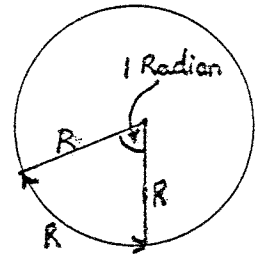


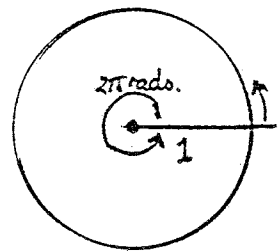
"RADIANS" - Circular Measurement

After travelling a distance around the circumference equal to the radius of the circle, the angle produced at the centre is an angle of ONE "radian".



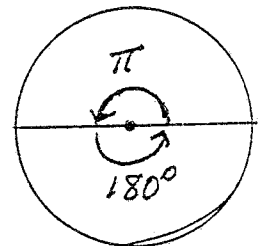
Note if we travel the complete length of the circumference, $2\pi R$, we have gone 2π radius's around the circle

\therefore 1 revolution is 2π radians



$$C = 2\pi$$

Thus remember π radians ($\frac{1}{2}$ way) is the same angle as 180 degrees.



To convert degrees to radians

eg. $75^\circ = 75^\circ \times \frac{\pi \text{ rads}}{180^\circ} = 1.31 \text{ rads}$

To convert radians to degrees

eg. $2 \text{ rads} = 2 \text{ rads} \times \frac{180^\circ}{\pi \text{ rads}} = 114.6^\circ$

Note

$$180^\circ = \pi \text{ rads}$$

$$30^\circ = \frac{\pi}{6} \text{ rads}$$

$$90^\circ = \frac{\pi}{2} \text{ rads}$$

$$60^\circ = \frac{\pi}{3} \text{ rads}$$

$$45^\circ = \frac{\pi}{4} \text{ rads}$$

$$120^\circ = \frac{2\pi}{3} \text{ rads}$$

etc.,

WORK SHEET (1) -

(A) Convert to radians:

① $120^\circ = 120^\circ \times \frac{\pi \text{ rads}}{180^\circ} = \frac{2\pi \text{ rads}}{3}$

② $20^\circ =$

③ $45^\circ =$

④ $60^\circ =$

⑤ $270^\circ =$

⑥ $300^\circ =$

⑦ $71.4^\circ =$

(B) Convert to degrees:-

⑧ $\frac{\pi}{2} = \frac{\pi}{2} \times \frac{180^\circ}{\pi} = 90^\circ$

⑨ $\frac{5\pi}{4} =$

⑩ $\frac{4\pi}{9} =$

⑪ $\frac{\pi}{6} =$

⑫ $\frac{3\pi}{10} =$

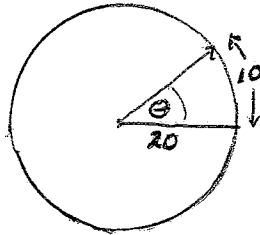
⑬ $2 \text{ rads} =$

⑭ $\frac{5\pi}{3} =$

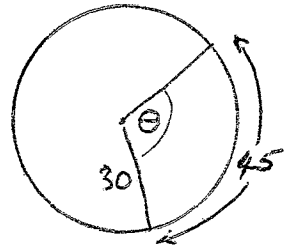
⑮ $0.85 \text{ rads} =$

(C) Find θ in radians.

