ROSE BAY SECONDARY COLLEGE

YEAR 12 TERM 2 EXAMINATION

2006

MATHEMATICS

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.
- Full marks may not be awarded for poor presentation, omission of reasons and incomplete arguments.

Total Marks – 84

- Attempt Questions 1 – 7.
- All questions are of equal value.

Question 1: (start a new page)

a) Find the primitive of $x^4 - 5$ 1

b) Convert $\frac{5\pi}{6}$ radians to degrees. 1

c) Solve $3^{x+2} = \frac{1}{9}$ 2

d) Find $\lim_{x\to 2}\frac{x^2 - x - 2}{x-2}$ 2

e) Solve $\cos\theta = -\frac{1}{\sqrt{2}}$ for $0 \le \theta \le 2\pi$ 2

f) Consider the parabola $x^2 = 12(y+4)$

i) State the co-ordinates of the vertex of the parabola. 1

ii) Determine the co-ordinates of the focus of the parabola and the equation of its directrix. 2

Question 2 (start a new page)

a) The co-ordinates of the points A, B and C are $(0,2)$, $(4,0)$ and (6,-4) respectively.

i) Show this information on a number plane 1

ii) Find the length and gradient of AB 2

iii) Show that the equation of the line L, drawn through C parallel to AB is $x + 2y + 2 = 0$ 2

iv) Find the co-ordinates of D, the point where L intersects the x axis. 1

v) Find the perpendicular distance of the point A from the line L. 2

b) The first three terms of an arithmetic sequence are 12, 8, 4, ……

i) Find the 15th term of this sequence 2

ii) Calculate the sum of the first 50 terms of the sequence. 2

Question 3: (start a new page)

a) Differentiate with respect to x:

i) $3x^3 - 4x + 1$ 1

ii) $\dfrac{x^2}{e^x}$ 2

iii) $\log_e(3-2x)$ 2

iv) $\sqrt{(3x^2-5)}$ 2

b) Find

i) $\int (2x-1)^4 \, dx$ 2

ii) $\int_0^3 \dfrac{dx}{3x+2}$ 3

Question 4: (start a new page)

a) Solve the equation $\log_{10} x - \log_{10}(x-1) = 1$ 2

b) Find the equation of the normal to the curve $y = 2e^{-2x}$ at the point where the curve cuts the y axis. 4

c) If α and β are the roots of the equation $2x^2 - 4x - 5$ evaluate

i) $\alpha + \beta$ 1

ii) $\alpha\beta$ 1

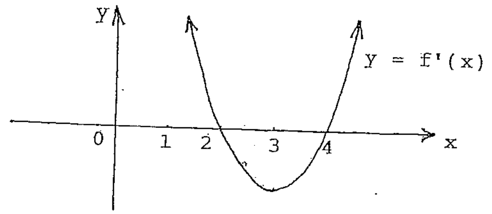
iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ 1

d) Use Simpsons Rule with five function values to estimate the value of

$$\int_1^5 \log_e x \, dx$$ 3

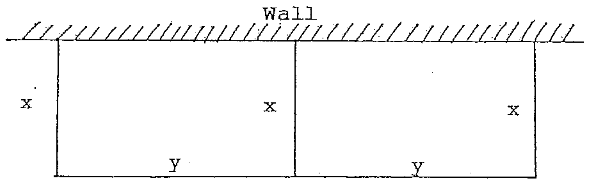
Question 5: (start a new page)

a) Copy the sketch of the following graph of $y = f'(x)$ onto your answer page then, on the same set of axes, draw a possible sketch for $y = f(x)$, given that $f(o) = -1$ 3



b) For what values of k does the equation $x^2 + (k-1)x - (2k+1) = 0$ have two real distinct roots? 4

c)



A farmer has 120 metres of fencing to make two identical rectangular enclosures using an existing wall as one side of each enclosure.

