
(i) Show that the x values of A and B (where the curves meet) are $\frac{\pi}{3}$ and $-\pi$ respectively. **2**

(ii) Hence or otherwise, find the exact area of the shaded region. **3**

Question 16 (15 marks) Use a SEPARATE writing booklet. Marks

(a) The limiting sum of the series $-\frac{1}{a}, \frac{1}{a^2}, -\frac{1}{a^3}, \ldots\ldots$ is equal to $4a$ $(a \neq 0)$. **3**
Find the value of a.

(b) **3**



The diagram above shows the area bounded by the curve $y = \frac{1}{\sqrt{e^{-x}}}$, $y = 3$ and the $y-$axis. Use Simpson's rule with three y function values to find an approximation of the volume when this area is rotated about the $y-axis$. *(leave your answer to 2 decimal places)*

(c) Raakhi's grandparents have set up a fund with a single investment of \$400,000 to provide financial support for her. She is granted an annual payment of \$25,000 from this fund at the end of each year. The fund accrues interest at a rate of 5% per annum compounded annually.

(i) Calculate the balance in the fund at the beginning of the second year. **1**

(ii) Let $\$A_n$ be the balance of the fund at the end of n years (after Raakhi receives her payment). Show that $A_n = 500,000 - 100,000(1.05)^n$. **2**

(iii) If this fund began at the beginning of 2000, in what year will the fund run out of money? **1**

(d) A particle is moving in a straight line. Initially, it is travelling to the left at 1 cm/min. Its acceleration as a function of time (t) is given by

$$a = \pi \cos \pi t + \pi \sin \pi t \text{ for } 0 \leq t \leq 2$$

where time and displacement are measured in minutes and cm respectively.

(i) Find when the particle changes direction. **2**

(ii) Find the exact total distance travelled in the first half a minute. **3**

END OF PAPER

Section I

Question	Marks	Answer	Outcomes Assessed	Targeted Performance Bands
1	1	A	P4	2-3
2	1	D	H5	2-3
3	1	B	H4	2-3
4	1	B	H5	2-3
5	1	A	H3	3-4
6	1	D	H5	3-4
7	1	C	H9	5-6
8	1	C	H5	3-4
9	1	C	H3	5-6
10	1	D	H6	4-5

Section II

Question 11 (15 marks)

11(a) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Obtains either $x \le 1$ *or* $x \ge \frac{1}{3}$	1
• Correct answer	1

Answer

$|3x-2| \le 1$

$3x-2 \le 1 \quad or \quad 3x-2 \ge -1$

$3x \le 3 \quad or \quad 3x \ge 1$

$x \le 1 \quad or \quad x \ge \frac{1}{3}$

$\frac{1}{3} \le x \le 1$

11(b) (i) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Obtains correct gradient for BC	1
• Correct answer	1

Answer

for $4x+3y-22=0 \quad m=-\frac{4}{3}$

for $BC: m=\frac{3}{4}, B=(6,3)$

$y-3=\frac{3}{4}(x-6)$

$4y-12=3x-18$

$3x-4y-6=0$

11(b) (ii) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Mark
• Correct working	1

Answer

Substitute $x=-2$ in $3x-4y-6=0$
$-6-4y-6=0$
$-12=4y$
$y=-3$
$\therefore C=(-2,-3)$

11(b) (iii) (1 mark)
Outcomes Assessed: P3
Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

$$\overline{BC}=\sqrt{8^2+6^2}$$
$$=\sqrt{100}$$
$$=10\ units$$

11(b) (iv) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct Substitution into formulae.	1
• Correct Answer.	1

Answer

$A(1,6)\quad 3x-4y-6=0$

$$\text{Distance}=\left|\frac{3-24-6}{\sqrt{9+16}}\right|$$
$$=\left|\frac{-27}{\sqrt{25}}\right|$$
$$=\frac{27}{5}\ units$$

11(b) (v) (1 mark)
Outcomes Assessed: H1
Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

