Randwick Boys' High School

Mathematics Department
Two Unit Mathematics
Yearly Examination

Year Eleven

September 2001

Examiner: F Archinal

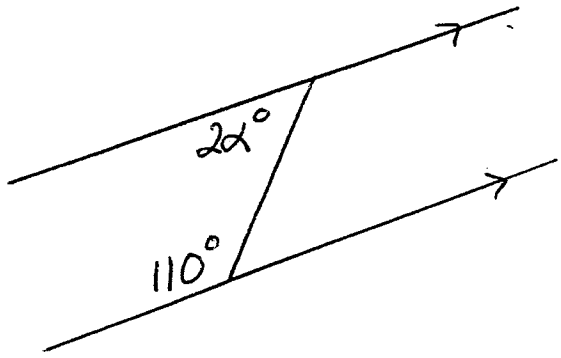
TIME ALLOWED 90 minutes

Candidates may attempt all questions.
Begin each Question on a NEW sheet of paper.
All necessary working should be shown in every question.
Full marks may not be awarded for careless or badly arranged work.
Approved calculators may be used.
Templates without formulae may be used.

Students Name: _____

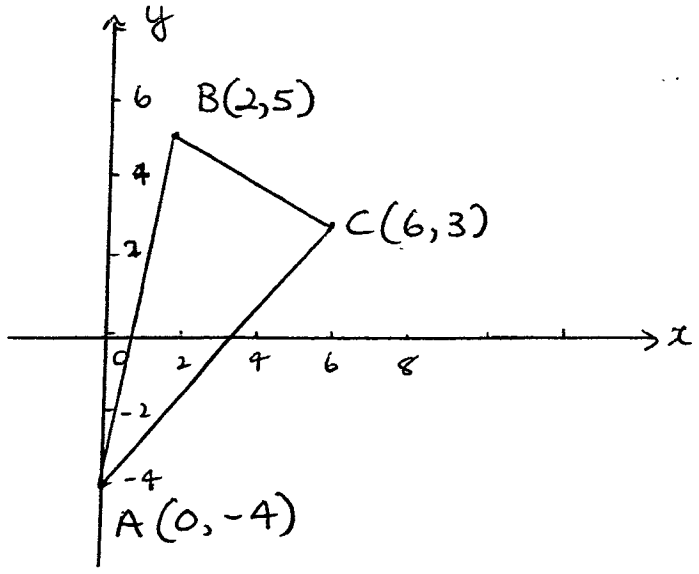
Teachers Name: _____

This paper must be handed in with your answers at the conclusion of the examination

QUESTION 1 (12 marks)	**Marks**
(a) Factorise fully $4x^2 - 36$	2
(b) Solve $5 - 2x \leq 3$ and graph the solution on a number line.	2
(c) $f(x)$ is defined by $f(x) = \begin{cases} x^2 & \text{when } x \leq -1 \\ 3 & \text{when } x > -1 \end{cases}$ Evaluate $f(-2) + f(2)$	2 2
(d)  Find the value of α, giving a reason	2
(e) Write down the exact value of $\sin 30^\circ - \cos 45^\circ$, giving your answer as a single fraction	2
(f) State the domain and range of the function $f(x) = \sqrt{1-x}^2$	2

QUESTION 2 (12 Marks)

Marks



A triangle has vertices A(0, - 4) B(2, 5) and C(6,3)

(a) Find the gradient of the line BC — 2

(b) Show that the equation of the line BC is $x + 2y - 12 = 0$ — 2

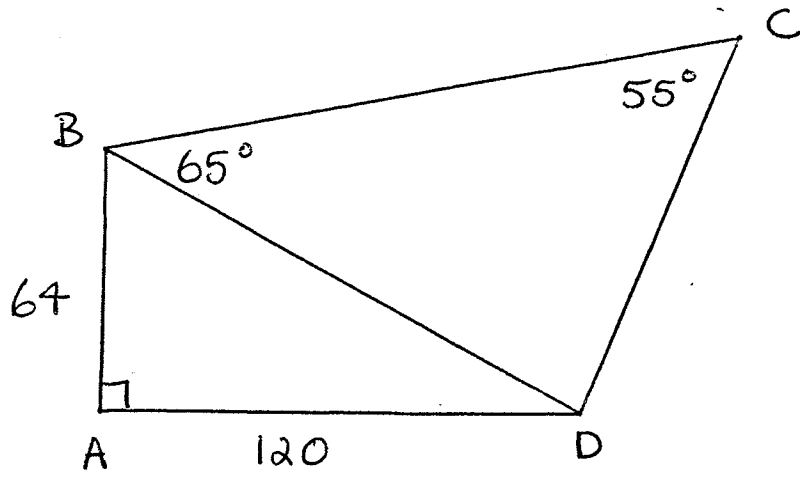
(c) If D is the midpoint of BC, find the coordinates of D. — 1

