

# YEAR 11 TEST - 3 UNIT - FUNCTIONS and GRAPHS

- ① Given  $f(x) = 4x - x^2$ , find: Name: \_\_\_\_\_  
Mark: \_\_\_\_\_
- a)  $f(-2)$    b)  $f(3x)$    c)  $f(a-b) - f(a)$

- ② Find the natural domain and range of:

a)  $y = \frac{1}{3x-6}$    b)  $y = \sqrt{x-1}$    c)  $y = \sqrt{9-x^2}$

d)  $y = \frac{1}{x^2}$    e)  $y = \sqrt{x-1} + \sqrt{x-2}$    (✓)

- ③ Find the centre and radius of the circle

$$x^2 + y^2 - 6x + 3 = 0.$$

- ④ Sketch each, showing intercepts clearly:

a)  $y = \frac{1}{x-2}$    b)  $y = -\sqrt{4-x^2}$    c)  $y = (x-1)^3$

d)  $y = 2 - (x+1)^2$    (x+1=2, x=1, x-1=2, x=3)   e)  $y = |x^2 - 4|$    f)  $y = -\sqrt{x-1}$

- ⑤ a) Sketch  $y = (x+2)^2 - 1$  in the domain  $-3 \leq x \leq 0$ .

b) Find the coordinates of the vertex

c) Find the range for the given domain.

- ⑥ Prove that  $y = \frac{2x}{x^2+4}$  is an odd function.

- ⑦ Sketch the function defined by:

a)  $y = 3x, x > 1$    b)  $y = \frac{x-1}{x^2-1}$   
 $= x^2, x \leq 1$

- ⑧ Sketch the intersection of the regions  $y > x^2 + x$ ,  $y \geq 1 - x$

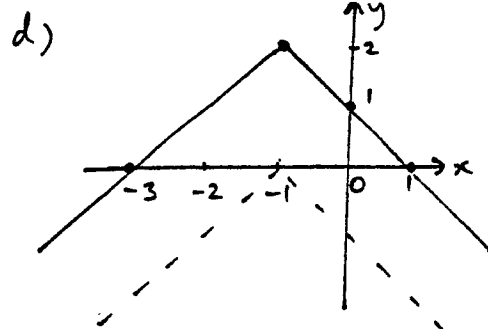
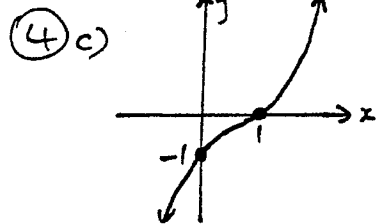
- ⑨ Sketch: a)  $y = \frac{|x|}{x^2}$  (Hint: Use the definition of  $|x|$ )

b)  $y = \frac{1}{x^2+2}$  (Hint: Use reciprocals).

1) a)  $f(-2) = -8 - (-2)^2$   
 $= -8 - 4$   
 $= -12$

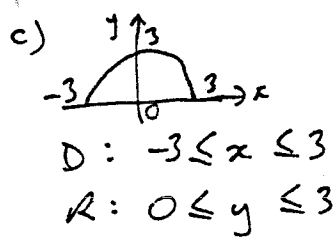
b)  $f(3x) = 12x - 9x^2$

c)  $f(a-b) - f(a) = 4(a-b) - (a-b)^2 - (4a - a^2)$   
 $= 4a - 4b - a^2 + 2ab - b^2 - 4a + a^2$   
 $= -4b + 2ab - b^2$



2) a) D: all  $x, x \neq 2$   
 R: all  $y, y \neq 0$

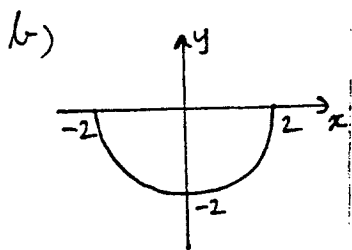
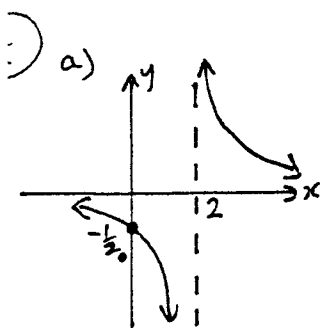
b) D:  $x \geq 1$   
 R:  $y \geq 0$



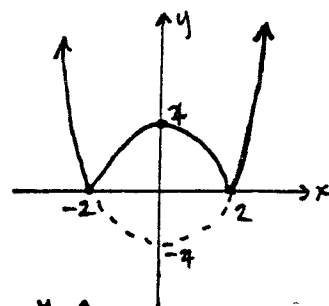
d) D: all  $x$ , except  $x=0$   
 R:  $y > 0$

e)  $x-1 \geq 0$  and  $x-2 \geq 0$   
 $\therefore x \geq 1$  and  $x \geq 2$   
 $\therefore D: x \geq 2$   
 R:  $y \geq 1$

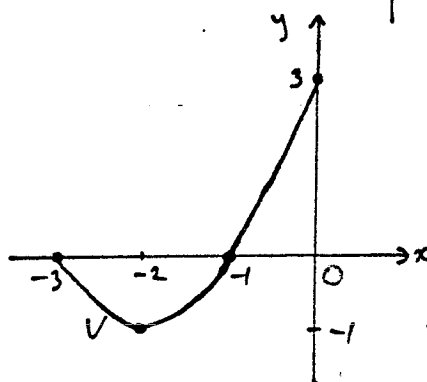
3)  $x^2 - 6x + y^2 = -3$   
 $(x^2 - 6x + 9) + y^2 = -3 + 9$   
 $(x-3)^2 + y^2 = 6$   
 $\therefore$  centre is  $(3, 0)$   
 radius is  $\sqrt{6}$  units.



f) See over



5) a)



b) Vertex is  $x = -2, y = (-2+2)^2 - 1 = -1$   
 $\therefore V(-2, -1)$

c) Range is  $-1 \leq y \leq 3$ .