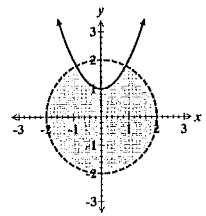
Answer

$$\text{Area}=\left[\frac{1}{2}\times 10\times\frac{27}{5}\right]$$
$$=27\,units^2$$

11(c) (3 marks)
Outcomes Assessed: P5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct graph of $x^2+y^2<4$	1
• Correct graph of $y\le x^2+1$	1
• Correct answer	1

Answer



11(d) (3 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Obtains $y'=\dfrac{2}{2x+1}$	1
• Obtains pt$(0,0)$ $m=2$	1
• Correct answer	1

Answer

$$y=\ln(2x+1)$$
$$y'=\frac{2}{2x+1}$$
$$f'(0)=2$$
$$f(0)=0$$
$$\text{pt}(0,0) \quad m=2$$
$$\therefore y=2x$$

Question 12 (15 marks)
12(a) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct answer	1

Answer

$$0.9\times\frac{\pi}{180}=51.5662$$
$$=51'34''$$

12(b) (i) (2 marks)
Outcomes Assessed: P7, H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Uses the chain rule	1
• Correct answer	1

Answer

$$\frac{d}{dx}(e^x+1)^3=3(e^x+1)^2\times e^x$$
$$=3e^x(e^x+1)^2$$

12(b) (ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Uses the quotient rule	1
• Correct answer	1

Answer

$$\frac{d}{dx}\left(\frac{\cos 3x}{x}\right)=\frac{x\times -3\sin 3x-\cos 3x}{x^2}$$
$$=\frac{-3x\sin 3x-\cos 3x}{x^2}$$

12(c) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Demonstrates knowledge of $\int \sec^2 x \quad dx=\tan x+c$	1
• Correct answer	1

Answer

$$\int \sec^2\frac{x}{2} \quad dx=2\tan\frac{x}{2}+c$$

12(d) (3 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 4-5

Criteria	Marks
• Obtains $\frac{1}{8}\left[\ln\left(2+2x^4\right)\right]_0^2$	1
• Correct substitution	1
• Correct simplification	1

Answer

$$\int_0^2 \frac{x^3}{2+2x^4} \quad dx=\frac{1}{8}\left[\ln\left(2+2x^4\right)\right]_0^2$$
$$=\frac{1}{8}[\ln 34-\ln 2]$$
$$=\frac{\ln 17}{8}$$

12(e) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct answer	1

Answer

4000g, 3920g, 3840g
a=4000 d=-80

$T_n = a + (n-1)d$

$T_{12} = 4000 + 11(-80)$

$= 3120g$

12(e) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct use of formulae	1
• Correct answer	1

Answer

$S_n = \frac{n}{2}\{a+l\}$

$S_{12} = \frac{12}{2}\{4000+3120\}$

$= 42720g$

$= 42.72Kg$

12(e) (iii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Obtains $73200 = 3030n - 30n^2$	1
• Correct answer	1

Answer

$73200 = \frac{n}{2}\{6000 + (n-1)(-60)\}$

$= \frac{n}{2}\{6060 - 60n\}$

$= n\{3030 - 30n\}$

$73200 = 3030n - 30n^2$

$n^2 - 101n + 2440 = 0$

$(n-40)(n-61) = 0$

$n = 40, 61$

$n = 40 \quad (n \le 50)$

Question 13 (15 marks)
13(a) (i) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 3-5

Criteria	Marks
• Notes $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	1
• Correct answer	1

Answer

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{4}\right)^2 - 2\left(-\frac{2}{4}\right)$

$= \frac{9}{16} + 1$

$= \frac{25}{16}$

13(a) (ii) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 3-5

Criteria	Marks
• Notes $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	1
• Correct answer	1