The dimensions of each enclosure are x metres by y metres as shown.

i) Show that $y = 60 - \dfrac{3}{2}x$ 1

ii) Show that the total area of the two enclosures (A m^2) is given by

$$A = 120x - 3x^2$$ 1

iii) Calculate the maximum value of this area 3

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QUESTION 1

a) $\frac{x^5}{5} - 5x + c$

b) $\pi^c = 180°$

$\frac{5}{6}\pi = \frac{5}{6} \times 180$

$= 150°$

c) $3^{x+2} = \frac{1}{9}$

$3^{x+2} = 3^{-2}$

$\therefore x+2 = -2$

$x = -4$

d) $\lim_{x\to 2} \frac{x^2-x-2}{x-2}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$

$= 2+1$

$= 3$

e) $\cos\theta = -\frac{1}{\sqrt{2}}$

cos is negative in quad 2,3

$\theta = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

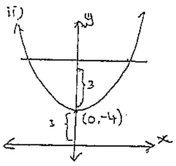
$= \frac{3\pi}{4}, \frac{5\pi}{4}$

f) $x^2 = 12(y+4)$

$x^2 = 4.3\,(y+4)$

i) $\therefore$ Vertex $(0,-4)$

ii)



From

$(x-x_1)^2 = 4a(y-y_1)$

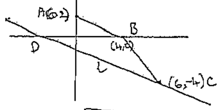
$a = 3$

Eq^n Directrix $y = -7$

Focus $(0,-1)$

QUESTION 2

a) i)



ii) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(0-4)^2+(2-0)^2}$

$= \sqrt{16+4}$

$= \sqrt{20} = 2\sqrt{5}$ units

$m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2-0}{0-4} = -\frac{2}{4} = -\frac{1}{2}$

iii) $grad_L = -\frac{1}{2}$ (since ∥ to AB)

Eq^n L

$(y-y_1) = m(x-x_1)$

$y+4 = -\frac{1}{2}(x-6)$

$-2y-8 = x-6$

$\therefore x+2y+2 = 0$ as req'd

iv) x intercept of L occurs where $y=0$

$x+2{\cdot}0+2 = 0$

$x = -2$

$\therefore D(-2,0)$

v) $d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$ $\quad A=1,\ B=2,\ C=2,\ x_1=0,\ y_1=2$

$= \frac{|1.0+2.2+2|}{\sqrt{1^2+2^2}}$

$= \frac{|6|}{\sqrt{5}} = \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$ units

b) i) $12, 8, 4, \ldots$

$a = 12$ $\quad T_n = a+(n-1)d$

$d = 8-12 = -4$ $\quad T_{15} = 12+(15-1)\times -4$

$n = 15$ $\quad = 12 + 14\times -4$

$T_{15} = ?$ $\quad = -44$

ii) $S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{50} = \frac{50}{2}(2.12+49.-4)$

$= -4{,}300$

QUESTION 3

a) i) $\frac{d}{dx}(3x^3-4x+1) = 9x^2-4$

ii) $\frac{d}{dx}\left(\frac{x^2}{e^x}\right)$ $\quad u = x^2 \quad v = e^x$ $\quad u' = 2x \quad v' = e^x$