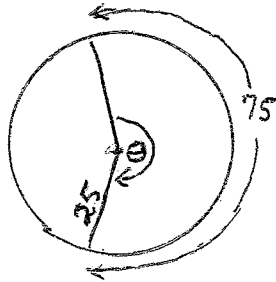
⑯



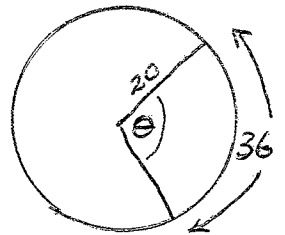
⑰



⑱

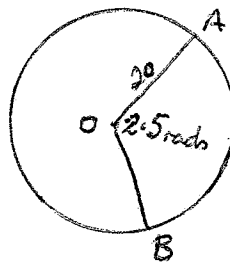


⑲

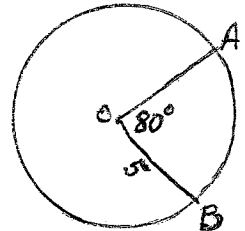


(D) For each case find (i) the arc length AB
(ii) Area of Sector OAB

⑳

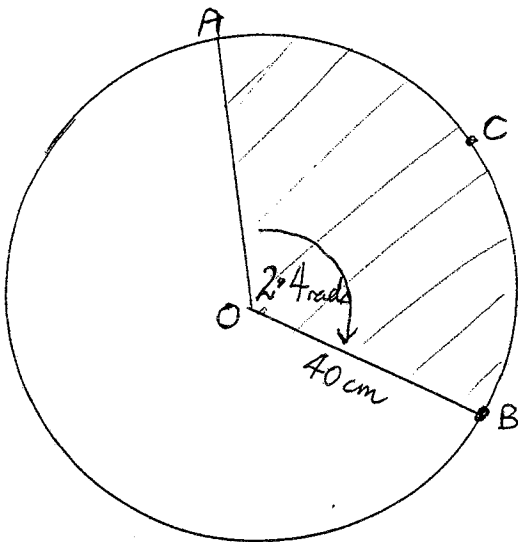


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WORKSHEET ②

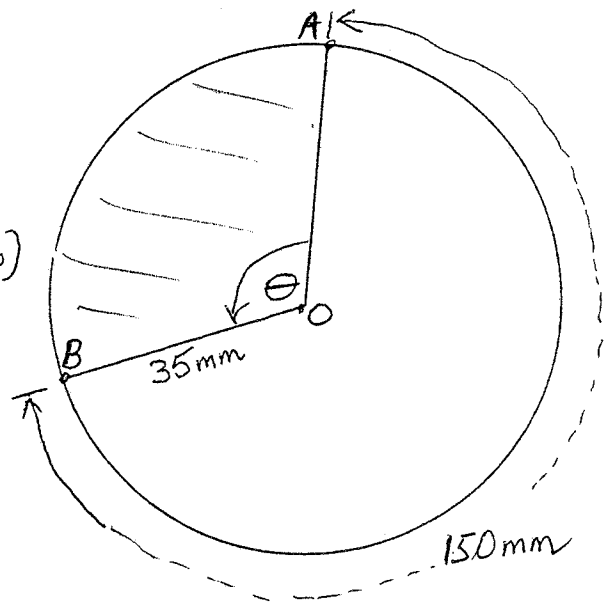
Qu ①



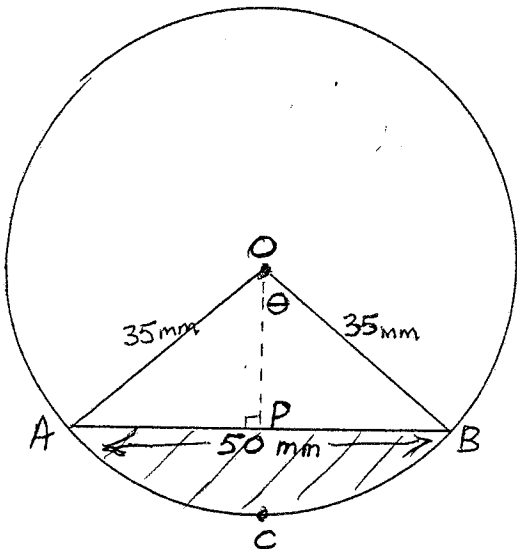
- (i) Use $l = r \times \theta$ to find the arc length ACB
- (ii) Find the shaded Area ($A = \frac{1}{2} r^2 \theta$ if θ is in "rads".)

Qu ②

- (i) Use $l = r \times \theta$ to find the "reflex" angle $\angle AOB$ in radians.
- (ii) Find θ in radians (1 revol = 2π rads)
- (iii) Find the shaded Area.