Answer

$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$$
$$=\left(\frac{3}{4}\right)\left[\left(\frac{25}{16}\right)-\left(-\frac{2}{4}\right)\right]$$
$$=\left(\frac{3}{4}\right)\left(\frac{33}{16}\right)$$
$$=\frac{99}{64}$$

13(b) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

Let F=In favour, A=Against and U=Unsure

$$P(FF)=(0.35)^2$$
$$=0.1225$$

13(b) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Notes $P(\text{One of each})=P(FA)+P(AF)$	1
• Correct answer.	1

Answer

$$P(\text{One of each})=P(FA)+P(AF)$$
$$=2[0.35\times0.55]$$
$$=0.385$$

13(b) (iii) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Notes $P(\text{at least one F})=P(FF)+P(FA)+P(FU)+P(AF)+P(UF)$	1
• Works positively towards answer	1
• Correct answer	1

Answer

$$P(\text{at least one F})=P(FF)+P(FA)+P(FU)+P(AF)+P(UF)$$
$$=(0.35)^2+(0.35\times0.55)+(0.35\times0.1)+(0.55\times0.35)+(0.1\times0.35)$$
$$=0.1225+0.1925+0.035+0.1925+0.035$$
$$=0.5775$$

OR Another method

$$P(\textit{at least one Favourable})=1-P(\tilde{F})$$
$$=1-(0.65\times0.65)$$
$$=0.5775$$

13(c) (i) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Uses RHS for proof	1
• Correct working with one mistake	1
• Correct proof	1

Answer

Prove ΔCDB ≅ ΔCEB

∠CDB=∠CEB (90° *given*)

∠DCB=∠*EBC* (base angles of an isosceles triangle *ABC* are equal)

BC is common

∴ ΔCDB ≅ ΔCEB (*AAS*)

13(c) (ii) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct explanation	1

Answer

$\angle ECB = \angle DBC$ (corresponding angles of congruent traingles are equal)

13(c) (iii) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct proof	1

Answer

$CE = DB$ (corresponding sides of congruent traingles are equal)
$FC = FB$ (sides of an isosceles triangle CFB are equal)
$\therefore DF = FE$

Question 14 (15 marks)
14(a) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-6

Criteria	Marks
• Obtains $\sin\theta\left(3\tan^2\theta - 1\right) = 0$	1
• Achieves some answers but not all	1
• Correct answer	1

Answer

$$3\sin\theta\tan^2\theta = \sin\theta$$
$$\sin\theta\left(3\tan^2\theta - 1\right) = 0$$
$$\sin\theta = 0 \quad or \quad \tan\theta = \pm\frac{1}{\sqrt{3}}$$
$$\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$$

14(b) (i) (1 mark)
Outcomes Assessed: H1
Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

$$y = x(x-3)^2$$
$$y = 0$$
$$x(x-3)^2 = 0$$
$$x = 0, 3$$

14(b) (ii) (3 marks)
Outcomes Assessed: H6
Targeted Performance Bands: 4-5

Criteria	Marks
• Correct differentiation	1
• Obtain stationary points	1
• Determines their nature	1

Answer

$$y = x(x-3)^2$$
$$y' = (x-3)^2 + x \times 2(x-3)$$
$$= (x-3)\left[(x-3) + 2x\right]$$
$$= (x-3)(3x-3)$$
$$= 3(x-3)(x-1)$$

Stationary points occur when $y'=0$
$$3(x-3)(x-1) = 0$$
$$x = 1, 3$$
$$ie\ (1,4) \quad (3,0)$$

Check concavity:
$y' = 3x^2 - 12x + 9$
$y'' = 6x - 12$
$y''(1) = -6$ $\therefore (1,4)$ is a maximum
$y''(3) = 6$ $\therefore (3,0)$ is a minimum

14(b) (iii) (1 mark)
Outcomes Assessed: H6
Targeted Performance Bands: 4-5

Criteria	Mark
• Correct answer	1

Answer

Decreases most rapidly when $y''=0$ *and* $y'<0$

$6x-12=0$

$x=2$

$f'(2)=-3<0$

Check:

