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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP1)
BINOMIAL THEOREM II

PAST EXAMINATION QUESTIONS:

HSC 05

(2) (b) Use the binomial theorem to find the term independent of x in the expansion of $\left(2x-\frac{1}{x^2}\right)^{12}$. **3**

126720

(6)

(a) There are five matches on each weekend of a football season. Megan takes part in a competition in which she earns one point if she picks more than half of the winning teams for a weekend, and zero points otherwise. The probability that Megan correctly picks the team that wins any given match is $\frac{2}{3}$.

(i) Show that the probability that Megan earns one point for a given weekend is 0.7901, correct to four decimal places. 2

(ii) Hence find the probability that Megan earns one point every week of the eighteen-week season. Give your answer correct to two decimal places. 1

0.01 (to 2 d.p.)

(iii) Find the probability that Megan earns at most 16 points during the eighteen-week season. Give your answer correct to two decimal places. **2**

0.92 (to 2 d.p.)

HSC 04

(4) (c) Katie is one of ten members of a social club. Each week one member is selected at random to win a prize.

(i) What is the probability that in the first 7 weeks Katie will win at least 1 prize? **1**

0.52

(ii) Show that in the first 20 weeks Katie has a greater chance of winning exactly 2 prizes than of winning exactly 1 prize. **2**

$\boxed{P(2\text{ prizes}) = 0.2852;\ P(\text{exactly 1 prize}) = 0.2702}$

(iii) For how many weeks must Katie participate in the prize drawing so that she has a greater chance of winning exactly 3 prizes than of winning exactly 2 prizes? **2**

$\boxed{n = 30}$

(7) (b) (i) Show that for all positive integers n, 1

$$x\left[(1+x)^{n-1}+(1+x)^{n-2}+\cdots+(1+x)^2+(1+x)+1\right]=(1+x)^n-1.$$

(ii) Hence show that for $1 \le k \le n$, 1

$$\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-3}{k-1}+\cdots+\binom{k-1}{k-1}=\binom{n}{k}.$$

(iii) Show that $n\binom{n-1}{k}=(k+1)\binom{n}{k+1}$. 1

(iv) By differentiating both sides of the identity in (i), show that for $1 \leq k < n$, **3**

$$(n-1)\binom{n-2}{k-1}+(n-2)\binom{n-3}{k-1}+\cdots+k\binom{k-1}{k-1}=k\binom{n}{k+1}.$$

HSC 03

(3) (c) (i) Explain why the probability of getting a sum of 5 when one pair of fair dice is tossed is $\frac{1}{9}$. **1**

(ii) Find the probability of getting a sum of 5 at least twice when a pair of dice is tossed 7 times. **2**

0.178 (to 3 d.p.)

HSC 02

(7) (b) The coefficient of x^k in $(1 + x)^n$, where n is a positive integer, is denoted by c_k (so $c_k = {}^nC_k$).

(i) Show that **3**

$$c_0 + 2c_1 + 3c_2 + \ldots + (n+1)c_n = (n+2)2^{n-1}.$$

(ii) Find the sum **3**

$$\frac{c_0}{1.2} - \frac{c_1}{2.3} + \frac{c_2}{3.4} - \ldots + (-1)^n \frac{c_n}{(n+1)(n+2)}\ .$$

Write your answer as a simple expression in terms of n.

$$\boxed{\frac{1}{n+2}}$$

***HSC 01** **Marks**

(5) (b) By using the binomial expansion, show that **3**

$$(q+p)^n-(q-p)^n=2\binom{n}{1}q^{n-1}p+2\binom{n}{3}q^{n-3}p^3+\ldots$$

What is the last term in the expansion when n is odd?
What is the last term in the expansion when n is even?

$2p^n$ for n is odd; $2nqp^{n-1}$ for n is even

(c) A fair six-sided die is randomly tossed n times.

(i) Suppose $0 \le r \le n$. What is the probability that exactly r 'sixes' appear in the uppermost position? **2**

$$\boxed{{}^{n}C_{r}\left(\frac{5}{6}\right)^{n-r}\left(\frac{1}{6}\right)^{r}}$$

(ii) By using the result of part (b), or otherwise, show that the probability that an odd number of 'sixes' appears is **2**

$\frac{1}{2}\left\{1-\left(\frac{2}{3}\right)^{n}\right\}$. (Hint: Use the result in part (b) above)

***HSC '99**

(7) (b) By considering $(1-x)^n\left(1+\frac{1}{x}\right)^n$, or otherwise, express

$\binom{n}{2}\binom{n}{0}-\binom{n}{3}\binom{n}{1}+\ldots+(-1)^n\binom{n}{n}\binom{n}{n-2}$ in the simplest form.

$\binom{n}{\frac{n+2}{2}}(-1)^{\frac{n+2}{2}}$ if n is even; 0 if n is odd.

HSC '98

(7) (a) (i) Use the binomial theorem to obtain an expansion for

$$(1+x)^{2n}+(1-x)^{2n}\text{, where } n \text{ is a positive integer.}$$

$$\boxed{2\left(1+{}^{2n}C_2x^2+{}^{2n}C_4x^4+\ldots.+{}^{2n}C_{2n}x^{2n}\right)}$$

(ii) Hence evaluate $1+{}^{20}C_2+{}^{20}C_4+\ldots.+{}^{20}C_{20}$.

$\boxed{2^{19}}$

HSC '97

(7) (b) (i) Simplify $n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}$. **3**

$\boxed{n\left(2^{n-1}-2\right)}$

(ii) Find the smallest positive integer n such that

$$n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}>20\ 000.$$

$\boxed{n=12}$

HSC '96

(7) (a) Using the fact that $(1+x)^4(1+x)^9=(1+x)^{13}$, show that **2**

$$ {}^4C_0\,{}^9C_4+{}^4C_1\,{}^9C_3+{}^4C_2\,{}^9C_2+{}^4C_3\,{}^9C_1+{}^4C_4\,{}^9C_0={}^{13}C_4 . $$

HSC '95

(3) (b) Find the value of the term that does not depend on x in the expansion of

$$\left(x^2+\frac{3}{x}\right)^6$$

1215

HSC '92

(5) (c) Consider the binomial expansion $1+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{n}x^n=(1+x)^n$.

(i) Show that $1-\binom{n}{1}+\binom{n}{2}-\ldots+(-1)^n\binom{n}{n}=0.$

(ii) Show that $1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\ldots+(-1)^n\frac{1}{n+1}\binom{n}{n}=\frac{1}{n+1}$.

HSC '90

(6) (c) (i) Show that $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n=(1+x)^{2n}$.

(ii) Hence prove that $1+\binom{n}{1}^2+\binom{n}{2}^2+\ldots+\binom{n}{n}^2=\binom{2n}{n}$.