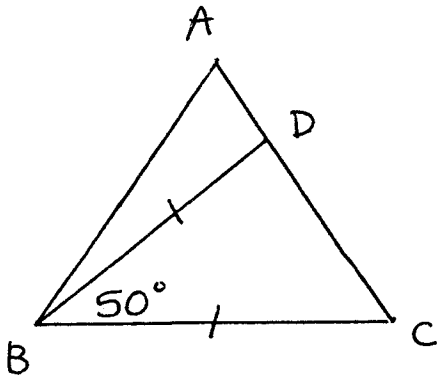
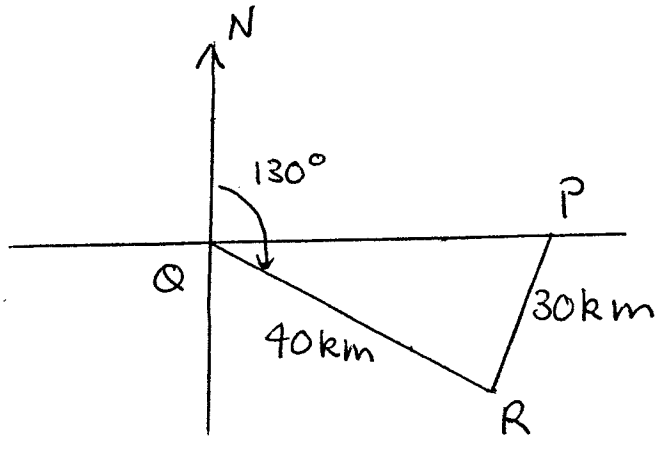
(d) Find the equation of the line through D and perpendicular to BC. — 2

(e) The line in (d) passes through A. What does this tell us about ΔABC. Why? — 2

(f) What is the acute angle between BC and the x-axis? (to the nearest degree) — 1

(g) Find the perpendicular distance from A to BC — 2

QUESTION 3 (12 Marks)	**Marks**
(a) Factorise $x^3 + 8$	1
(b) If $x = \frac{1}{1+\sqrt{2}}$, Show that $x + \frac{1}{x} = 2\sqrt{2}$	3
(c) Sketch the function $y = \lvert x \rvert - 1$ showing x and y intercepts	2
(d) i) Factorise $6x^2 + 5x - 6$ ii) Hence, solve $6x^2 + 5x - 6 = 0$	3
(e)  ABCD is a quadrilateral in which AB = 64cm, AD = 120cm, ∠BAD = 90°, ∠DBC = 65°, ∠BCD = 55°. i) Show that the length of BD is 136cm. ii) Find the length of side DC correct to four significant figures	3

QUESTION 4 (12 Marks)	**Marks**
(a) If the point (-1, 2) is on the line $ax - 4y + 11 = 0$, find the value of a	2
(b) Solve $1 + 2\sin\theta = 0$ for $0° \leq \theta° \leq 360°$	3
(c)  In the triangle above AB = AC BD = BC $\angle$ DBC = 50° i) Find $\angle$ BCD, giving reasons ii) Hence, find $\angle$ BAC, giving reasons	4
(d)  i) Use the sine rule in the Δ PQR to find $\angle$ QPR. (to the nearest degree) ii) Hence, find the bearing of P from R.	3

QUESTION 5 (14 Marks) — **Marks**

(a) Show that $f(x) = \dfrac{3x}{16 + x^2}$ is an odd function., — 3

(b) Find the values of x and y if $x + y\sqrt{3} = (2 - \sqrt{3})^2$ — 3

(c) What values of x are excluded from the domain of $f(x) = \dfrac{3}{x^2 - x}$ — 2

