Randwick Boys'
Technology High School

Mathematics Department
Three Unit Half Yearly Examination
Year Eleven.

April 1991

Candidates may attempt all questions.
All necessary working should be shown in every question.
Full marks may not be awarded for careless or badly arranged work.

Time allowed : 2 Hours.

Number each question clearly.
Start each question on a new page.
Write your name on every page.

QUESTION 1. (15 Marks)

a. Simplify: $(-3x^4y)^3$

b. Express: $\sqrt{512}$ in the form: $a\sqrt{b}$

c. $F = \frac{9C}{5} + 32$. Find C when F = 77.

d. Find the exact value of: $\frac{2}{15} - \frac{27}{1000}$ as a fraction in its lowest terms.

e. What is the domain of the following:

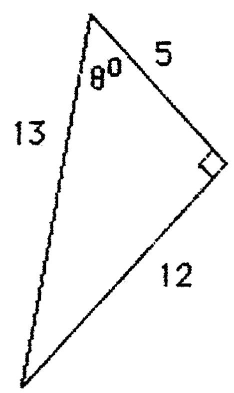
(i) $y = \sqrt{3 - x}$

(ii) $y = \frac{1}{x^2 - 1}$

f. Find the number of sides of a polygon if each exterior angle is 15^0.

g. The equation of a circle is: $x^2 + y^2 - 2x + 4y - 20 = 0$
Find its centre and radius.

h. Write down the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ from the following:



i. Find the size of the angle z^0 if it is four times its supplement.

QUESTION 2. (18 Marks)

a. Simplify: $\dfrac{a}{x - y} - \dfrac{3a}{5x - 5y}$

b. Simplify: $(a + b)^2 - (a - b)^2$

c. A function, $f(x)$ is defined by: $f(x) = 6$ for $x < 3$
$f(x) = 2x$ for $x \geq 3$

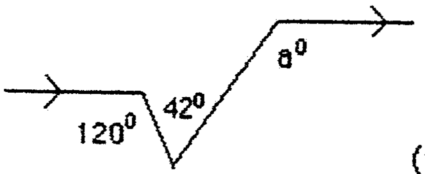
(i) find $f(-2) + f(4)$

(ii) sketch the function for $-2 \leq x \leq 6$

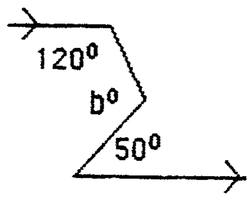
(iii) What other information is required if you were asked to find $f(2a)$?

d. Find the size of each angle marked with a pronumeral. Show working out but do not give reasons.

(i)



(ii)



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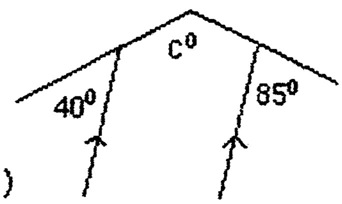

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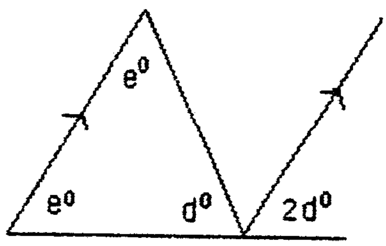


QUESTION 3. (17 Marks)

a. Simplify: $16 - |5 - 12|$

b. Solve: $x + x\sqrt{3} = 8$ leaving your answer in surd form with rational denominator.

c. Factorise: $3x^2 + 4x - 15$

d. Sketch the following curves on separate diagrams showing their main features.

(i) $y = \frac{4}{x - 1}$ (ii) $y = \sqrt{4 - x^2}$

(iii) $y = x(4 - x)$ (iv) $y = |x| + 4$

(v) $y = 4^{-x}$

QUESTION 4. (17 Marks)

a. Solve for x by completing the square. Give your answer correct to 2 decimal places.

$$5x^2 + 4x - 1 = 0$$

b. Simplify: $\frac{x^2 - x}{3x^3 + 3x^2} \div \frac{x^2 - 1}{6x}$

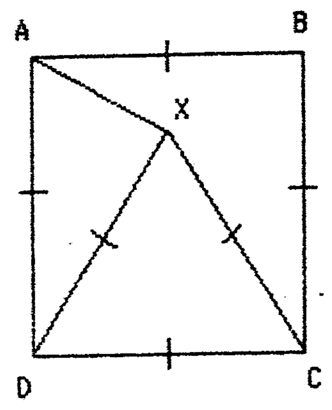
c. Illustrate clearly on a diagram the region of the number plane where the following inequalities are simultaneously true:

$y \le 4 - x^2$, $y \ge 0$, and $y > 2x$.

