NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP2)
PROJECTILE MOTION
(SIMPLER QUESTIONS)

PAST SIMPLER EXAMINATION QUESTIONS:

HSC 07

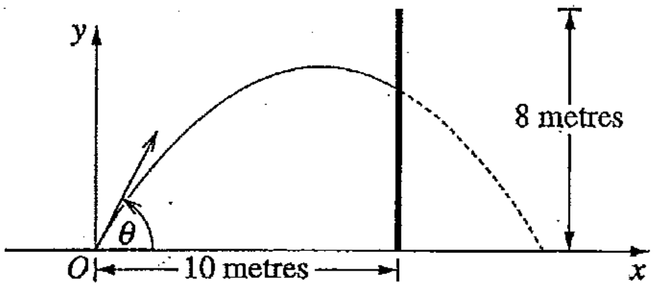
(7)(b) A small paintball is fired from the origin with initial velocity 14 metres per second towards an eight-metre high barrier. The origin is at ground level, 10 metres from the base of the barrier.

The equations of motion are

$$x = 14t\cos\theta$$

$$y = 14t\sin\theta - 4.9t^2$$

where θ is the angle to the horizontal at which the paintball is fired and t is the time in seconds. (Do NOT prove these equations of motion.)



(i) Show that the equation of trajectory of the paintball is **2**

$$y = mx - \left(\frac{1+m^2}{40}\right)x^2, \quad \text{where } m = \tan\theta.$$

(ii) Show that the paintball hits the barrier at height h metres when **2**

$$m = 2 \pm \sqrt{3 - 0.4h}\,.$$

Hence determine the maximum value of h.

7.5m

(iii) There is a large hole in the barrier. The bottom of the hole is 3.9 metres above the ground and the top of the hole is 5.9 metres above the ground. The paintball passes through the hole if m is in one of two intervals. One interval is $2.8 \le m \le 3.2$. **2**

Find the other interval.

$0.8 \le m \le 1.2$ or $2.8 \le m \le 3.2$

(iv) Show that, if the paintball passes through the hole, the range is **3**

$$\frac{40m}{1+m^2} \text{ metres.}$$

Hence find the widths of the two intervals in which the paintball can land at ground level on the other side of the barrier.

Width $=12.6696...-11.3879...=1.28\,\text{m}$ (to 2 d.p.);
Width $=20-19.5121...=0.49\,\text{m}$ (to 2 d.p.)

HSC 04

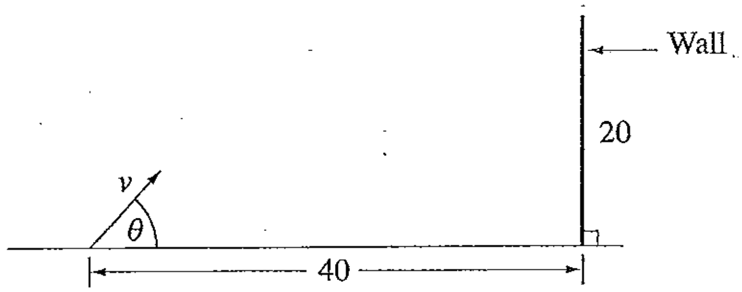
(6) (b) A fire hose is at ground level on a horizontal plane. Water is projected from the hose. The angle of projection, θ, is allowed to vary. The speed of the water as it leaves the hose, v metres per second, remains constant. You may assume that if the origin is taken to be the point of projection, the path of the water is given by the parametric equations

$$x = vt\cos\theta$$
$$y = vt\sin\theta - \frac{1}{2}gt^2$$

where $g\ \mathrm{m\,s^{-2}}$ is the acceleration due to gravity. (Do NOT prove this.)

(i) Show that the water returns to ground level at a distance $\dfrac{v^2\sin 2\theta}{g}$ metres from the point of projection. **2**

This fire hose is now aimed at a 20 metre high thin wall from a point of projection at ground level 40 metres from the base of the wall. It is known that when the angle θ is 15°, the water just reaches the base of the wall.