(d) Graph the following functions, showing important features. — 6

i) $y = 4 - x^2$

ii) $y = 2^x$

iii) $y = \begin{cases} x^2 & x \le 0 \\ x & 0 < x \le 1 \\ 1 & x > 1 \end{cases}$

QUESTION 6 (12 Marks) — **Marks**

(a) — 5

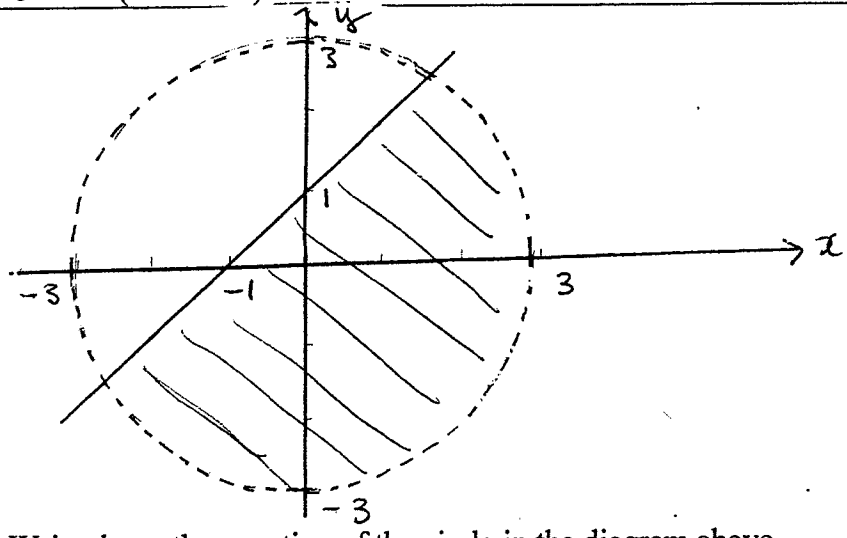


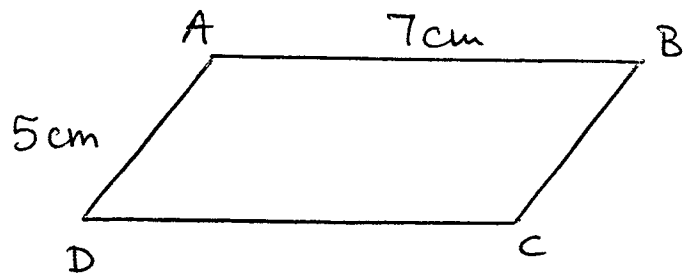
Diagram not to scale

i) Write down the equation of the circle in the diagram above

ii) The equation of the given line is $y = x + 1$. What are the coordinates of the points of intersection of the line and the circle? (in surd form)

iii) Write two inequalities to describe the shaded region

(b)



4

Two adjacent sides of a parallelogram are 5cm and 7cm in length. The shorter diagonal AC is 6cm long.

i) Find the size of $\angle ADC$ (to the nearest minute)

ii) Find the area of the parallelogram. (to the nearest centimetre)

(c) Show that

3

$$\frac{\sin\theta}{1-\cos\theta} - \frac{\sin\theta}{1+\cos\theta} = 2\cot\theta$$

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Question 1

a. $4x^2-36 = 4(x^2-9)$

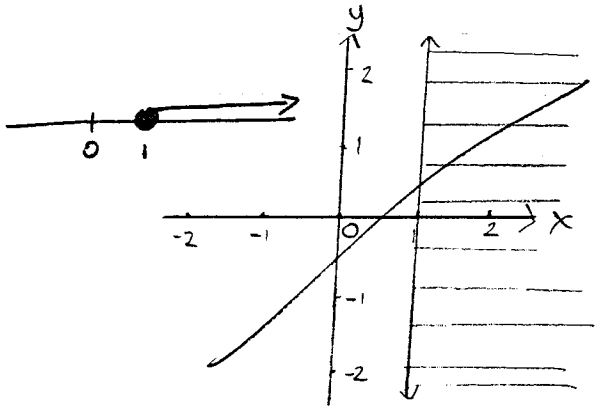
$= 4(x+3)(x-3)$ ✓

b. $5-2x \leq 3$

$-2x \leq 3-5$

$-2x \leq -2$

$x \geq 1$ ✓



c. $f(-2) = x^2 = (-2)^2 = 4$

$f(2) = 3$

$f(-2)+f(2) = 4+3 = 7$ ✓

d. $2x^\circ + 110^\circ = 180^\circ$ (cointerior $\angle$s of parallel line)