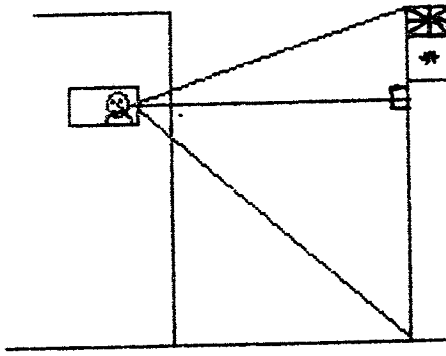
d. ABCD is a square and CDX is an equilateral triangle. Find the size of:

(i) $\angle AXB$

(ii) the reflex $\angle AXC$



e. Looking out of a window, 5m above a level stretch of ground, an observer finds that the angle of elevation to the top of a flag pole is $8^0\ 37'$, and the angle of depression to the foot of the pole is $10^0\ 31'$.



(i) Draw a neat sketch on your answer paper showing all the given information.

(ii) Calculate, to the nearest metre, the distance of the pole from the observer.

(iii) Calculate, to the nearest metre, the height of the pole.

<u>QUESTION 5.</u> (18 Marks)

a. Solve: $\frac{x}{3} - \frac{3x - 4}{2} = 6$

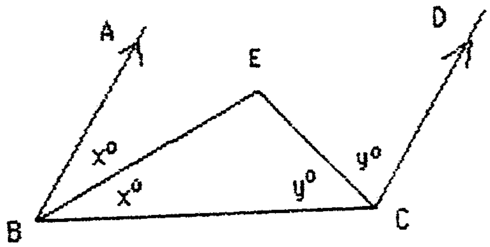
b. Solve: $\frac{2}{x - 3} \geq \frac{1}{x}$

c. Find the equation of the locus of a point, P (x,y) , which moves so that its distance from the point, A (-4,2) , is equal to its distance from the point, B (5,-1) .

Give a geometrical description of this locus.

d. A ship sails due East from a lighthouse, L, for a distance of 25 nm. It then turns and sails due South for 20 nm to a point P. What is the bearing of P from L ? (Draw a diagram first.)

e. Find the size of angle BEC giving reasons.



QUESTION 6. (15 Marks)

a. Solve: $\left| \dfrac{1}{x - 2} \right| \geq 2$

b. If $1 + a < \dfrac{1}{1 - a}$ show that $a < 1$.

c. Is the function: $f(x) = x^2 - x$ odd, even or neither? Give reasons.

d. A wheel makes 20 revolutions per minute. Through what angle does a spoke turn in one second?

e. P is any point on the side AB of a square ABCD. The line from A perpendicular to DP cuts BC at Q. AQ and DP intersect at X.

(i) Draw a diagram showing the above information.

(ii) Prove that DP = AQ.

(1a) $\sqrt{\dfrac{7.849 \times (6.27)^2}{19.863}} = 3.94$ ✓✓

(b) $\dfrac{3x-2}{5} = \dfrac{x}{4} + 3$

$= \dfrac{3x-2}{5} = \dfrac{x+12}{4}$

$4(3x-2) = 5(x+12)$ ✓

$12x - 8 = 5x + 60$

$7x = 68$

$x = 68/7$ ✓

(c) $x^2 - 6x = 0$

$x(x-6) = 0$

$\therefore x = 6$ or 0 ✓

(d) $(2.7 \times 10^{-23}) \times (8 \times 10^{29})$

$= 2.16 \times 10^7$ ✓

(e) $t^4 - 2t^2 + 1$ when $t = 2\sqrt{3}$

$(2\sqrt{3})^4 - 2(2\sqrt{3})^2 + 1 = 0$

$= 144 - 24 + 1$

$= 121$ ✓

(f) $\dfrac{(x-5)(x+4)}{(x-5)(x+5)} \times \dfrac{x(x+5)}{x+1}$ ✓ $=$

$= \dfrac{x(x+4)}{x+1}$ ✓

(2a) $(8/27)^{1/3} \times (4/9)^{-1/2}$

$= 2/3 \times 3/2$ ✓

$= 1$ ✓

(2b) $\dfrac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} \times \dfrac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$

$3\sqrt{2} = \sqrt{18}$

$2\sqrt{3} = \sqrt{12}$

$= \dfrac{18 - 2\sqrt{216} + 12}{(3\sqrt{2})^2 - (2\sqrt{3})^2} = \dfrac{30 - 12\sqrt{6}}{(3\sqrt{2})^2 - (2\sqrt{3})^2} = \dfrac{30 - 12\sqrt{6}}{18 - 12}$