Qu ③



- (i) Use trigonometry to find θ
- (ii) Find angle $\angle AOB$
- (iii) Find Area of Sector OACB
- (iv) Find length of OP (use pythagorus)
- (v) Find Area of triangle $\triangle AOB$
- (vi) Find shaded Area (lens)

ANSWERS

Qu ① (i) 96 cm (ii) 1920 cm² Qu ② (i) 4.286 rads (ii) $\theta = 1.997$ rads (iii) 1223 mm²
Qu ③ (i) 0.7956 rads (ii) 1.5912 rads (iii) 974.6 mm² (iv) 24.5 (v) 612.4 mm² (vi) 362.2 mm²

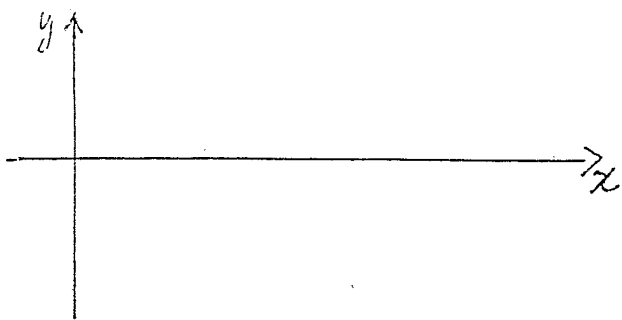
Quest ①

For each of the following TRIG functions $y = a \text{TRIG}(nx)$

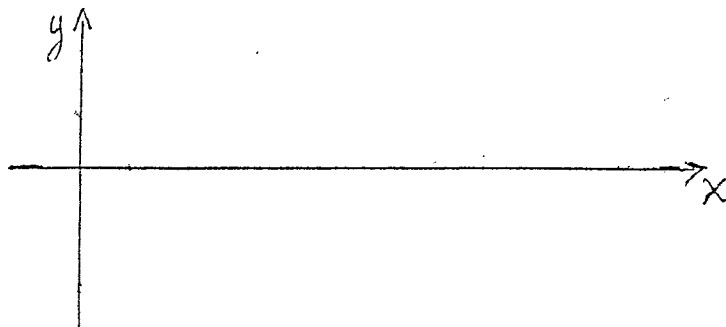
(a) Find the amplitude, "a" (b) Find the period $T = \frac{2\pi}{n}$

(c) Sketch exactly one cycle of their graphs, showing the above information.