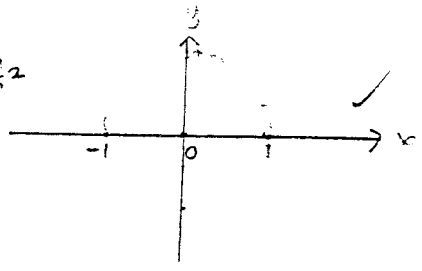
$2x^\circ = 70^\circ$

$x = 35^\circ$ ✓

e. $\sin 30^\circ = \frac{1}{2}$ $\quad \sin 30^\circ - \cos 45^\circ = \frac{1}{2} - \frac{1}{\sqrt{2}}$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad = \frac{1}{2} - \frac{\sqrt{2}}{2} = \frac{1-\sqrt{2}}{2}$ ✓

F. $y = \sqrt{4-x^2}$



✓

Domain : $-1 \leq x \leq 1$ ✓

Range : $0 \leq y \leq 1$ ✓

Question 2

a. B(2,5) and C(6,3) $\quad m_{BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-5}{6-2} = \frac{-2}{4} = -\frac{1}{2}$ ✓

b. $y-y_1 = m(x-x_1)$

$y-5 = -\frac{1}{2}(x-2)$ ✓

c $y-5 = -\frac{1}{2}x+1$ $\quad \times 2$

$2y-10 = -x+2$ ✓

$x+2y-12 = 0$

c. Midpoint of BC :

$x = \frac{2+6}{2} = 4$

$y = \frac{5+3}{2} = 4$

$\therefore D(4,4)$ ✓

d. $M_{BC} = -\frac{1}{2} \rightarrow M_1 = -\frac{1}{2}$

$M_2 = 2$ (the line are perpendicular $M_1 \times M_2 = -1$)

$y - y_1 = m(x - x_1)$

$y - 4 = 2(x-4)$

$y - 4 = 2x - 8$

$0 = 2x - y - 4$ ✓

e. The $\triangle ABC$ is divided into 2 congruent $\triangle$s.

$\triangle ADB$, $\triangle ADC$

S : $BD = CD$ (D is the midpoint of BC)

A : $\angle BDA = \angle CDA$ ($AD \perp BC$)

S : AD is common

$\therefore \triangle ADB \equiv \triangle ADC$ (SAS Test)

Isosceles $\triangle$

$\therefore \triangle ABC$ is divided into 2 congruent $\triangle$s $\rightarrow$ $\triangle ADB$ and $\triangle ADC$

F. $\tan\theta = \left|-\frac{1}{2}\right|$

Acute $\theta = 26°34'$

$\theta = 27°$ ✓

g. Equation of BC : $x + 2y - 12 = 0$

$A = (0,-4)$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|1x + 2y - 12|}{\sqrt{1+4}}$ ✓

$= \frac{|0 - 8 - 12|}{\sqrt{5}} = \frac{|-20|}{\sqrt{5}} = \frac{20}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{20\sqrt{5}}{5}$

$= 4\sqrt{5}$ ✓

Question 3

a. $x^3 + 8 = (x+2)(x^2 - 2x + 4)$ ✓

b. $x = \frac{1}{1+\sqrt{2}} = \frac{1}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{-1}$

$\frac{1}{x} = \frac{1}{\frac{1}{1+\sqrt{2}}} = 1 \times \frac{1+\sqrt{2}}{1} = 1+\sqrt{2}$