$\therefore a = 30, \quad b = 12$

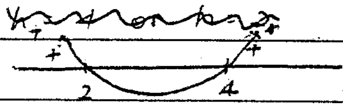
$= \dfrac{30 - 12\sqrt{6}}{6}$

$a = 5, \quad b = -2$

$= 5 - 2\sqrt{6}$

(c) $h^2 - 6h + 8 \geq 0$

$(h-4)(h-2) \geq 0$



~~$h \leq 4$~~

$h \leq 2$

$h \geq 4$ or $h \leq 2$

(d) $3x^2 - x - 3 = 0$

$x = \dfrac{1 \pm \sqrt{1 - 4.3.-3}}{6}$ ✓

$= \dfrac{1 \pm \sqrt{37}}{6}$ ✓

(e) $\dfrac{a^{-1} + b^{-1}}{a+b} = \left(\dfrac{1}{a} + \dfrac{1}{b}\right) \times \dfrac{1}{a+b}$

$= \dfrac{b+a}{ab} \times \dfrac{1}{a+b}$

$= \dfrac{1}{ab}$ ✓

(f) $2x + 3y = 5$ — (i) $\times 3$

$3x + 4y = 6$ — (ii) $\times 2$

$6x + 9y = 15$ — (i)

$6x + 8y = 12$ — (ii)

$y = 3$ ✓

Sub y into (i):

$2x + 3(3) = 5$

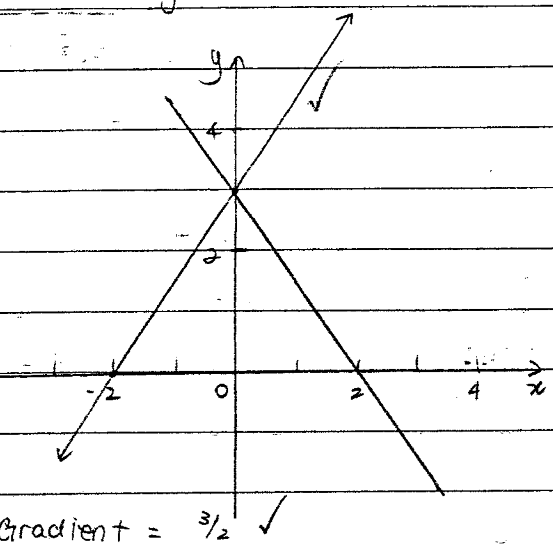
$2x = -4$

$x = -2$ ✓

(i) $3x - 2y + 6 = 0$

$-2y = -3x - 6$

$y = \frac{3}{2}x + 3$ ✓



) Gradient $= \frac{3}{2}$ ✓

i) cuts Y-axis at ~~-2~~ 3

) $m = \frac{1}{2}$

$\perp m = -2$

∴ Eq^n of line:

$y - 3 = -2(x - 2)$ ✓

$y = -2x + 7$

⇒ $2x + y - 7 = 0$ ✓

) $3x - 4y + 15 = 0$, tangent to $x^2 + y^2 = 9$

$-4y = -3x - 15$ $\quad y = \sqrt{9 - x^2}$

$y = \frac{3}{4}x + \frac{15}{4}$

$x^2 + y^2 = 9$ has centre $(0, 0)$

radius 3

$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

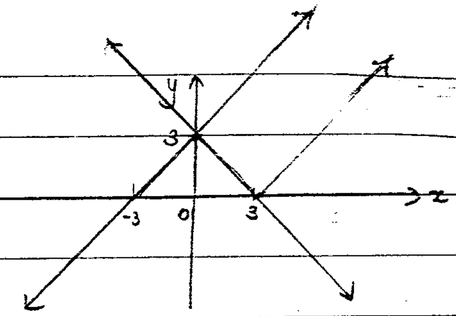
$= \frac{|3(0) - 4(0) + 15|}{\sqrt{3^2 + 4^2}}$

$= \frac{|15|}{5}$

$= 3 =$ radius.

∴ $3x - 4y + 15 = 0$ is a tangent.

(d)



(e) $f(x) = 2x^3 - x$

$f(-x) = -2x^3 + x$

$= -f(x)$ Since $f(-x) = -f(x)$, then

∴ This is an ODD function.