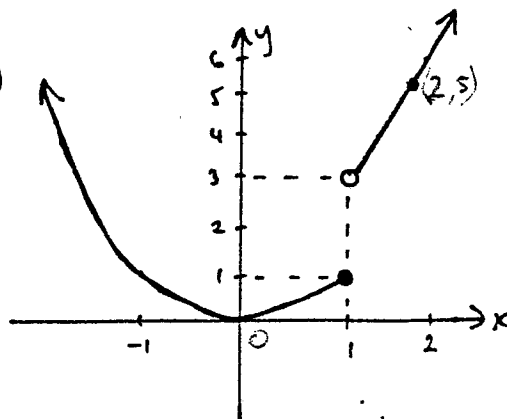
6) "Odd" function when  $f(x) = -f(x)$

$$f(x) = \frac{2(x)}{(-x)^2 + 4}, \quad -f(x) = -\left(\frac{2x}{x^2 + 4}\right)$$

$$= \frac{-2x}{x^2 + 4} = \frac{-2x}{x^2 + 4}$$

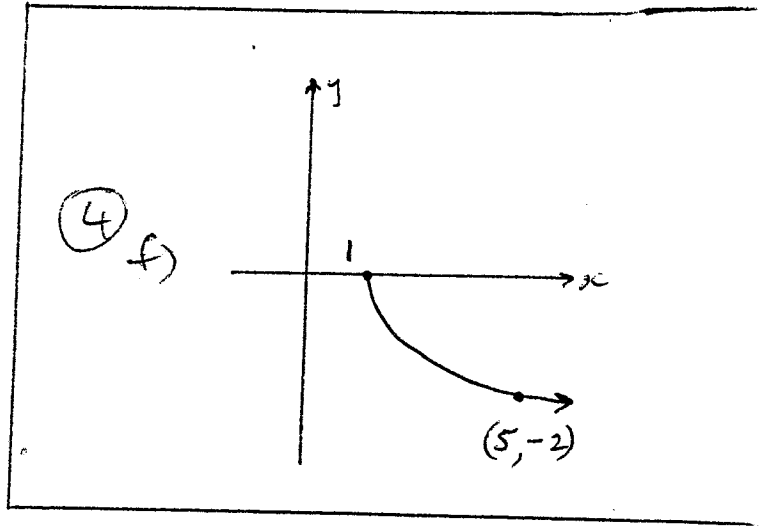
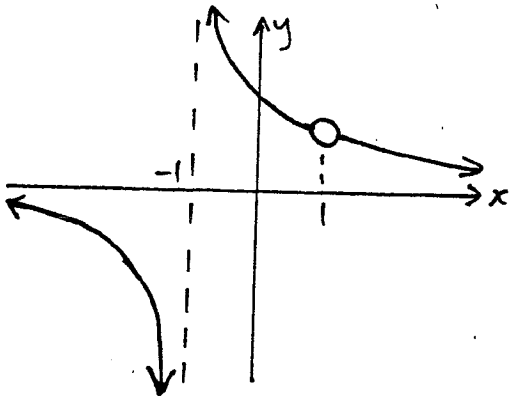
$\therefore$  an odd function.  $= f(-x)$

7) a)

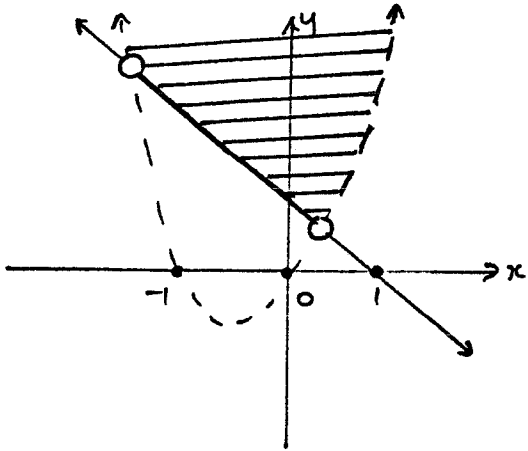


7) b)  $y = \frac{x-1}{(x-1)(x+1)} \quad (x \neq \pm 1)$

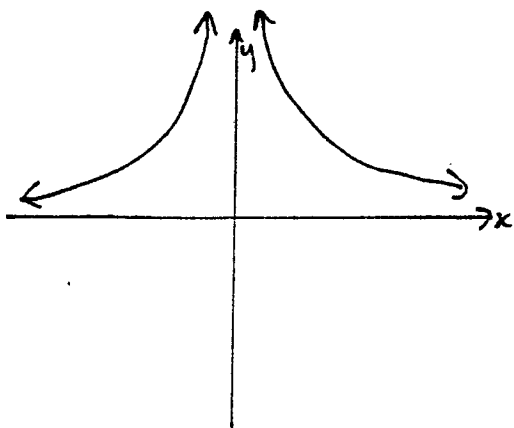
$= \frac{1}{x+1} \quad (x \neq \pm 1)$



8)  $y > x(x+1)$        $y \geq 1-x$



9) a)  $y = \frac{|x|}{x^2} \begin{cases} = \frac{x}{x^2} = \frac{1}{x} \text{ (if } x > 0) \\ = \frac{-x}{x^2} = -\frac{1}{x} \text{ (if } x < 0) \end{cases}$



b) First, sketch  $y = x^2 + 2$ .  
then take reciprocal  $y$  values