$= \frac{vu'-uv'}{v^2}$

$= \frac{e^x\times 2x - x^2\times e^x}{(e^x)^2}$

$= \frac{xe^x(2-x)}{e^{2x}}$

$= \frac{x(2-x)}{e^x}$

iii) $\frac{d}{dx}\log_e(3-2x) = \frac{1}{3-2x}\times -2$

$= \frac{-2}{3-2x}$

iv) $\frac{d}{dx}\left(\sqrt{3x^2-5}\right)$

$= \frac{d\,(3x^2-5)^{\frac{1}{2}}}{dx}$

$= \frac{1}{2}(3x^2-5)^{-\frac{1}{2}}\times 6x$

$= 3x\,(3x^2-5)^{-\frac{1}{2}}$

$= \frac{3x}{\sqrt{3x^2-5}}$

b) i) $\int (2x-1)^4\,dx$

$= \frac{(2x-1)^5}{2\times 5} + c$

$= \frac{1}{10}(2x-1)^5 + c'$

ii) $\int_0^3 \frac{dx}{3x+2} = \frac{1}{3}\int_0^3 \frac{3\,dx}{3x+2}$

$= \frac{1}{3}\left[\log_e(3x+2)\right]_0^3$

$= \frac{1}{3}(\log_e 11 - \log_e 2)$

$= \frac{1}{3}\left(\log_e\left(\frac{11}{2}\right)\right)$

QUESTION 4

a) $\log_{10}x - \log_{10}(x-1) = 1$ $\quad x>1$ *

$\log_{10}\left(\frac{x}{x-1}\right) = 1$

$\therefore \frac{x}{x-1} = 10^1$

$x = 10x-10$

$9x = 10$

$x = 1\frac{1}{9}$

Check or consider restriction *

b) $y = 2e^{-2x}$

$y' = -4e^{-2x}$

Curve cuts y axis where $x=0$

ie $y = 2\times e^0 = 2$

$\therefore$ At $x=0$ $\quad y' = m_1 = -4e^0 = -4$

$\therefore$ Gradient of normal

$m_2 = \frac{1}{4}$ $\quad (m_1.m_2 = -1)$

Eq^n of normal

$y-y_1 = m(x-x_1)$

$y-2 = \frac{1}{4}(x-0)$

$4y-8 = x$

$\therefore x-4y+8 = 0$

or $y = \frac{1}{4}x+2$

c) $2x^2-4x-5 = 0$ $\quad a=2,\ b=-4,\ c=-5$

i) $\alpha+\beta = \frac{-b}{a}$

$= \frac{-(-4)}{2}$

$= 2$

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-5}{2}$

iii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$= \frac{2}{-\frac{5}{2}}$

$= -\frac{4}{5}$

d)

x	1	2	3	4	5
$\log_e x$	ln1	ln2	ln3	ln4	ln5
	y_0	y_1	y_2	y_3	y_4

$h = 1$

$\therefore \int_1^5 \log_e x\,dx = \frac{h}{3}(y_0+y_4+2y_2+4(y_1+y_3))$

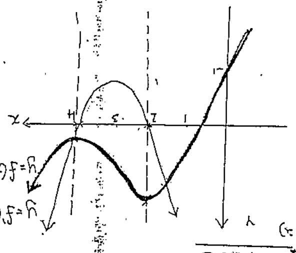
$= \frac{1}{3}(\ln1+\ln5+2\ln3+4(\ln2+\ln4))$

$= 4.041476\ldots$

$\doteq 4.04$ (3 s.f.)

QUESTION 5

i)



b) $x^2+(k-1)x-(2k+1) = 0$

Eq^n has real and distinct roots when $\Delta > 0$.

$\Delta = b^2-4ac$

$= (k-1)^2-4\times1\times-(2k+1)$

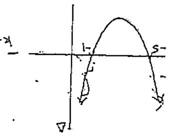
$= k^2-2k+1+8k+4$

$= k^2+6k+5$

$\therefore$ For what k is

$k^2+6k+5 > 0$

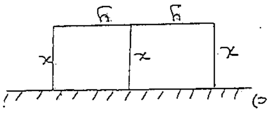
$(k+5)(k+1) > 0$



$\therefore k > -1$

$k < -5$

c)



i) $3x+2y = 120$

$2y = 120-3x$

$y = 60-\frac{3}{2}x$

ii) $A = xy\times 2$

$= 2x(60-\frac{3}{2}x)$

$= 120x-3x^2$

iii) Max Area occurs when

$A' = 0$

$A' = 120-6x$

$A'' = -6$

$A' = 120-6x = 0$

$6x = 120$

$x = 20$

$A'' < 0$ for all x

$\therefore$ Curve concave down for all x

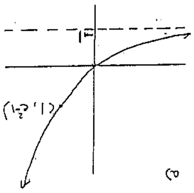
Now since the function is quadratic (ie curve is a parabola AND concave down) $x = 20$ will give THE Max. Area

ie Max Area $= 120\times20-3\times20^2$

$= 1200\,m^2$

QUESTION 6

a)



b) $x+20, x-4, x-20, \ldots$

i) Since terms are in GP

thus

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{x-4}{x+20} = \frac{x-20}{x-4}$

$(x-4)^2 = (x+20)(x-20)$

$x^2-8x+16 = x^2-400$

$8x = 416$

$x = 52$

ii) $\therefore T_1 = 52+20$

$= 72$

iii) $T_2 = 52-4 = 48$

$T_3 = 52-20 = 32$

$r = \frac{48}{72} = \frac{2}{3}$

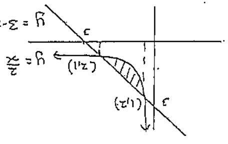
iv) $S_\infty = \frac{a}{1-r}$ $\quad S_\infty$ exists since $-1<r<1$

$= \frac{72}{1-\frac{2}{3}}$

$= \frac{72}{\frac{1}{3}}$

$= 216$

c)



i) $A = \int_1^2 (3-x)-\frac{2}{x}\,dx$

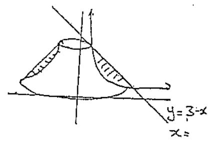
$= \left[3x-\frac{x^2}{2}-2\ln x\right]_1^2$

