Year 12 Extension 2 – Mechanics | **Time: 45 mins**

Term 3, 2015 | **MARKS: 22**

Name: ____________________ | **Teacher: RDS HRK**

Question 1 **Start a new Booklet** **8 Marks**

a) A helicopter is hovering 1000 metres above the ground. The crew throw the annoying co-pilot directly towards the ground at a speed of u ms^{-1} where $u < \sqrt{\frac{mg}{k}}$.

The co-pilot experiences a resistive force proportional to the square of his velocity.

i) Draw a force diagram to represent this situation. **1**

ii) Show that the co-pilot's velocity can be related to the distance he has fallen, x metres, by the equation: **4**

$$v^2 = \frac{mg - \left(mg - ku^2\right)e^{-\frac{2kx}{m}}}{k}$$

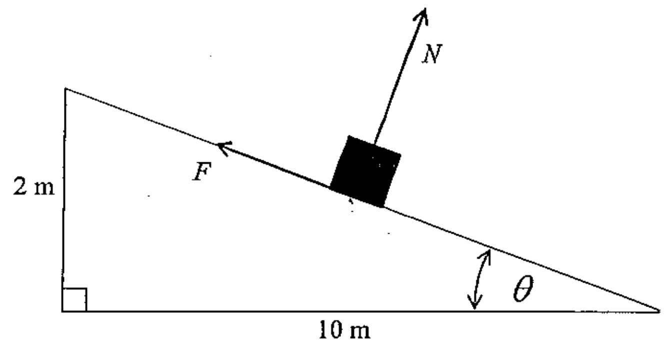
iii) Explain why his velocity cannot exceed $\sqrt{\frac{mg}{k}}$. **1**

iv) The co-pilot has a suit that will save him if he hits the ground with a velocity less than 200 ms^{-1}. Taking $g = 9.8$ ms^{-2}, the pilot's mass to be 100kg and the coefficient of resistance to be $\frac{1}{10000}$ find the fastest possible initial velocity he can survive to 3 significant figures. **2**

Question 2 **Start a new booklet** **7 Marks**

A car is moving around a track banked at an angle of θ to the horizontal. The track has a radius of $10\sqrt{26}$ metres, a width of 10 metres and the height of the outer edge of the track is 2 metres. The car weighs 1.4 tonnes and is moving at v ms^{-1}.

The car experiences a frictional force F up the track and a normal reaction force N.



i) If the frictional force is positive, is the car moving up or down the track? Explain your answer. **1**

ii) By resolving forces vertically and horizontally show that: **4**

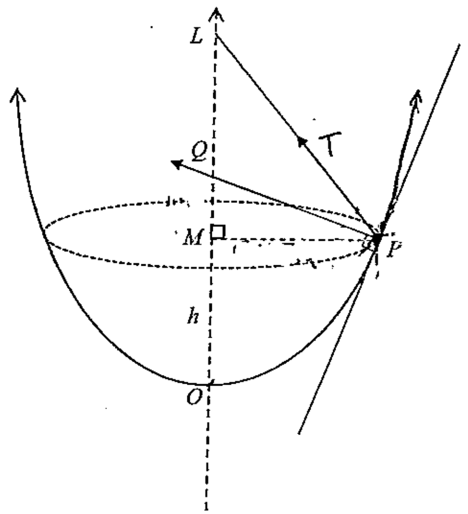
$$N = \frac{70v^2 + 3500\sqrt{26}g}{13}$$

$$F = \frac{700\sqrt{26}g - 350v^2}{13}$$

iii) Taking $g = 9.8$ ms^{-2}, find the optimal speed at which the car can travel around the banked track correct to 2 decimal places. **2**

Question 3 **Start a new booklet** **7 Marks**

a) A point of unit mass is moving in uniform circular motion around the inside of a parabolic bowl whose surface is formed by rotating the curve $x^2 = 4y$. The mass is attached to a light, inelastic string and moves in a circle, centre M, at a height h above the vertex of the parabola at 1 radian per second. It experiences both a tension force, T, and a normal reaction force, N. The string is attached to a point L, 3 units above the centre of motion.



i) Show that the radius of the motion is $2\sqrt{h}$. **1**

ii) By finding the equation of the normal at P or otherwise show that

$$\angle QPM = \tan^{-1}\frac{1}{\sqrt{h}}$$ **2**

iii) Resolve the forces horizontally and vertically and then show that

$$T = (g-2)\sqrt{9+4h}$$ **4**

Ext 2 Term 3 Mechanics Solutions 2015

Q1 i)