(ii) Show that $v^2 = 80g$. **1**

(iii) Show that the cartesian equation of the path of the water is given by **2**

$$y = x\tan\theta - \frac{x^2\sec^2\theta}{160}.$$

(iv) Show that the water just clears the top of the wall if **2**

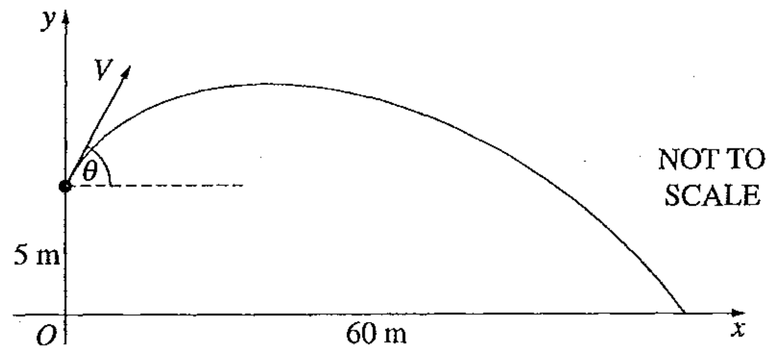
$$\tan^2\theta - 4\tan\theta + 3 = 0.$$

(v) Find all values of θ for which the water hits the front of the wall. **2**

Show that $\theta = 15^0$ or 75^0; hence $15^0 < \theta < 45^0$ or $72^0 < \theta < 75^0$

HSC 02

(6)(a)



An angler casts a fishing line so that the sinker is projected with a speed $V \text{ m s}^{-1}$ from a point 5 metres above a flat sea. The angle of projection to the horizontal is θ, as shown.

Assume that the equations of motion of the sinker are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10,$$

referred to the coordinate axes shown.

(i) Let (x, y) be the position of the sinker at time t seconds after the cast, and before the sinker hits the water. **2**

It is known that $x = Vt\cos\theta$.

Show that $y = Vt\sin\theta - 5t^2 + 5$.

(ii) Suppose the sinker hits the sea 60 metres away as shown in the diagram. **3**

Find the value of V if $\theta = \tan^{-1}\dfrac{3}{4}$.

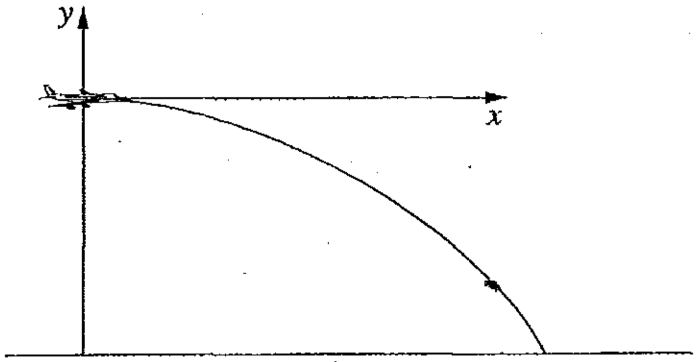
$$V = \frac{15\sqrt{10}}{2}$$

(iii) For the cast described in part (ii), find the maximum height above sea level that the sinker achieved. **2**

$15\frac{1}{8}$ m

HSC 01

(4) (b) An aircraft flying horizontally at $V\,\text{ms}^{-1}$ releases a bomb that hits the ground 4000 m away, measured horizontally. The bomb hits the ground at an angle of 45^0 to the vertical. **4**



Assume that, t seconds after release, the position of the bomb is given by

$$x = Vt, \quad y = -5t^2.$$

Find the speed V of the aircraft.

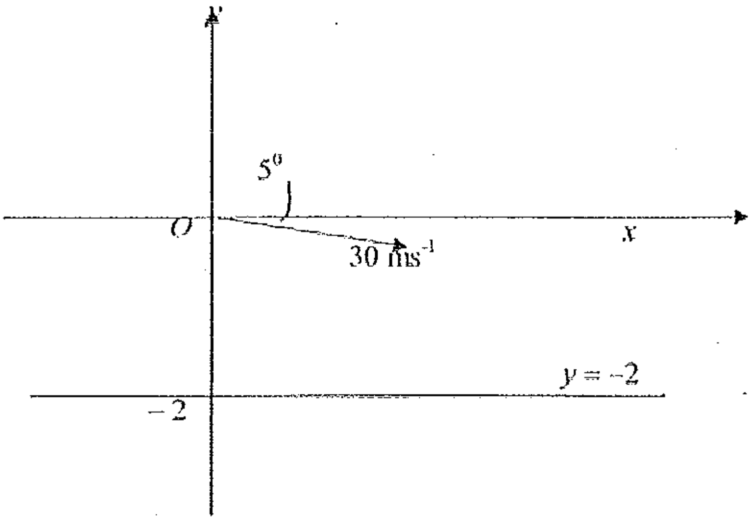
200 ms^{-1} (to the nearest integer)

HSC '99

(7) (a) A cricket ball leaves the bowler's hand 2 metres above the ground with a velocity of 30 ms^{-1} at angle of 5^0 below the horizontal. **8**

The equations of motion for the ball are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10.$$



Take the origin to be the point where the ball leaves the bowler's hand.