$= (3.2-\frac{2^2}{2}-2\ln2)-(3-\frac{1}{2}-2\ln1)$

$= 6-2-\ln4-(2\frac{1}{2}-0)$

$= (1\frac{1}{2}-\ln4)\ u^2$

ii)



Given $y = 3 - x$

$x = 3 - y$

and $x = \frac{2}{y}$

$V = \pi\int_1^2 (3-y)^2\,dy - \pi\int_1^2 \left(\frac{2}{y}\right)^2 dy$

$= \pi\int_1^2 [(3-y)^2 - 4y^{-2}]\,dy$

$= \pi\left[\frac{(3-y)^3}{-1\times3} - \frac{4y^{-1}}{-1}\right]_1^2$

$= \pi\left[-\frac{1}{3}(3-y)^3 + \frac{4}{y}\right]_1^2$

$= \pi\left[\left(-\frac{1}{3}\times1^3 + \frac{4}{2}\right) - \left(-\frac{1}{3}\times2^3 + 4\right)\right]$

$= \pi\left[-\frac{1}{3} + 2 + \frac{8}{3} - 4\right]$

$= \frac{\pi}{3}\,u^3$

QUESTION 7

1a i) $y = 3x^4 - 8x^3 + 6$

$y' = 12x^3 - 24x^2$

$y'' = 36x^2 - 48x$

stat pts occur when $y' = 0$

$12x^3 - 24x^2 = 0$

$12x^2(x-2) = 0$

$x = 0, 2$

$y = 6, -10$

at $x = 0$

$y'' = 0$ possible horizontal point of inflexion

x	-1	0	-1
y''	$+$	0	$-$

y'' changes sign $\therefore$

$(0,6)$ is a horizontal point of inflexion

at $x = 2$

$y'' = 36\times2^2 - 48\times2 > 0$ ∪

$\therefore (2,-10)$ is a Min T.P.

ii) Points of inflexion where $y'' = 0$ (if y'' changes sign either side of pt)

$36x^2 - 48x = 0$

$12x(3x-4) = 0$

$x = 0, \frac{4}{3}$

$y = -6, -3.4814\ldots \left(-3\frac{13}{27}\right)$

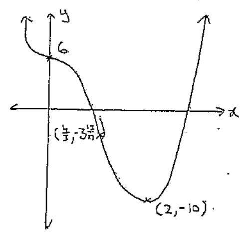
$(0,-6)$ is a horizontal pt inflex^n

at $x = \frac{4}{3}$

x	1	$4/3$	2
y''	$-$	0	$+$

y'' changes sign

$\therefore \left(\frac{4}{3}, -3.4\right)$ $\left(-3\frac{13}{27}\right)$ is a pt inflex.



Not To Scale

iv) Maximum value ?

when $x = -1$ $y = 17$

when $x = 3$ $y = 33$

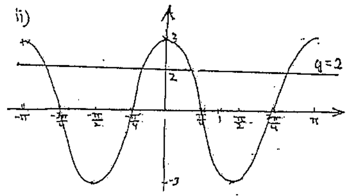
$\therefore$ The maximum value in this domain is 33

b) i) Amplitude = 3

Period $= \frac{2\pi}{2}$

$= \pi$

ii)



iii) 4 solutions within this domain.

(since there are 4 points of intersection)

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QUESTION 1

a) $\frac{x^5}{5} - 5x + c$

b) $\pi^c = 180^\circ$

$\frac{5}{6}\pi = \frac{5}{6} \times 180$

$= 150^\circ$

c) $3^{x+2} = \frac{1}{9}$

$3^{x+2} = 3^{-2}$

$\therefore x+2 = -2$

$x = -4$

d) $\lim_{x \to 2} \frac{x^2-x-2}{x-2}$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$

$= 2+1$

$= 3$

e) $\cos\theta = -\frac{1}{\sqrt{2}}$

cos is negative in quad 2,3

$\theta = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

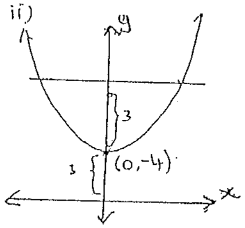
$= \frac{3\pi}{4}, \frac{5\pi}{4}$

f) $x^2 = 12(y+4)$

$x^2 = 4.3(y+4)$

(i) $\therefore$ Vertex $(0,-4)$

ii)