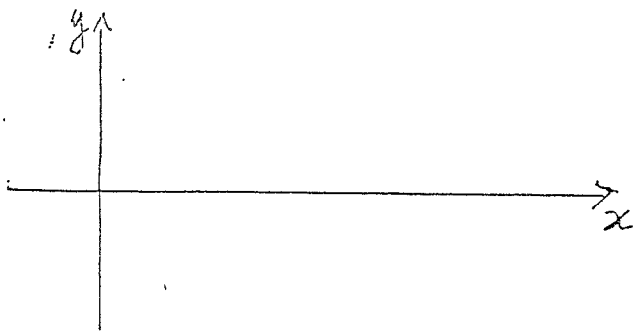
(i) $y = 3 \sin 4x$



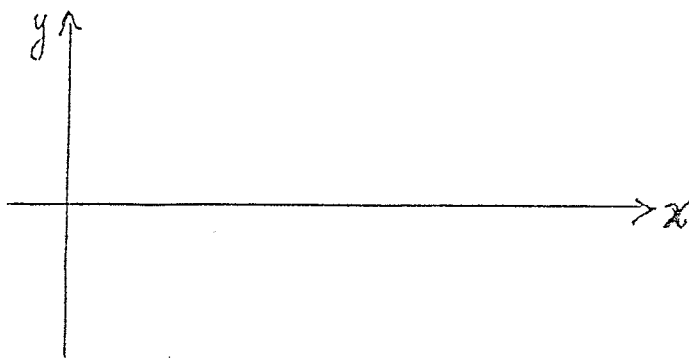
(ii) $y = 10 \cos 2x$



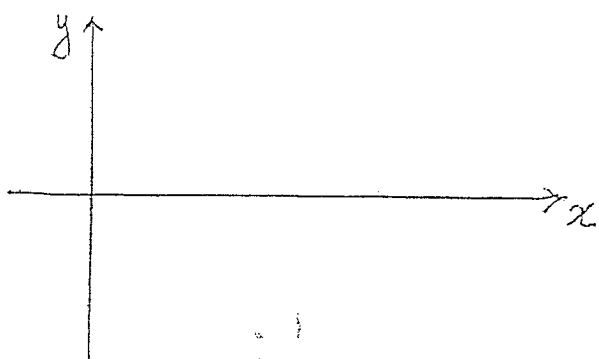
(iii) $y = 4 \sin \frac{x}{10}$



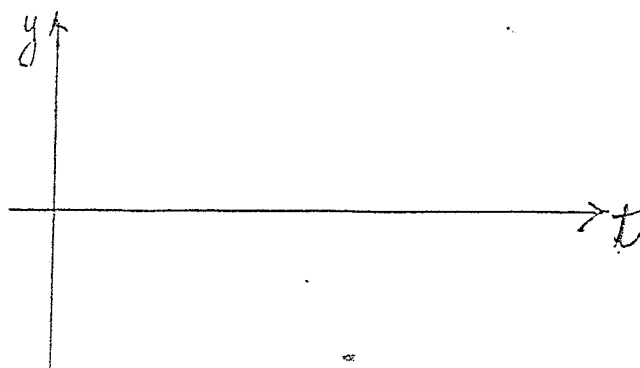
(iv) $y = 5 \sin \pi x$

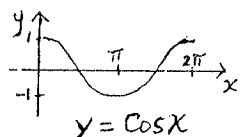
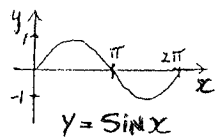