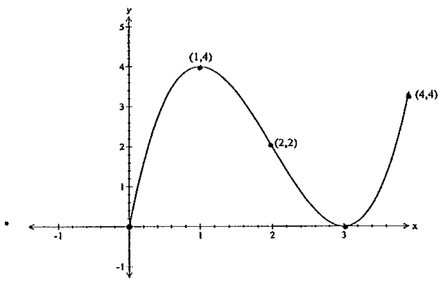
x	1	2	3
y"	-	0	+

At t=2 the curve is decreasing most rapidly.

14(b) (iv) (2 marks)
Outcomes Assessed: H6
Targeted Performance Bands: 4-5

Criteria	Marks
• Notes turning point and point of inflexion	1
• Correct diagram	1

Answer



14(c) (i) (2 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct working for A_0	1
• Correct working for k	1

Answer

$A=A_0e^{-kt}$

$t=0 \quad A=0.24$

$\therefore A_0=0.24$

$now\ t=28 \quad A=0.001$

$\therefore 0.001=0.24e^{-28k}$

$0.004167=e^{-28k}$

$k=\dfrac{\ln(0.004167)}{-28}$

$k=0.1957$

14(c) (ii) (2 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct substitution into equation	1
• Correct answer	1

Answer

$\therefore 0.05=0.24e^{-0.1957t}$

$0.2083=e^{-0.1957t}$

$t=\dfrac{\ln(0.2083)}{-0.1957}$

$t=8.015$

$t=8$

$\therefore \text{time}=\text{Sunday 7am}$

14(c) (iii) (1 mark)
Outcomes Assessed: H6
Targeted Performance Bands: 4-5

Criteria	Mark
• Correct answer	1

Answer

$A = 0.24e^{-0.1957t}$

$A' = -0.046968e^{-0.1957t}$

$A'(9) = -0.00807$

rate of decrease $= 0.00807$ *mg / ml per hour*

Question 15 (15 marks)
15(a) (1 mark)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct answer	1

Answer

$\log_2(x+4) = 4$

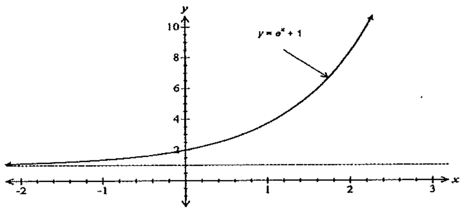
$2^4 = x+4$

$x = 12$

15(b) (i) (1 mark)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct graph	1

Answer



15(b) (ii) (3 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 5-6

Criteria	Marks
• Obtains $y^2 = e^{2x} + 2e^x + 1$	1
• Obtains $V = \pi\left[\frac{1}{2}e^{2x} + 2e^x + x\right]_0^{\ln 2}$	1
• Correct answer	1

Answer

$$y = e^x + 1$$

$$y^2 = (e^x+1)^2$$

$$= e^{2x} + 2e^x + 1$$

$$v = \pi\int_0^{\ln 2} e^{2x} + 2e^x + 1\,dx$$

$$= \pi\left[\frac{1}{2}e^{2x} + 2e^x + x\right]_0^{\ln 2}$$

$$= \pi\left[\left(\frac{1}{2}e^{2\ln 2} + 2e^{\ln 2} + \ln 2\right) - \left(\frac{1}{2} + 2 + 0\right)\right]$$

$$= \pi\left[2 + 4 + \ln 2 - 2\frac{1}{2}\right]$$

$$= \pi\left(3\frac{1}{2} + \ln 2\right)\ units^3$$

15(c) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Mark
• Correct answer	1

Answer

$$2r + r\theta + 2r\theta = 12$$

$$2r + 3r\theta = 12$$

$$r(2+3\theta) = 12$$

$$r = \frac{12}{2+3\theta}$$

15(c) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Obtains $A=\frac{3}{2}r^2\theta$	1
• Correct working	1