From $(x-x_1)^2 = 4a(y-y_1)$

$a = 3$

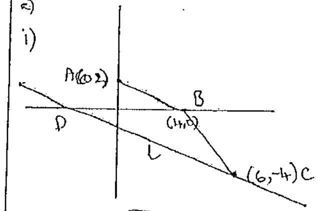
Eq^n Directrix $y = -7$

Focus $(0,-1)$

QUESTION 2

a)

i)



ii) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(0-4)^2+(2-0)^2}$

$= \sqrt{16+4}$

$= \sqrt{20} = 2\sqrt{5}$ units

$m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2-0}{0-4} = -\frac{2}{4} = -\frac{1}{2}$

iii) grad$_L = -\frac{1}{2}$ (since ∥ to AB)

Eq^n L

$(y-y_1) = m(x-x_1)$

$y+4 = -\frac{1}{2}(x-6)$

$-2y-8 = x-6$

$\therefore x+2y+2 = 0$ as req'd

iv) x intercept of L occurs where $y=0$

$x+2.0+2=0$

$x = -2$

$\therefore D(-2,0)$

v) $d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$ A=1, B=2, C=2, $x_1=0$, $y_1=2$

$= \frac{|1.0+2.2+2|}{\sqrt{1^2+2^2}}$

$= \frac{|6|}{\sqrt{5}} = \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$ units

b) i) 12, 8, 4, . . .

$a = 12$ $T_n = a+(n-1)d$

$d = 8-12 = -4$ $T_{15} = 12+(15-1)\times -4$

$n = 15$ $= 12 + 14 \times -4$

$T_{15} = ?$ $= -44$

ii) $S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{50} = \frac{50}{2}(2.12+49.-4)$

$= -4,300$

QUESTION 3

a) i) $\frac{d}{dx}(3x^3-4x+1) = 9x^2-4$

ii) $\frac{d}{dx}\left(\frac{x^2}{e^x}\right)$ $\begin{matrix}u\\v\end{matrix}$ $u = x^2$ $v = e^x$ $u' = 2x$ $v' = e^x$

$= \frac{vu'-uv'}{v^2}$

$= \frac{e^x \times 2x - x^2 \times e^x}{(e^x)^2}$

$= \frac{xe^x(2-x)}{e^{2x}}$

$= \frac{x(2-x)}{e^x}$

iii) $\frac{d}{dx}\log_e(3-2x) = \frac{1}{3-2x} \times -2$

$= \frac{-2}{3-2x}$

iv) $\frac{d}{dx}\left(\sqrt{3x^2-5}\right)$

$= \frac{d}{dx}(3x^2-5)^{\frac{1}{2}}$

$= \frac{1}{2}(3x^2-5)^{-\frac{1}{2}} \times 6x$

$= 3x(3x^2-5)^{-\frac{1}{2}}$

$= \frac{3x}{\sqrt{3x^2-5}}$

b) i) $\int (2x-1)^4 dx$

$= \frac{(2x-1)^5}{2 \times 5} + c$

$= \frac{1}{10}(2x-1)^5 + c'$

ii) $\int_0^3 \frac{dx}{3x+2} = \frac{1}{3}\int_0^3 \frac{3\,dx}{3x+2}$

$= \frac{1}{3}\left[\log_e(3x+2)\right]_0^3$

$= \frac{1}{3}(\log_e 11 - \log_e 2)$

$= \frac{1}{3}\left(\log_e\left(\frac{11}{2}\right)\right)$

QUESTION 4

a) $\log_{10}x - \log_{10}(x-1) = 1$ $x > 1$ *

$\log_{10}\left(\frac{x}{x-1}\right) = 1$

$\therefore \frac{x}{x-1} = 10^1$

$x = 10x-10$

$9x = 10$

$x = 1\frac{1}{9}$

Check or consider restriction *

b) $y = 2e^{-2x}$

$y' = -4e^{-2x}$

Curve cuts y axis where $x=0$

$\therefore$ ie $y = 2 \times e^0 = 2$

$\therefore$ At $x=0$ $y' = m_1 = -4e^0 = -4$

$\therefore$ Gradient of normal

$m_2 = \frac{1}{4}$ $(m_1.m_2 = -1)$

Eq^n of normal

$y-y_1 = m(x-x_1)$

$y-2 = \frac{1}{4}(x-0)$

$4y-8 = x$

$\therefore x-4y+8 = 0$

or $y = \frac{1}{4}x+2$

c) $2x^2 - 4x - 5 = 0$ $\quad a = 2,\ b = -4,\ c = -5$

i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{-(-4)}{2}$

$= 2$

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-5}{2}$

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{2}{-\frac{5}{2}}$

$= -\frac{4}{5}$

d)

x	1	2	3	4	5
$\log_e x$	$\ln 1$	$\ln 2$	$\ln 3$	$\ln 4$	$\ln 5$
	y_0	y_1	y_2	y_3	y_4