(v) $y = 0.2 \cos 200\pi x$



(vi) $y = \cos 50\pi t$



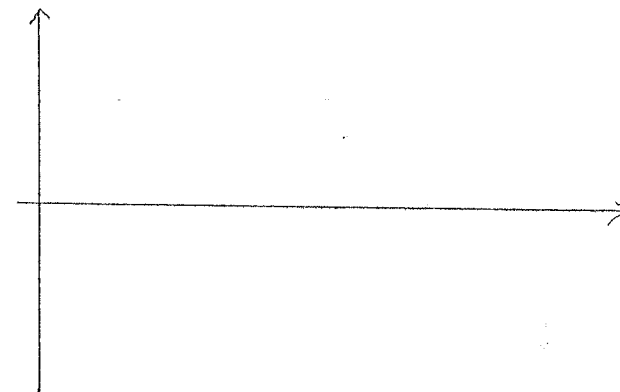
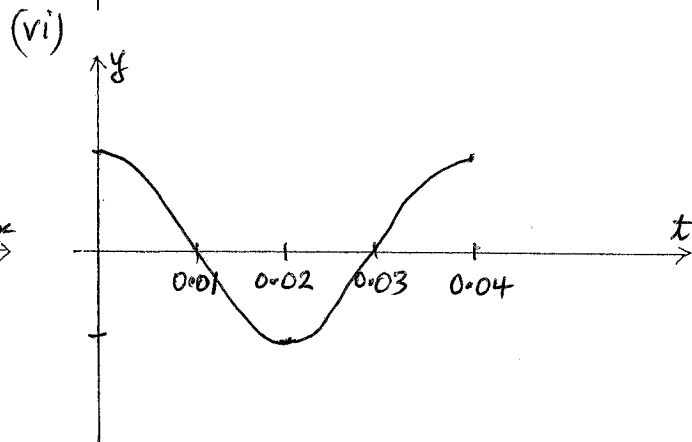
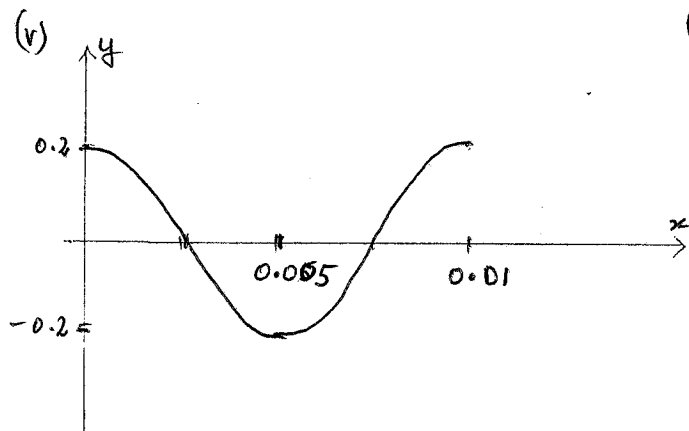
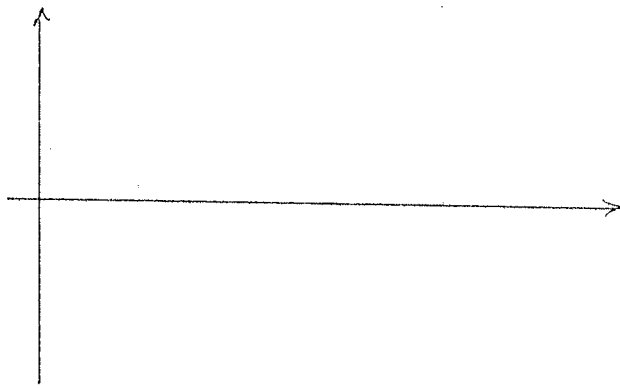
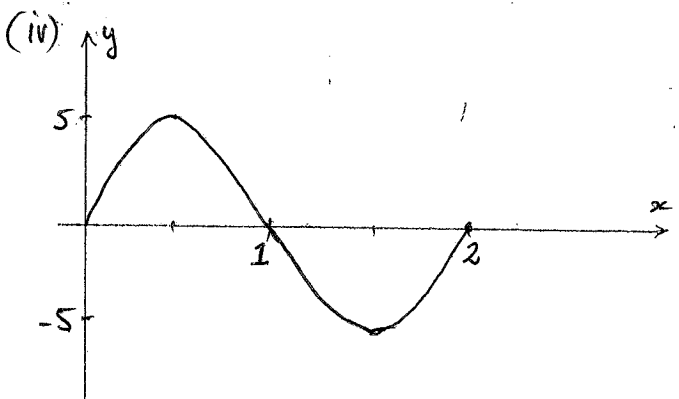
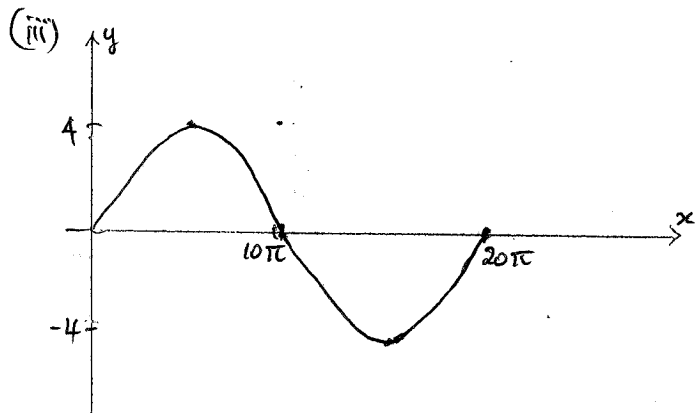
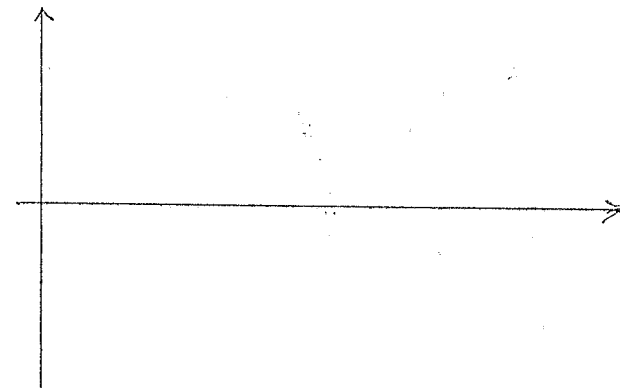
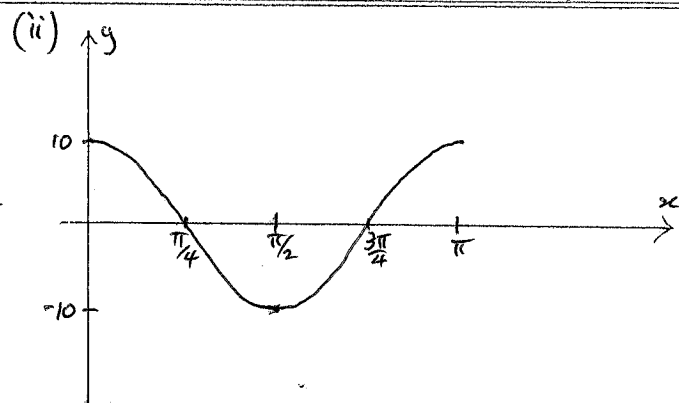
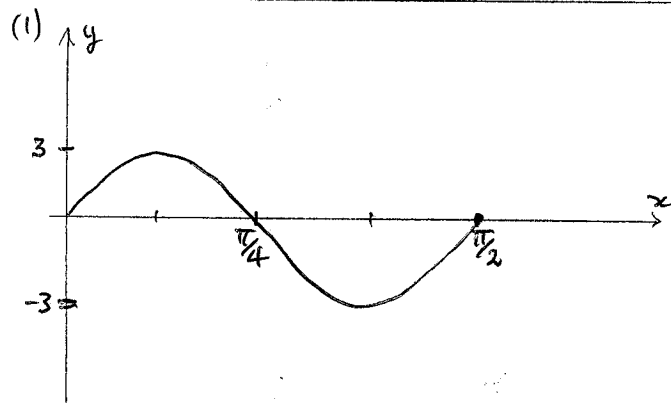