$x + \frac{1}{x} = \frac{1-\sqrt{2}}{-1} + \frac{1+\sqrt{2}}{1}$

$= \frac{(1)(1-\sqrt{2}) - 1(1+\sqrt{2})}{(-1)(1)}$ ✓

$= \frac{1 - \sqrt{2} - 1 - \sqrt{2}}{-1}$

$= \frac{-2\sqrt{2}}{-1}$

$= 2\sqrt{2}$

c. $y = |x| - 1$ x-intercept, $y = 0$

$0 = |x| - 1$

$1 = |x|$

$x = \pm 1$

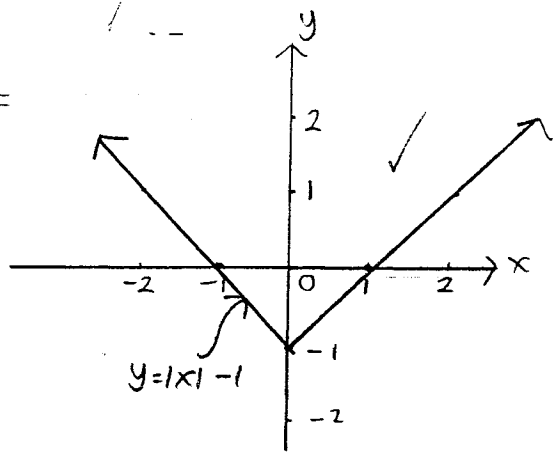
x	-2	-1	0	1	2
y	1	0	-1	0	1

y-intercept, $x = 0$

$y = |x| - 1$

$y = 0 - 1$

$y = -1$



d. (i) $6x^2+5x-6$

$p = -36x^2$ $\quad 6x^2+9x-4x-6$

$s = 5x$ $\quad 3x(2x+3)-2(2x+3)$

$f = (9x)(-4x)$ $\quad (3x-2)(2x+3)$ ✓

(ii). $6x^2+5x-6=0$

$(3x-2)(2x+3)=0$

$3x-2=0$ or $2x+3=0$

$3x=2$ $\quad 2x=-3$

$x=\frac{2}{3}$ ✓ $\quad x=-\frac{3}{2}$ ✓

e. (i) $BD^2 = AB^2+AD^2$

$= (64)^2+(120)^2$

$= 4096+14400$

$BD^2 = 18496$

$BD = 136$ cm ✓

(ii) $\frac{DC}{\sin 65^\circ} = \frac{BD}{\sin 55^\circ}$

$\frac{DC}{\sin 65^\circ} = \frac{136}{\sin 55^\circ}$ ✓

$DC = \frac{136}{\sin 55^\circ} \times \sin 65^\circ$

$= 150.5$ cm ✓

Question 4

a. $ax-4y+11=0$

$a(-1)-4(2)+11=0$

$-a-8+11=0$

$-a+3=0$

$a=3$ ✓

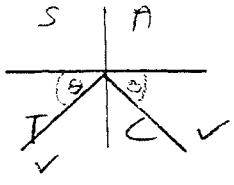
b. $1+2\sin\theta=0$

$2\sin\theta=-1$

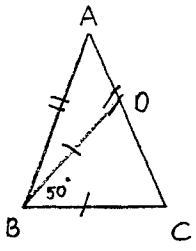
$\sin\theta=-\frac{1}{2}$

(Acute $\theta = 30^\circ$)

$\theta = 210^\circ, 330^\circ$ ✓



✓ ✓

c. 

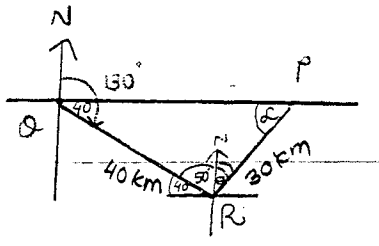
(i) $\angle BCD = \frac{180^\circ-50^\circ}{2}$ ($\triangle BCD$ is an isosceles $\triangle$, sum of $\angle$ in $\triangle$)

$= 65^\circ$ ✓

(ii) $\angle ABC = 65^\circ$ (AB = AC ⇒ given ∴ base $\angle$s of isosceles $\triangle$ are equal)

$\angle BAC = 180^\circ-65^\circ-65^\circ$ (sum of $\angle$ in $\triangle$)

$= 50^\circ$ ✓

d. 