$h = 1$

$\therefore \int_1^5 \log_e x\, dx = \frac{h}{3}\left(y_0 + y_4 + 2y_2 + 4(y_1 + y_3)\right)$

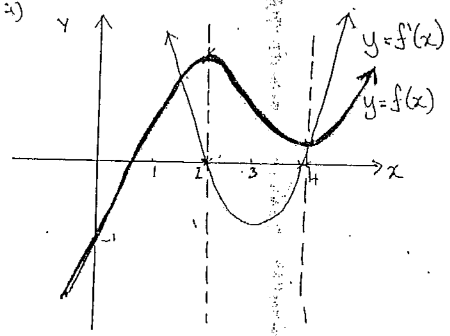
$= \frac{1}{3}\left(\ln 1 + \ln 5 + 2\ln 3 + 4(\ln 2 + \ln 4)\right)$

$= 4.041476\ldots$

$\doteqdot 4.04$ (3 sf)

QUESTION 5

i)



b) $x^2 + (k-1)x - (2k+1) = 0$

Eq^n has real and distinct roots when $\Delta > 0$

$\Delta = b^2 - 4ac$

$= (k-1)^2 - 4 \times 1 \times -(2k+1)$

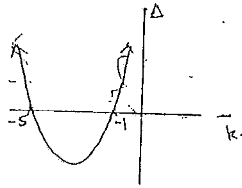
$= k^2 - 2k + 1 + 8k + 4$

$= k^2 + 6k + 5$

$\therefore$ For what k is

$k^2 + 6k + 5 > 0$

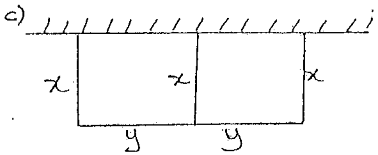
$(k+5)(k+1) > 0$



$\therefore k > -1$

$k < -5$

c)



i) $3x + 2y = 120$

$2y = 120 - 3x$

$y = 60 - \frac{3}{2}x$

ii) $A = xy \times 2$

$= 2x(60 - \frac{3}{2}x)$

$= 120x - 3x^2$

iii) Max Area occurs when

$A' = 0$

$A' = 120 - 6x$

$A'' = -6$

$A' = 120 - 6x = 0$

$6x = 120$

$x = 20$

$A'' < 0$ for all x

$\therefore$ Curve concave down for all x

Now since the function is quadratic (ie curve is a parabola AND concave down)

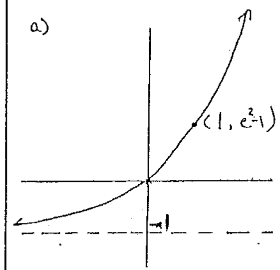
$x = 20$ will give THE Max. Area

ie Max Area $= 120 \times 20 - 3 \times 20^2$

$= 1200\, m^2$

QUESTION 6

a)



b) $x + 20,\ x - 4,\ x - 20, \ldots$

i) Since terms are in GP then

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{x-4}{x+20} = \frac{x-20}{x-4}$

$(x-4)^2 = (x+20)(x-20)$

$\cancel{x^2} - 8x + 16 = \cancel{x^2} - 400$

$8x = 416$

$x = 52$

ii) $\therefore T_1 = 52 + 20$

$= 72$

iii) $T_2 = 52 - 4 = 48$

$T_3 = 52 - 20 = 32$

$r = \frac{48}{72} = \frac{2}{3}$

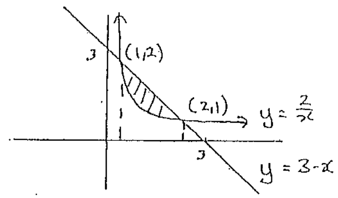
iv) $S_\infty = \frac{a}{1-r}$ $\quad S_\infty$ exists since $-1 < r < 1$