$$y = A \sin Nx$$

$A = \text{amplitude}$

$N = \text{no. of waves to fit between } 0 \rightarrow 2\pi$

ANSWERS:- worksheet(3)



WORKSHEET (4)

Qu(1): Convert the following angles to "Radian" measure.

$$\begin{array}{llll} \text{(i) } 80^\circ & \text{(ii) } 45^\circ & \text{(iii) } 120^\circ & \text{(iv) } 315^\circ \\ = 80^\circ \times \frac{\pi}{180^\circ} & = & & \\ = 4 \times \frac{\pi}{9} & = & & \\ = \frac{4\pi}{9} & = & & \end{array}$$

Qu(2): Convert the following angles to degrees:-

$$\begin{array}{llll} \text{(i) } \frac{\pi}{3} \text{ radians} & \text{(ii) } \frac{3\pi}{2} \text{ radians} & \text{(iii) } \frac{7\pi}{9} \text{ rads.} & \text{(iv) } 2 \text{ rads.} \\ = \frac{\pi}{3} \times \frac{180^\circ}{\pi} & = & = & = \\ = & = & = & = \end{array}$$

Qu(3) Evaluate (use "RAD" mode!)

$$\begin{array}{llll} \text{(i) } \tan 1.8 & \text{(ii) } \cos 3.7 & \text{(iii) } \sin 0.4 & \text{(iv) } \cot 2.6 \\ = & = & = & = \end{array}$$

Qu(4) Solve; leaving answers in "radians" to 3 significant figures.

$$\text{(i) } \cos x = 0.8 \quad \text{(ii) } \tan x = 1.85 \quad \text{(iii) } \sec x = 1.85$$

Qu(5) Sketch graphs of the following:-

$$\text{(a) } y = 10 \sin 2x \quad \text{(b) } y = 4 \cos\left(\frac{x}{2}\right)$$

Qu(6) Simplify the following trig. expressions:-

$$\text{(a) } \frac{\cos^2 x + \sin^2 x}{\sin x} \quad \text{(b) } \tan \theta \times \sin(90-\theta) \quad \text{(c) } \frac{1}{4 - 4 \cos^2 A}$$

WORKSHEET (5)

Quest 1: Find the a) Amplitude & b) Period of the following waves.

(i) $y = 5 \sin 6x$ (ii) $y = \cos \frac{x}{2}$ (iii) $y = \frac{\sin(40\pi x)}{4}$

Quest 2 Convert into RADIANS!

(i) 210° (ii) 315° (iii) 111° (iv) $56^\circ 25'$

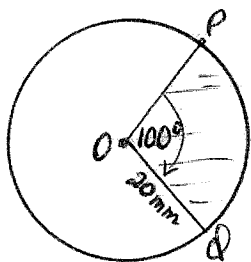
Quest 3 Convert into DEGREES!

(i) $\frac{3\pi}{4}$ rads (ii) $\frac{5\pi}{3}$ rads (iii) 4 rads. (iv) 0.555 rads

Quest 4 Solve, find θ in RADIANS! (ie $0 \leq \theta < 2\pi$)

(i) $\sin \theta = \frac{1}{2}$ (ii) $\cos \theta = 0.123$ (iii) $\tan \theta = 1$

Quest 5



A circle of radius 20 mm subtends an angle of 100 degrees at its centre to form a Sector.