2(4a) $\left(\frac{-1 + x}{2}, \frac{2 + y}{2}\right) = (3, 5)$

∴ $x = 7$, $y = 8$ ✓

∴ coods B $(7, 8)$

(c) $\tan\theta = 135°$

$m = -1$

∴ Eq^n:

$y - 2 = -1(x + 3)$ ✓

$y = -x - 1$

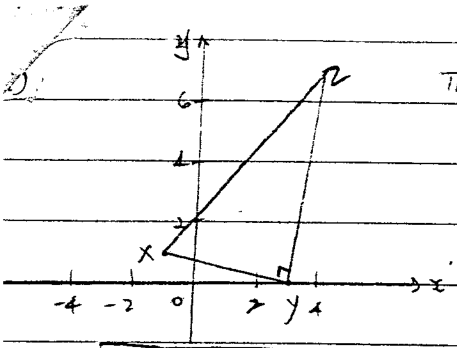
⇒ $x + y + 1 = 0$ ✓

(b) $P(1, 1)$, $Q(0, 4)$, $R(2, 10)$.

$m_{PQ} = \frac{4 - 1}{0 + 1} = 3$ ✓ $\quad m_{QR} = \frac{10 - 4}{2 - 0} = 3$

∴ PQR are collinear ✓

(d)



Try again.

$|yz| = \sqrt{(4-3)^2 + (7-0)^2}$

$= \sqrt{50}$ units $= 5\sqrt{2}$

$xy = \sqrt{(0-1)^2 + (2-1)^2}$

$= \sqrt{5}$ units

$A = \frac{1}{2} bh$

$= \frac{1}{2} \times 5\sqrt{2} \times \sqrt{5}$

$= \frac{1}{2} \times \sqrt{250}$

$= 7.91$ units^2 ✓

$\cos\theta = \frac{1}{2}$

$\theta = 60°$

is +ve in Quad I & IV:

$\theta = 60°,\ 360° - 60°$

$= 60°,\ 300°$ ✓

$AC^2 = 1^2 + (1+\sqrt{2})^2$

$= 1 + (1 + 2\sqrt{2} + 2)$

$= 4 + 2\sqrt{2}$ ✓

$AC = \sqrt{4+2\sqrt{2}}$

(5b) Area $= l^2$

$= (\sqrt{4+2\sqrt{2}})^2$

$= 4 + 2\sqrt{2}$ units^2 ✓

(c) $\angle BCD = 48°$

(ii) $\angle ADC = 180 - 48°$ (coint ∠s), $BC \parallel AD$)

$= 132°$ ✓

$\therefore \angle BAD = 48°$ (coint ∠s) ✓

$\angle EAB = 60°$ (∠ of equilateral $\Delta = 60°$)

$\therefore \angle EAD = 60° + 48°$

$= 108°$ ✓

(iii) $\angle EAD = 108°$

$EA = AD$

$\therefore \Delta EAD$ is isosceles. ✓

$\therefore \angle EDA = (180 - 108°) \div 2$

$= 36°$ ✓

(Q6a) $t_1 = a = 3$

$t_2 = a + d = 7$

$\therefore d = 4$

$t_n = a + (n-1)d$

$\therefore 71 = 3 + (n-1)4$

$71 = 3 + 4n - 4$

$4n = 72$

$n = 18$

(b) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

LHS $= (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta) + (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$

$= 2\sin^2\theta + 2\cos^2\theta$ ✓

$= 2(\sin^2\theta + \cos^2\theta)$

$(\sin^2\theta + \cos^2\theta = 1)$ ✓

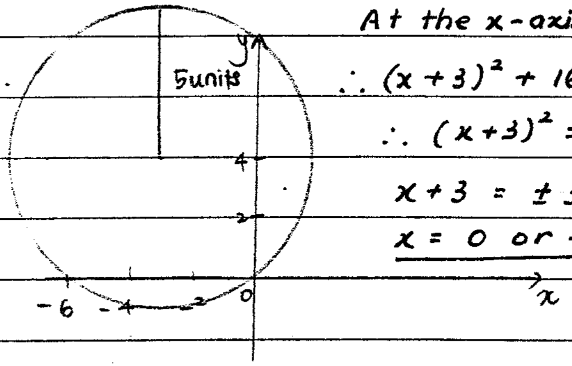
$\therefore = 2$

$=$ RHS.

(i) radius = 5 , centre (−3, 4)

eq^{n}: $(x+3)^2 + (y-4)^2 = 5^2$ ✓

(ii).



At the x-axis, y=0

$\therefore (x+3)^2 + 16 = 25$

$\therefore (x+3)^2 = 9$

$x+3 = \pm 3$

$\underline{x = 0 \text{ or } -6}$

Cross x-axis at pt −6 and 0.