Answer

$$A=\frac{1}{2}(2r)^2\theta-\frac{1}{2}r^2\theta$$
$$=2r^2\theta-\frac{1}{2}r^2\theta$$
$$=\frac{3}{2}r^2\theta$$
$$=\frac{3}{2}\left[\frac{12}{2+3\theta}\right]^2\theta$$
$$=\frac{216\theta}{(2+3\theta)^2}$$

15(c) (iii) (2 marks)
Outcomes Assessed: H6
Targeted Performance Bands: 4-5

Criteria	Marks
• Notes $\frac{216(2-3\theta)}{(2+3\theta)^3}=0$	1
• Correct answer	1

Answer

maximum occurs when A'=0 and concave down

$$\frac{216(2-3\theta)}{(2+3\theta)^3}=0$$

$$\theta=\frac{2}{3}\text{radians}$$

Check:

x	1/3	2/3	1
y'	+	0	-

15(d) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Proves A	1
• Proves B	1

Answer

Substitute $x=\frac{\pi}{3}$ and $x=-\pi$ into both equations

$x=\frac{\pi}{3}$

$y=\cos\frac{\pi}{6}$ $\quad$ $y=\sin\frac{\pi}{3}$

$y=\frac{\sqrt{3}}{2}$ $\quad$ $y=\frac{\sqrt{3}}{2}$

$x=-\pi$

$y=\cos\left(-\frac{\pi}{2}\right)$ $\quad$ $y=\sin(-\pi)$

$y=\cos\frac{3\pi}{2}$ $\quad$ $y=\sin\pi$

$y=0$ $\quad$ $y=0$

15(d) (ii) (3 marks)

Outcomes Assessed: H8

Targeted Performance Bands: 4-5

Criteria	Marks
• Obtains $A=\int_{-\pi}^{\frac{\pi}{3}}\cos\frac{x}{2}-\sin x\,dx$	1
• Obtains $A=\left[2\sin\frac{x}{2}+\cos x\right]_{-\pi}^{\frac{\pi}{3}}$	1
• Correct answer	1

Answer

$$A=\int_{-\pi}^{\frac{\pi}{3}}\cos\frac{x}{2}-\sin x\,dx$$

$$=\left[2\sin\frac{x}{2}+\cos x\right]_{-\pi}^{\frac{\pi}{3}}$$

$$=\left[2\left(\frac{1}{2}\right)+\frac{1}{2}-(-2-1)\right]$$

$$=4\frac{1}{2}\ units^2$$

Question 16 (15 marks)

16(a) (3 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Obtains 1st term and ratio	1
• Substitutes correctly into formulae	1
• Correct answer	1

Answer

$$S_\infty=\frac{a}{1-r}$$

$$4a=\frac{-\frac{1}{a}}{1+\frac{1}{a}}$$

$$4a\left[1+\frac{1}{a}\right]=-\frac{1}{a}$$

$$4a^2\left[1+\frac{1}{a}\right]=-1$$

$$4a^2+4a=-1$$

$$4a^2+4a+1=0$$

$$(2a+1)^2=0$$

$$a=-\frac{1}{2}$$

16(b) (3 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 5-6

Criteria	Marks
• Obtains $x^2 = 4(\ln y)^2$	1
• Uses the Simpson's rule correctly with one mistake	1
• Correct answer	1

Answer

$y = \dfrac{1}{\sqrt{e^{-x}}}$

$y^2 = \dfrac{1}{e^{-x}}$

$y^2 = e^x$

$\ln y^2 = x$

$x = 2\ln y$

$x^2 = (2\ln y)^2$

$x^2 = 4(\ln y)^2$

y	f(y)	weight	Result
1	0	1	0
2	$4(\ln 2)^2$	4	7.6872
3	$4(\ln 3)^2$	1	4.8278
Total			**12.515**