(i) $\angle PQR = 130^\circ-90^\circ = 40^\circ$

$\frac{40\text{ km}}{\sin R} = \frac{30\text{ km}}{\sin 40^\circ}$

$\sin R = \frac{40\text{ km}}{1} \times \frac{\sin 40^\circ}{30\text{ km}}$ ✓

$R = 58^\circ 59' = 59^\circ$

(ii) $\angle QRP = 180^\circ-40^\circ-59^\circ$

$= 81^\circ$

The bearing of P from R =

$81^\circ-50^\circ = 031^\circ T$ ✓

Question 5

a. $f(x) = \frac{3x}{16+x^2}$

$f(-x) = \frac{3(-x)}{16+(-x)^2}$

$= \frac{-3x}{16+x^2}$ ✓

$= -\frac{3x}{16+x^2} = -f(x)$

∴ It is an odd function ✓

b. $(2-\sqrt{3})^2 = (2-\sqrt{3})(2-\sqrt{3})$

$= 4 - 4\sqrt{3} + 3$

$= 7 - 4\sqrt{3}$

$x + y\sqrt{3} = 7 - 4\sqrt{3}$ ✓

$x = 7$

$y = -4$ ✓

c. $f(x) = \frac{3}{x^2 - x}$

$x^2 - x \neq 0$

$x(x-1) \neq 0$

$x \neq 0$ OR $x \neq 1$ ✓

d. (i). $y = 4 - x^2$

x-intercept, $y = 0$

$0 = 4 - x^2$

$x^2 = 4$

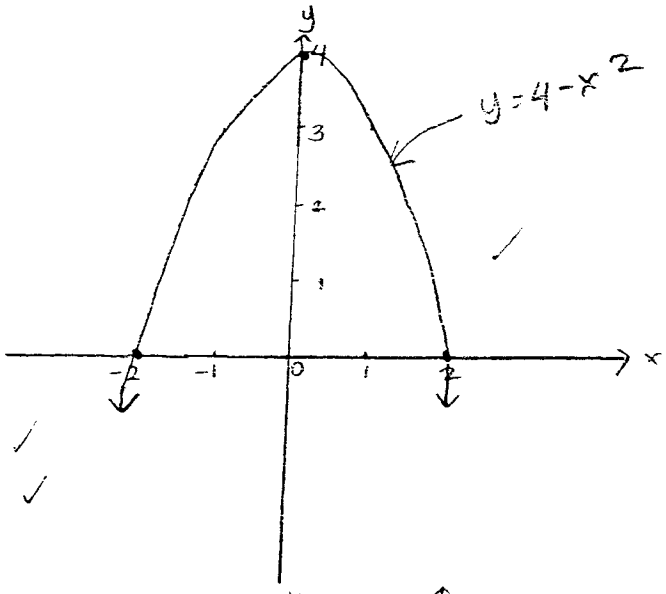
$x = \pm 2$

y-intercept, $x = 0$

$y = 4$

Domain : all real x ✓

Range : $y \leq 4$ ✓



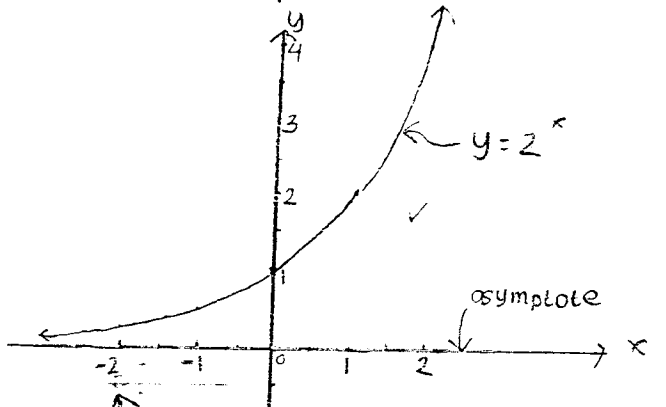
(ii). $y = 2^x$

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

Horizontal asymptote

Range : $y > 0$ ✓

Domain : all real x ✓

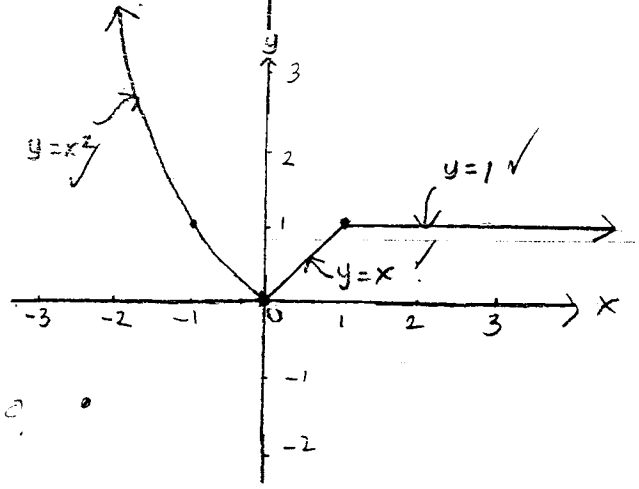


(iii)