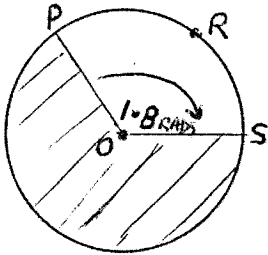
- Find (i) The length of the "arc PQ"
(ii) The Area of the Sector POQ

WORKSHEET 6

Quest 1 Find the equation of the tangent to $y = \sin\left(\frac{x}{2}\right)$ at $P\left(\frac{\pi}{3}, \frac{1}{2}\right)$

Quest 2 Find the area between $y = \sin 2x$ & the x -axis from $x=0$ to $x = \frac{\pi}{2}$

Quest 3

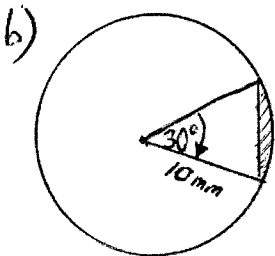


(i) Find the shaded area of this circle centre, O with radius of 50 mm.

(ii) Find the length of the "Arc PRS"

Quest 4

a) Use $l = r\theta$ to find the angle at the centre of a circle of radius 20 cm, if the arc it produces is 5 cm long.



Find the (shaded) area of a segment of a circle of radius 10 mm, ~~subtended~~ made by an angle of 30° at its centre.

ANSWERS - Radian Measure

Worksheet ①

(A) 1. $\frac{2\pi}{3}$ 2. $\frac{\pi}{9}$ 3. $\frac{\pi}{4}$ 4. $\frac{\pi}{3}$ 5. $\frac{3\pi}{2}$ 6. $\frac{5\pi}{3}$ 7. 1.25 rads

(B) 8. 90° 9. 225° 10. 80° 11. 30° 12. 54° 13. 114.6° 14. 300° 15. 48.7°

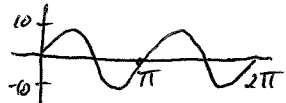
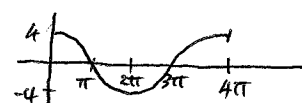
(C) 16. 0.5 rads 17. 1.5 rads 18. 3 rads 19. 1.8 rads

(D) 20. i) $l = 50 \text{ cm}$; $A = 500 \text{ cm}^2$ 21. i) $l = 7 \text{ cm}$; $A = 17.45 \text{ cm}^2$

Worksheet ④

① (i) $\frac{4\pi}{9}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{2\pi}{3}$ (iv) $\frac{7\pi}{4}$ ② (i) 60° (ii) 270° (iii) 140° (iv) 114.6°

③ (i) -4.286 (ii) -0.848 (iii) 0.389 (iv) -1.662 ④ (i) 0.6435 rads (ii) 1.075 rads (iii) 0.9997 rads

⑤ (a)  (b)  ⑥ (a) $\text{cosec } x$ (b) $\sin x$ (c) $\frac{1}{4} \text{cosec}^2 A$.

Worksheet ⑤

① (i) $A = 5$, $T = \frac{\pi}{3}$ (ii) $A = 1$, $T = 4\pi$ (iii) $A = \frac{1}{4}$, $T = 0.05$

② (i) $\frac{7\pi}{6}$ (ii) $\frac{\pi}{4}$ (iii) 1.937 rads (iv) 0.9847 rads

③ (i) 135° (ii) 300° (iii) $229^\circ 11'$ (iv) $31^\circ 48'$

④ (i) $\theta = \frac{\pi}{6}$, $\frac{5\pi}{6}$ (ii) $\theta = 1.4475$ or 4.8357 rads (iii) $\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

⑤ (i) $l = 34.9 \text{ mm}$ (ii) $A = 349.07 \text{ mm}^2$.

Worksheet ⑥

① $y'(\frac{\pi}{3}) = \frac{\sqrt{3}}{4} = m$, Equ: $3x - 4\sqrt{3}y + (2\sqrt{3} - \pi) = 0$

② 1 square unit.

③ (i) 5604 mm^2 (ii) 90 mm

④ (a) $\theta = 0.25$ rads. (b) $A = 50(\frac{\pi}{6} - \frac{1}{2}) \approx 1.18 \text{ mm}^2$