(i) Using calculus, prove that the coordinates of the ball at time t are given by

$$x = 30t\cos\left(5^0\right), \text{ and } \quad y = -30t\sin\left(5^0\right) - 5t^2.$$

(ii) Find the time at which the ball strikes the ground.

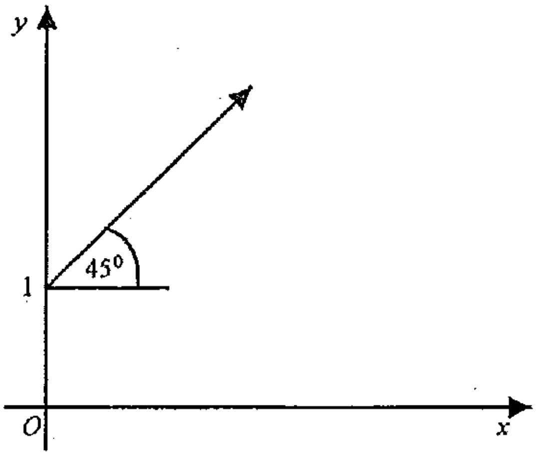
0.42 sec (to 2 d.p.)

(iii) Calculate the angle at which the ball strikes the ground.

13^0

HSC '98 **Marks**

(6) (a) **8**



A particle is projected from the point (0, 1) at an angle of 45^0 with a velocity of V metres per second. The equations of motion of the particle are

$\ddot{x} = 0$ and $\ddot{y} = -g$.

(i) Using calculus, derive the expressions for the position of the particle at time t. Hence show that the path of the particle is given by

$$y = 1 + x - g\frac{x^2}{V^2}.$$

A volleyball player serves a ball with initial speed V metres per second and angle of projection 45^0. At the moment the bottom of the ball is 1 metre above the ground and its horizontal distance from the net is 9.3 metres. The ball just clears the net, which is 2.3 m high.

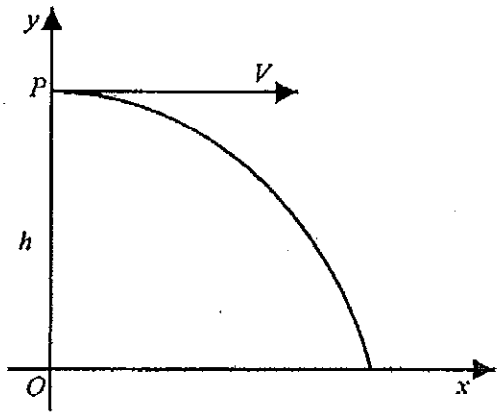
(ii) Show that the initial speed of the ball is approximately 10.3 metres per second. (Take $g = 9.8\ \text{ms}^{-2}$)

(iii) What is the horizontal distance from the net to the point where the ball lands?

2.4 m (1 d.p.)

HSC '97

(7) (a) **9**



A particle is projected horizontally from a point P, h metres above O, with a velocity of V metres per second.

The equations of motion of the particle are $\ddot{x} = 0$ and $\ddot{y} = -g$.

(i) Using calculus, show that the position of the particle at time t is given by

$$x = Vt \qquad \text{and} \qquad y = h - \frac{1}{2}gt^2 .$$

A canister containing a life raft is dropped from a plane to a stranded sailor.
The plane is traveling at a constant velocity of 216 km/hr, at a height of 120 metres above sea level, along a path that passes above the sailor.

(ii) How long will the canister take to hit the water? (Take $g = 10$ m/s^2).

$t = 2\sqrt{6}$ s

(iii) A current is causing the sailor to drift at a speed of 3.6 km/h in the direction as the plane is travelling. The canister is dropped from the plane when the horizontal distance from the plane to the sailor is D metres. What values can D take if the canister lands at most 50 metres from the stranded sailor?