Try this algebraically!

(6d) sum of interior $\angle$s $= (2n-4) \times 90°$

$= 180°n - 360° = 1980°$ ✓

$180n = 2340$

$n = 13$ ✓

(8a) $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$ ✓

$= \dfrac{5.6^2 + 4.8^2 - 8.6^2}{2 \times 5.6 \times 4.8}$

$A = 111°20'$ ✓

7ai) $\sum_{r=1}^{n} (5r-4)$

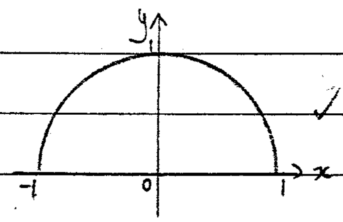
$(5-4) + (10-4) + (15-4)$

$= 1 + 6 + 11$

(ii) $d = 6-1 = 11-6 = 5$.

$\therefore$ it is an AP.

(bi)

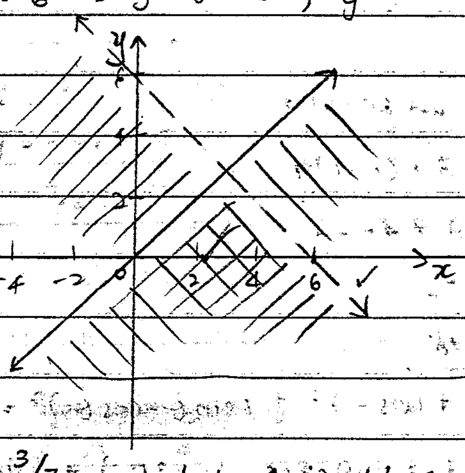


✓

D: $-1 \le x \le 1$ ✓

R: $0 \le x \le 1$ ✓

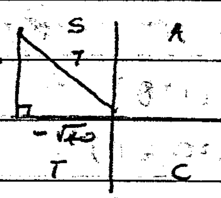
b) $x+y \le 6 \Rightarrow y < 6 - x$, $y \le x$



c) $\sin A = 3/7$ but A is obtuse!

$x^2 = 7^2 - 3^2$

$x = \sqrt{40}$



$\tan A = -3/\sqrt{40}$

a) $\angle WXB = 100°$

$\angle WXA = 180° - 100°$

$= 80°$ ✓

$\therefore \angle WXA = \angle WYC$ (AB // CD) Corresponding angles

$\therefore \angle x° = 80°$ ✓

(10b) $\cos(180 - \theta) \cdot \tan(360° + \theta) \cdot \text{cosec}\,\theta$

$-\cos\theta \times \tan\theta \times \frac{1}{\sin\theta}$ ✓

$= -\cos\theta \times \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta}$ ✓

$= -1$

b) $r = 1/3$

$S_8 = \frac{a(1 - r^n)}{1 - r}$ ✓

$= \frac{27(1 - 1/3^8)}{1 - 1/3}$

$= 40.5$ ✓

(c) $t_4 = ar^3 = 64$ — (i)

$t_7 = ar^6 = 8$ — (ii)

(i) ÷ (ii)

$r = 8$

$r = -\sqrt{8}$

$=$

(ii) ÷ (i)

$r^3 = \frac{1}{8}$

$r = \frac{1}{2}$

Sub r into (i)

$t_3 = a(\ )^3 = 64$

$\therefore a = 64$

$a =$

$\therefore a = 64 \times 8$

$= 512$

(ii) $T_n = 2^{10-n}$

$= ar^{n-1}$

$=$

$=$

$T_n = ar^{n-1}$

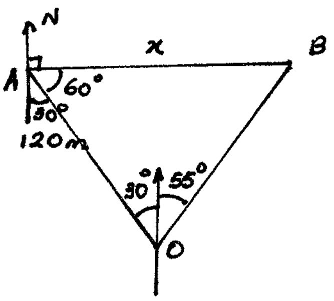
$= 512\left(\frac{1}{2}\right)^{n-1}$

$= 2^9 \cdot 2^{-n+1}$

$= 2^{9-n+1}$

$= 2^{10-n}$

c)



In Δ ABO, using Sine rule

$\frac{x}{\sin 85°} = \frac{120}{\sin 35°}$

$\therefore x = \frac{120 \sin 85°}{\sin 35°}$

$= 208$ m (to nearest m).

9a) LS of $2/3 - 4/9 + 8/27 - \ldots$

$r = -2/3$ ✓

$LS = \frac{2/3}{1 + 2/3}$

$= 2/5$ ✓