$V = \pi\left[\dfrac{1}{3}(12.515)\right]$

$= 13.1056\ units^3$

$= 13.11\ units^3$

16(c) (i) (1 mark)
Outcomes Assessed: H9
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct answer	1

Answer

$\text{Balance} = 400,000(1.05)^1 - 25,000$

16(c) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Achieves $A_n = 400,000(1.05)^n - 25,000\left[\dfrac{1((1.05)^n - 1)}{0.05}\right]$	1
• Correct working	1

Answer

$A_2 = \left[400,000(1.05)^1 - 25,000\right](1.05)^1 - 25,000$

$= 400,000(1.05)^2 - 25,000[1 + 1.05]$

$A_n = 400,000(1.05)^n - 25,000\left[1 + 1.05 + 1.05^2 + \ldots\ldots 1.05^{n-1}\right]$

$= 400,000(1.05)^n - 25,000\left[\dfrac{1((1.05)^n - 1)}{0.05}\right]$

$= 400,000(1.05)^n - 500,000((1.05)^n - 1)$

$= 500,000 - 100,000(1.05)^n$

16(c) (iii) (1 mark)
Outcomes Assessed: H5, H9
Targeted Performance Bands: 4-5

Criteria	Mark
• Correct answer	1

Answer

$0 = 500,000 - 100,000(1.05)^n$

$n = \dfrac{\ln 5}{\ln(1.05)}$

$= 32.987$

The fund will run out of money in 2033.

16(d) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Obtains correct expression for velocity	1
• Correct answer	1

Answer

$v = \sin \pi t - \cos \pi t + c$

$v = -1 \quad t = 0$

$-1 = 0 - 1 + c$

$c = 0$

$v = \sin \pi t - \cos \pi t$

changes direction when v=0

$\sin \pi t - \cos \pi t = 0$

$\tan \pi t = 1$

$t = \frac{1}{4}, \frac{5}{4}$ minutes $\quad (0 \le t \le 2)$

16(d) (ii) (3 marks)

Outcomes Assessed: H7, H9

Targeted Performance Bands: 5-6

Criteria	Marks
• Notes Distance travelled $= \left\| \int_0^{\frac{1}{4}} \sin \pi t - \cos \pi t \, dt \right\| + \int_{\frac{1}{4}}^{\frac{1}{2}} \sin \pi t - \cos \pi t \, dt$ that	1
• Works towards answer successfully	1
• Correct answer	1

Answer

$$\text{Distance travelled} = \left| \int_0^{\frac{1}{4}} \sin \pi t - \cos \pi t \, dt \right| + \int_{\frac{1}{4}}^{\frac{1}{2}} \sin \pi t - \cos \pi t \, dt$$

$$= \left| \left[-\frac{1}{\pi} \cos \pi t - \frac{1}{\pi} \sin \pi t \right]_0^{\frac{1}{4}} \right| + \left[-\frac{1}{\pi} \cos \pi t - \frac{1}{\pi} \sin \pi t \right]_{\frac{1}{4}}^{\frac{1}{2}}$$

$$= \left| \left[-\frac{1}{\pi} (\cos \pi t + \sin \pi t) \right]_0^{\frac{1}{4}} \right| + \left[-\frac{1}{\pi} (\cos \pi t + \sin \pi t) \right]_{\frac{1}{4}}^{\frac{1}{2}}$$

$$= \left| -\frac{1}{\pi} \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (1+0) \right] \right| - \frac{1}{\pi} \left[(0+1) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right]$$

$$= \left| -\frac{1}{\pi} \left(\sqrt{2} - 1 \right) \right| - \frac{1}{\pi} \left(1 - \sqrt{2} \right)$$

$$= \frac{\left(\sqrt{2} - 1 \right)}{\pi} + \frac{\left(\sqrt{2} - 1 \right)}{\pi}$$

$$= \frac{2\left(\sqrt{2} - 1 \right)}{\pi} cm$$