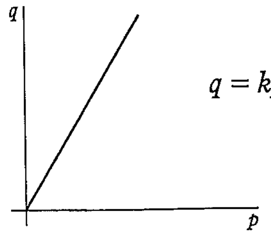
Types of proportionality

Direct and inverse proportionality

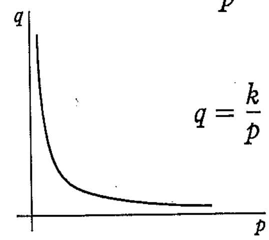
When two variables p and q are **directly proportional** ($p \propto q$), the graph looks like this.



The multiplier for q = the multiplier for p.

The ratio $\frac{q}{p}$ is constant and equal to the gradient of the line.

When two variables p and q are **inversely proportional** ($q \propto \frac{1}{p}$), the graph looks like this.



The multiplier for $q = \frac{1}{(\text{multiplier for } p)}$.

The product pq is constant.

Proportional quantities and their graphs ► page 10

Example

Complete this table given that p is inversely proportional to q.

p	3	6	60	15
q	20			

Start by working out the multipliers for successive values of p.

Then use the corresponding multipliers to work out the values of q.

		$\times 2$		$\times 10$		$\times \frac{1}{4}$	
p	3		6		60		15
q	20		10		1		4
		$\times \frac{1}{2}$		$\times \frac{1}{10}$		$\times 4$	

Other types of proportionality

There are three more types of proportion which you need to know. Notice the effect on the values in the tables.

$q \propto p^2$ $\quad q = kp^2 \quad$ multiplier for q = (multiplier for p)^2

		$\times 2$		$\times 3$		$\times 5$	
p	2		4		12		60
q	10		40		360		9000
		$\times 4$		$\times 9$		$\times 25$	

$q \propto p^3$ $\quad q = kp^3 \quad$ multiplier for q = (multiplier for p)^3

		$\times 2$		$\times 3$		$\times \frac{1}{2}$	
p	1		2		6		3
q	3		24		648		81
		$\times 8$		$\times 27$		$\times \frac{1}{8}$	

$q \propto \frac{1}{p^2}$ $\quad q = \frac{k}{p^2} \quad$ multiplier for $q = \frac{1}{(\text{multiplier for } p)^2}$

		$\times 2$		$\times 10$		$\times \frac{1}{5}$	
p	1		2		20		4
q	2		$\frac{1}{2}$		$\frac{1}{200}$		$\frac{1}{8}$
		$\times \frac{1}{4}$		$\times \frac{1}{100}$		$\times 25$	

You can find the value of k by substituting a pair of known values in the relevant equation: in the last example, q is 2 when p is 1, so $k = qp^2 = 2$.

Fitting functions to data ► page 40

1 Copy and complete the table of values on the assumption that
(a) y is proportional to x^2, (b) y is proportional to $\frac{1}{x}$.

x	6	24	48	120
y		100		

2 y is inversely proportional to the square of x, and $y = 10$ when $x = 2$.
(a) Write down an equation connecting y and x.
(b) Calculate (i) y when $x = 4$, (ii) x when $y = 40$.

3 The Highway Code gives this table for the braking distance of cars.

Speed in miles per hour (x)	30	40	50	60	70
Braking distance in feet (y)	45	80	125	180	245

(a) y is proportional to x^2. Write an equation connecting y and x^2.
(b) Use your equation to find the braking distance in feet for a speed of 75 m.p.h.
(c) What is the speed in miles per hour when the braking distance is 400 feet?

4 The density of a gas is inversely proportional to its volume.
What happens to the density when the volume is increased by a factor of 1·5?

5 The energy stored in a battery is proportional to the square of the diameter of the battery, for batteries of the same height.
One battery has a diameter of 2·5 cm and stores 1·6 units of energy.
Another has a diameter of 1·5 cm.
Calculate the energy stored in the second battery. ULEAC

6 (a) Copy and complete the following table if $d \propto t^2$ and $F \propto \frac{1}{d}$.

t	3	30	15	5	50	100
d	180					
F	1000					

(b) Write down equations connecting (i) d and t, (ii) F and d.