$u\,ms^{-1}\downarrow$ $\quad \downarrow mg \quad \uparrow kv^2 \quad \downarrow +$ (1)

ii) $\sum F = m\ddot{x}$

$\therefore m\ddot{x} = mg - kv^2$

$\ddot{x} = \dfrac{mg - kv^2}{m}$

(1) $v \cdot \dfrac{dv}{dx} = \dfrac{mg - kv^2}{m}$

$\dfrac{dv}{dx} = \dfrac{mg - kv^2}{mv}$

$\dfrac{dx}{dv} = \dfrac{mv}{mg - kv^2}$

$\displaystyle\int_0^x dx = \int_u^v \frac{mv}{mg - kv^2}\, dv$

$\displaystyle x = \frac{-m}{2k}\int_u^v \frac{-2kv}{mg - kv^2}\, dv$

(1)

$x = \dfrac{-m}{2k}\left[\ln|mg - kv^2| - \ln|mg - ku^2|\right]$ (1)

$-\dfrac{2kx}{m} = \ln\left|\dfrac{mg - kv^2}{mg - ku^2}\right|$

$\dfrac{mg - kv^2}{mg - ku^2} = e^{-\frac{2kx}{m}}$

(1) (correct rearrangement)

$mg - kv^2 = (mg - ku^2)e^{-\frac{2kx}{m}}$

$kv^2 = mg - (mg - ku^2)e^{-\frac{2kx}{m}}$

$v^2 = \dfrac{mg - (mg - ku^2)e^{-\frac{2kx}{m}}}{k}$

iii) Terminal velocity is the fastest speed an object can fall. It occurs when $\sum F = 0$

$\therefore 0 = mg - kv^2$

$kv^2 = mg$

$v = \sqrt{\dfrac{mg}{k}}$

NB// this is a limit, v can never be $\sqrt{\frac{mg}{k}}$

iv) $kv^2 = mg - (mg - ku^2)e^{-\frac{2kx}{m}}$

$$(mg - ku^2) = (mg - kv^2)e^{\frac{2kx}{m}}$$ (1) Rearrange

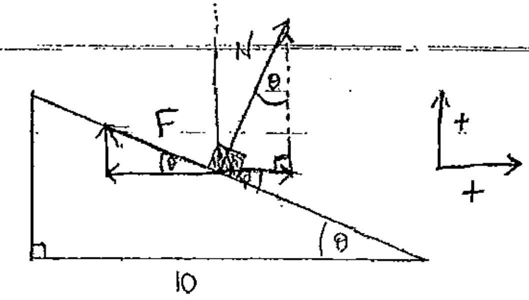
$$ku^2 = mg - (mg - kv^2)e^{\frac{2kx}{m}}$$

$$u^2 = \frac{mg - (mg - kv^2)e^{\frac{2kx}{m}}}{k}$$

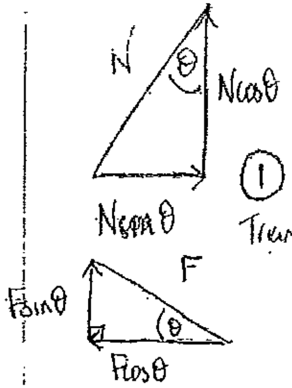
$\therefore u = 143 \text{ ms}^{-1}$ (3 sig figs)

(1) Correct answer.

Q2



i) It will be moving down the track. Frictional force opposes movement. So a positive friction (up the slope) will be caused by movement down the slope.



(1) Triangles

$$F_H = \frac{mv^2}{r} = N\sin\theta - F\cos\theta$$ (1)

$$F_V = 0 = N\cos\theta + F\sin\theta - mg$$

$$\therefore N\sin\theta - F\cos\theta = \frac{mv^2}{r} \quad \ldots (1)$$

$$N\cos\theta + F\sin\theta = mg \quad \ldots (2)$$

(1) × sinθ $\quad N\sin^2\theta - F\cos\theta\sin\theta = \frac{mv^2}{r}\sin\theta$

(+)

(2) × cosθ $\quad N\cos^2\theta + F\sin\theta\cos\theta = mg\cos\theta$

$$\therefore N = \frac{mv^2}{r}\sin\theta + mg\cos\theta$$

iii) Optimal velocity occurs when $F=0$

$$\frac{700\sqrt{26}\,g}{\cancel{13}} = \frac{350v^2}{\cancel{13}}$$

$$v^2 = 99.940\ldots$$

$$v = \cancel{10.0ms^{-1}} \ (1dp)$$

$$10.00 \ (2dp)$$

$① \times \cos\theta \quad N\sin\theta\cos\theta - F\cos^2\theta = \frac{mv^2}{r}\cos\theta$

