

NOTE: No Calculators.

1. Express the following integers as products of their prime factors in index form:

(a) 36,

(b) 540.

2. Solve for n :

(a) $3^n = 81$,

(b) $2^n = 128$.

3. Simplify these expressions, removing any brackets:

(a) $3x^2 \times 5x^3$,

(d) $x^3(x^5 - 3x^2)$,

(b) $a^3b^5 \div ab$,

(e) $(2m^2n^3)^4 \div (2m^3n^2)^2$,

(c) $(4ab^3)^2$,

(f) $(x^3 - 1)(x^3 + 1)$.

4. Express these numbers without zero or negative indices:

(a) $(3x)^0$,

(d) $3x^{-2}$,

(b) $7a^0 + 7^0a^0 + 7^0a$,

(e) $(xy)^{-1}$,

(c) $(\frac{2}{5})^{-1}$,

(f) $(x^3)^{-2}$.

5. Write in standard notation, correct to 4 significant figures:

(a) $56\ 298\ 123$,

(b) $0.000\ 037\ 448$.

6. Calculate these numbers, giving your answers in standard notation:

(a) $(3 \times 10^9) \times (4 \times 10^{15})$.

(b) $(2 \times 10^{-16}) \div (5 \times 10^{-20})$.

7. Evaluate:

(a) $9^{\frac{1}{2}}$,

(c) $8^{\frac{2}{3}}$,

(b) $4^{-\frac{1}{2}}$,

(d) $(\frac{25}{4})^{-\frac{3}{2}}$

8. Simplify fully:

(a) $x^{\frac{3}{4}} \times x^{\frac{1}{4}}$,

(c) $(7^{\frac{2}{3}})^3$,

(b) $(8m^3)^{\frac{1}{3}}$,

(d) $(0.04)^{-\frac{1}{2}}$.

9. Simplify each of the following expressions, giving your answer with positive indices.

(a) $(xb^{-\frac{2}{3}})^{\frac{1}{3}} \times (x^{\frac{1}{4}}b^{-3})^{\frac{1}{3}}$

(b) $\frac{(\frac{1}{2}a^3b^{-2})^{-1}}{(8a^{-9}b^6)^{\frac{1}{3}}}$

1. These are factorings into primes.

$$(a) 36 = 2^2 \times 3^2.$$

$$(b) 540 = 2^2 \times 3^3 \times 5.$$

$$2. (a) 3^n = 81$$

$$n = 4 \quad (\text{because } 3^4 = 81).$$

$$(b) 2^n = 128$$

$$n = 7 \quad (\text{because } 2^7 = 128).$$

$$3. (a) 3x^2 \times 5x^3 = 15x^5.$$

$$(b) a^3b^5 \div ab = a^2b^4.$$

$$(c) (4ab^3)^2 = 16a^2b^6.$$

$$(d) x^3(x^5 - 3x^2) = x^8 - 3x^5.$$

$$(e) (2m^2n^3)^4 \div (2m^3n^2)^2 \\ = 16m^8n^{12} \div 4m^6n^4 \\ = 4m^2n^8.$$

$$(f) (x^3 - 1)(x^3 + 1) = x^6 - 1.$$

$$4. (a) (3x)^0 = 1.$$

$$(b) 7a^0 + 7^0a^0 + 7^0a \\ = 7 \times 1 + 1 \times 1 + 1 \times a \\ = 7 + 1 + a \\ = 8 + a.$$

$$(c) \left(\frac{2}{5}\right)^{-1} = \frac{5}{2}.$$

$$(d) 3x^{-2} = \frac{3}{x^2} \quad (\text{the index } -2 \text{ applies only} \\ \text{to the } x \text{ and not to the } 3).$$

$$(e) (xy)^{-1} = \frac{1}{xy}.$$

$$(f) (x^3)^{-2} = x^{-6} \\ = \frac{1}{x^6}.$$

5. Answers given to 4 significant figures.

$$(a) 56298123 \div 5.630 \times 10^7.$$

$$(b) 0.000037448 \div 3.745 \times 10^{-5}.$$

$$6. (a) (3 \times 10^9) \times (4 \times 10^{15})$$

$$= 12 \times 10^{24}$$

$$= 1.2 \times 10 \times 10^{24}$$

$$= 1.2 \times 10^{25}.$$

$$(b) (2 \times 10^{-16}) \div (5 \times 10^{-20})$$

$$= (20 \times 10^{-17}) \div (5 \times 10^{-20})$$

$$= 4 \times 10^3.$$

$$7. (a) 9^{\frac{1}{2}} = 3.$$

$$(b) 4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} \\ = \frac{1}{2}.$$

$$(c) 8^{\frac{2}{3}} = 2^2 \\ = 4.$$

$$(d) \left(\frac{25}{4}\right)^{-\frac{3}{2}} = \left(\frac{4}{25}\right)^{\frac{3}{2}} \\ = \left(\frac{2}{5}\right)^3 \\ = \frac{8}{125}.$$

$$8. (a) x^{\frac{3}{4}} \times x^{\frac{1}{4}} = x.$$

$$(b) (8m^3)^{\frac{1}{3}} = 2m.$$

$$(c) \left(7^{\frac{2}{3}}\right)^3 = 7^2 \\ = 49.$$

$$(d) (0.04)^{-\frac{1}{2}} = \left(\frac{1}{25}\right)^{-\frac{1}{2}} \\ = 25^{\frac{1}{2}} \\ = 5.$$

$$9. (a) \frac{x^{\frac{5}{2}}}{b^{\frac{1}{9}}}$$

$$(b) 1$$