$$y = \begin{cases} x^2 & x \leq 0 \\ x & 0 < x \leq 1 \\ 1 & x > 1 \end{cases}$$

For x^2

x	0	-1	-2	-3
y	0	1	4	9



Question 6

a. (i). $x^2+y^2=9$ ✓

(ii). The intersection of the line and the circle:

$x^2+y^2=9$ .... (i)

$y=x+1$ ..... (ii)

Subs (ii) into (i)

$x^2+(x+1)^2=9$

$x^2+x^2+2x+1=9$

$2x^2+2x-8=0$

$x^2+x-4=0$

$x_{12}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-1\pm\sqrt{1+16}}{2}$

$=\frac{-1\pm\sqrt{17}}{2}$

$x_1=\frac{-1+\sqrt{17}}{2}$ $\quad x_2=\frac{-1-\sqrt{17}}{2}$

Subs x_1 into (ii) | Sub x_2 into (ii)

$y_1=\frac{-1+\sqrt{17}}{2}+\frac{2}{2}$ | $y_2=\frac{-1-\sqrt{17}}{2}+\frac{2}{2}$

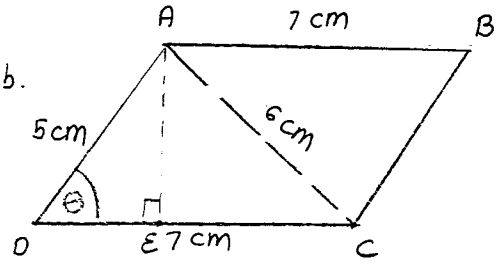
$=\frac{1+\sqrt{17}}{2}$ | $=\frac{1-\sqrt{17}}{2}$

$\therefore\left(\frac{-1+\sqrt{17}}{2},\frac{1+\sqrt{17}}{2}\right)$ ✓ | $\therefore\left(\frac{-1-\sqrt{17}}{2},\frac{1-\sqrt{17}}{2}\right)$ ✓

(iii). ~~$y\leq\sqrt{9-x^2}$~~ and $y\leq x+1$ ✓

~~$x^2+y^2\leq 9$~~ ✓

b.



(i) Let $\angle ADC=\theta$

$AC^2=AD^2+DC^2-2(AD)(DC)\cos\theta$

$36=25+49-2(5)(7)\cos\theta$ ✓

$36=74-70\cos\theta$

$70\cos\theta=38$

$\cos\theta=\frac{38}{70}$ ✓

$\theta=57°7'$ $\quad\therefore\angle ADC=57°7'$ ✓

(ii). $\frac{AE}{\sin 57°7'}=\frac{AD}{\sin 90°}$

$\frac{AE}{\sin 57°7'}=\frac{5}{1}$

$AE=5\times\sin 57°7'$ ✓

$=4.199$ cm

Area of parallelogram $=DC\times AE$ ✓

$=7\text{ cm}\times 4.199\text{ cm}$

$=29.392\text{ cm}^2$

$=29\text{ cm}^2$ ✓

c. $\frac{\sin\theta}{1-\cos\theta}-\frac{\sin\theta}{1+\cos\theta}=2\cot\theta$

LHS: $\frac{\sin\theta(1+\cos\theta)-\sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$=\frac{\sin\theta+\sin\theta\cos\theta-\sin\theta+\sin\theta\cos\theta}{1-\cos^2\theta}$ ✓

$=\frac{2\sin\theta\cos\theta}{1-(1-\sin^2\theta)}$

$=\frac{2\sin\theta\cos\theta}{\sin^2\theta}$ ✓

$=\frac{2\cos\theta}{\sin\theta}$

$=2\cot\theta$

$=$ RHS