$(-)$ $② \times \sin\theta \quad N\cos\theta\sin\theta + F\sin^2\theta = mg\sin\theta$

$$-F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

$$F = mg\sin\theta - \frac{mv^2}{r}\cos\theta$$ ① Both F & N

$m = 1400$, $\sin\theta = \frac{1}{\sqrt{26}}$, $\cos\theta = \frac{5}{\sqrt{26}}$, $\tan\theta = \frac{1}{5}$, $r = 10\sqrt{26}$

$$\therefore N = \frac{140\cancel{0}v^2}{1\cancel{0}\sqrt{26}} \cdot \frac{1}{\sqrt{26}} + 1400g \times \frac{5}{\sqrt{26}}$$

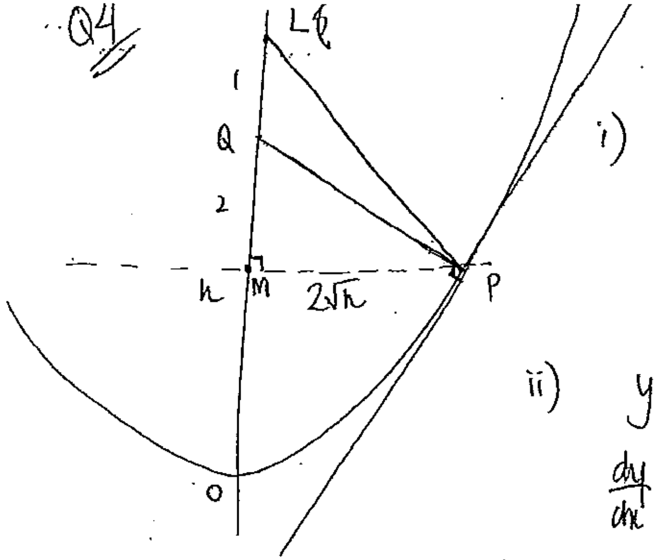
$$= \frac{70v^2}{13} + \frac{1400g \times 5\sqrt{26}}{26}$$

$$= \frac{70v^2}{13} + \frac{3500g\sqrt{26}}{13}$$

$$F = 1400g \times \frac{1}{\sqrt{26}} - \frac{1400v^2}{10\sqrt{26}} \times \frac{5}{\sqrt{26}}$$

$$= \frac{700g\sqrt{26}}{13} - \frac{350v^2}{13}$$ ①

Q4



i) $x^2 = 4y$

$y = h$

$x = 2\sqrt{h}$

ii) $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{x}{2}$

@ $y=h$ $\frac{dy}{dx} = \frac{2\sqrt{h}}{2}$

$= \sqrt{h}$

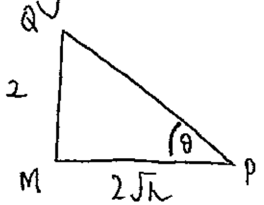
$\therefore m_N = -\frac{1}{\sqrt{h}}$

$\therefore$ Normal $-\frac{1}{\sqrt{h}} = \frac{y-h}{x-2\sqrt{h}}$

y intercept $-\frac{1}{\sqrt{h}} = \frac{y_{INT} - h}{-2\sqrt{h}}$

$2 = y_{INT} - h$

$y_{INT} = 2 + h$

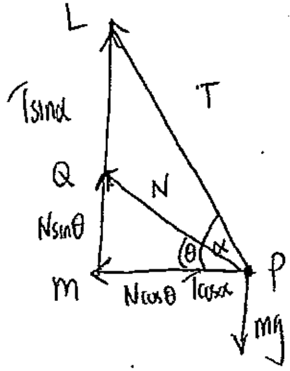


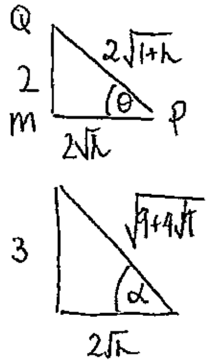
$\therefore \tan\theta = \frac{2}{2\sqrt{h}}$

$\theta = \tan^{-1}\frac{1}{\sqrt{h}}$

iii) $F_H = mr\omega^2 = N\cos\theta + T\cos\alpha$

$F_V = 0 = N\sin\theta + T\sin\alpha - mg$





$F_H = N\left(\frac{\cancel{2\sqrt{h}}}{\cancel{2}\sqrt{1+h}}\right) + T\left(\frac{2\sqrt{h}}{\sqrt{9+4h}}\right) = 1 \times 1 \times 2\sqrt{h}$

$\frac{N}{\sqrt{1+h}} + \frac{2T}{\sqrt{9+4h}} = 2$

$F_V = N\left(\frac{\cancel{2}}{\cancel{2}\sqrt{1+h}}\right) + T\left(\frac{3}{\sqrt{9+4h}}\right) = g$

$\frac{N}{1+h} + \frac{3T}{\sqrt{9+4h}} = g$

$$\frac{3T - 2T}{\sqrt{9+4h}} = g-2$$

$$T = (g-2)\sqrt{9+4h}$$