$= \frac{72}{1 - \frac{2}{3}}$

$= \frac{72}{\frac{1}{3}}$

$= 216$

c)



i) $A = \int_1^2 (3 - x) - \frac{2}{x}\, dx$

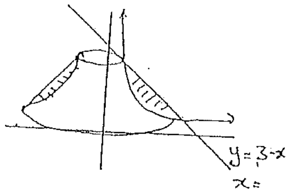
$= \left[3x - \frac{x^2}{2} - 2\ln x\right]_1^2$

$= \left(3 \cdot 2 - \frac{2^2}{2} - 2\ln 2\right) - \left(3 - \frac{1}{2} - 2\ln 1\right)$

$= 6 - 2 - \ln 4 - (2\tfrac{1}{2} - 0)$

$= (1\tfrac{1}{2} - \ln 4)\ u^2$

ii)



Given $y = 3-x$

$x = 3-y$

and $x = \frac{2}{y}$

$$V = \pi\int_1^2 (3-y)^2\,dy - \pi\int_1^2 \left(\frac{2}{y}\right)^2 dy$$

$$= \pi\int_1^2 \left[(3-y)^2 - 4y^{-2}\right] dy$$

$$= \pi\left[\frac{(3-y)^3}{-1\times 3} - \frac{4y^{-1}}{-1}\right]_1^2$$

$$= \pi\left[-\frac{1}{3}(3-y)^3 + \frac{4}{y}\right]_1^2$$

$$= \pi\left[\left(-\frac{1}{3}\times 1^3 + \frac{4}{2}\right) - \left(-\frac{1}{3}\times 2^3 + 4\right)\right]$$

$$= \pi\left[-\frac{1}{3} + 2 + \frac{8}{3} - 4\right]$$

$$= \frac{\pi}{3}\ u^3$$

QUESTION 7

1a i) $y = 3x^4 - 8x^3 + 6$

$y' = 12x^3 - 24x^2$

$y'' = 36x^2 - 48x$

stat pts occur when $y' = 0$

$12x^3 - 24x^2 = 0$

$12x^2(x-2) = 0$

$x = 0, 2$

$y = 6, -10$

at $x = 0$

$y'' = 0$ possible horizontal point of inflexion

x	-1	0	-1
y''	$+$	0	$-$

y'' changes sign $\therefore$

$(0,6)$ is a horizontal point of inflexion

at $x = 2$

$y'' = 36\times 2^2 - 48\times 2 > 0$ ∪

$\therefore (2,-10)$ is a Min T.P.

ii) Points of inflexion where

$y'' = 0$ (if y'' changes sign either side of pt)

$36x^2 - 48x = 0$

$12x(3x-4) = 0$

$x = 0, \frac{4}{3}$

$y = -6, -3.4814\ldots\left(-3\frac{13}{27}\right)$

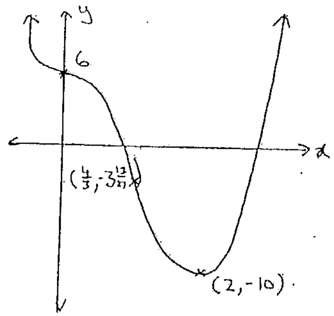
$(0,-6)$ is a horizontal pt inflex^n

at $x = \frac{4}{3}$

x	1	$4/3$	2
y''	$-$	0	$+$

y'' changes sign

$\therefore \left(\frac{4}{3}, -3.4\right)$ $-3\frac{13}{27}$ is a pt inflex.



Not To Scale

iv) Maximum value?

when $x = -1$ $y = 17$

when $x = 3$ $y = 33$

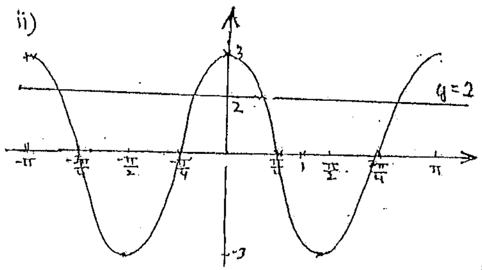
$\therefore$ The maximum value in this domain is 33

b) i) Amplitude $= 3$

Period $= \frac{2\pi}{2}$

$= \pi$

ii)



iii) 4 solutions within this domain.
(since there are 4 points of intersection)