7 The frequency of sound is inversely proportional to the wavelength.
The lowest audible sound has a frequency of 20 Hertz and a wavelength of 16·5 metres.
(a) A sound has wavelength 1 metre.
What is its frequency?
(b) The highest audible sound has a frequency of 15 000 Hertz.
What is its wavelength? MEG (SMP)

8 In the table Q is proportional to the cube of P.
Calculate s and t.

P	0·8	t	6
Q	s	13·5	108

Answers and hints ► page 104

Types of proportionality (page 12)

1 (a)

x	6	24	48	120
y	6·25	100	400	2500

(b)

x	6	24	48	120
y	400	100	50	20

2 (a) *Substitute known values in* $y = \frac{k}{x^2}$ *to find the value of* k.

$10 = \frac{k}{4}$

$k = 40$

So $y = \frac{40}{x^2}$ or $yx^2 = 40$

(b) (i) $y = 2{\cdot}5$ (ii) $x = 1$ or $^-1$

3 (a) *Substitute a pair of values in* $y = kx^2$ *to find the value of* k.

$45 = 30^2k = 900k$

$k = \frac{45}{900} = \frac{1}{20}$

So $y = \frac{x^2}{20}$ or $20y = x^2$

(b) When $x = 75$, $20y = 75^2 = 5625$

$y = 281{\cdot}25$

The braking distance is 280 feet (to 2.s.f.).

(c) When $y = 400$, $20 \times 400 = x^2$

$x^2 = 8000$

$x = 89{\cdot}4$

The speed is 89 m.p.h. (to 2 s.f.).

4 The density is reduced by a factor of 1·5 (or equivalent answer).

5 $1{\cdot}6 = k \times 2{\cdot}5^2$

$k = \frac{1{\cdot}6}{2{\cdot}5^2}$

Energy stored in second battery

$= \frac{1{\cdot}6}{2{\cdot}5^2} \times 1{\cdot}5^2 = 0{\cdot}576$ units of energy

It is usually best to leave all the calculation to the end, especially if rounding is involved.

6 (a)

t	3	30	15	5	50	100
d	180	18000	4500	500	50000	200000
F	1000	10	40	360	3·6	0·9

(b) *Start by calculating the constant of proportionality,* k.

(i) $d = kt^2$

$k = \frac{180}{9} = 20$

$d = 20t^2$

(ii) $F = \frac{k}{d}$

$k = Fd$

$= 180000$

$Fd = 180\,000$ or $F = \frac{180\,000}{d}$

7 (a) The constant of proportionality is $20 \times 16{\cdot}5 = 330$.

The frequency for a 1 m wavelength is $330 \div 1 = 330$ Hertz.

(b) Wavelength $= \frac{330}{15000} = 0{\cdot}022$ metres

8 *Use the pair of values you know to find the constant of proportionality,* k, *and then use this to find* s *and* t *by substitution.*

$k = \frac{108}{6^3} = 0{\cdot}5$

$Q = kP^3 = 0{\cdot}5P^3$

So $s = 0{\cdot}5 \times 0{\cdot}8^3 = 0{\cdot}256$

Similarly, $13{\cdot}5 = 0{\cdot}5t^3$, so

$t^3 = 13{\cdot}5 \div 0{\cdot}5 = 27$ and

$t = \sqrt[3]{27} = 3$

More help or practice

Direct and inverse proportionality
(using the multiplier rule and constant ratio rule)
► Book Y3 pages 68 to 74, Book Y4 pages 62 to 63

Other types of proportionality ($p \propto q^2, p \propto q^3, p \propto \frac{1}{q^2}$)
► Book Y4 pages 64 to 67