$239\text{ m} \leq D \leq 339\text{ m}$

SOLUTIONS

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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP2) PROJECTILE MOTION (SIMPLER QUESTIONS)

TERRIFIC EFFORT

See corrections on pg 5

HSC 04

(6) (b) A fire hose is at ground level on a horizontal plane. Water is projected from the hose. The angle of projection, θ, is allowed to vary. The speed of the water as it leaves the hose, v metres per second, remains constant. You may assume that if the origin is taken to be the point of projection, the path of the water is given by the parametric equations

$$x = vt\cos\theta$$
$$y = vt\sin\theta - \frac{1}{2}gt^2$$

where g ms^{-2} is the acceleration due to gravity. (Do NOT prove this.)

(i) Show that the water returns to ground level at a distance $\frac{v^2\sin 2\theta}{g}$ metres from the point of projection. 2

$y=0$: $0 = vt\sin\theta - \frac{1}{2}gt^2$

$t(v\sin\theta - \frac{1}{2}gt) = 0$

$\frac{1}{2}gt = v\sin\theta$ ✓

$t = \frac{2v\sin\theta}{g}$

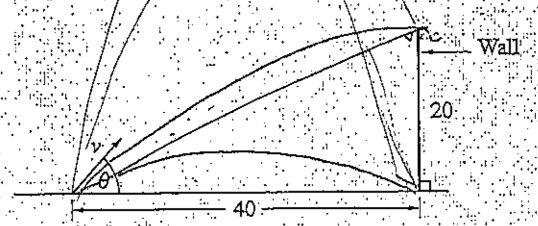
$x = vt\cos\theta$

$= \frac{v\cos\theta \times 2v\sin\theta}{g}$ ✓

$= \frac{v^2}{g} \times 2\sin\theta\cos\theta$

$= \frac{v^2\sin 2\theta}{g}$ ✓ m

This fire hose is now aimed at a 20 metre high thin wall from a point of projection at ground level 40 metres from the base of the wall. It is known that when the angle θ is 15°, the water just reaches the base of the wall.



(ii) Show that $v^2 = 80g$. **1**

$x = 40$

$\theta = 15$

$x = \dfrac{v^2 \sin 2\theta}{g}$ ✓

$40 = \dfrac{v^2}{g}(\sin 30)$

$80 = \dfrac{v^2}{g}$

$v^2 = 80g$ ✓

(iii) Show that the cartesian equation of the path of the water is given by **2**

$$y = x\tan\theta - \frac{x^2\sec^2\theta}{160}$$

$t = \dfrac{x}{v\cos\theta}$

$y = vt\sin\theta - \frac{1}{2}gt^2$ ✓

$= \dfrac{vx\sin\theta}{v\cos\theta} - \dfrac{g}{2}\left[\dfrac{x^2}{v^2\cos^2\theta}\right]$

$= x\tan\theta - \dfrac{x^2 g}{2v^2}\sec^2\theta$

$= x\tan\theta - \dfrac{x^2\sec^2\theta}{2v^2}\left(\dfrac{v^2}{80}\right)$

$\therefore = x\tan\theta - \dfrac{x^2\sec^2\theta}{160}$ ✓

(iv) Show that the water just clears the top of the wall if **2**

$$\tan^2\theta - 4\tan\theta + 3 = 0.$$

$y = 20 \quad x = 40$

$20 \overset{(>)}{=} 40\tan\theta - \dfrac{1600}{160}\sec^2\theta$

$\overset{(>)}{=} 40\tan\theta - 10(\tan^2\theta + 1)$

$\overset{(>)}{=} 40\tan\theta - 10\tan^2\theta - 10.$

$10\tan^2\theta - 40\tan\theta + 30 \overset{>}{=} 0$

$\tan^2\theta - 4\tan\theta + 3 \overset{>}{=} 0$ ✓

(v) Find all values of θ for which the water hits the front of the wall. **2**

$\tan^2\theta - 4\tan\theta + 3 > 0$

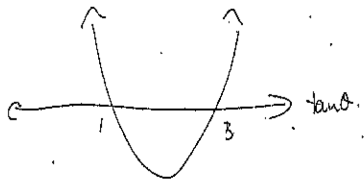
$(\tan\theta - 3)(\tan\theta - 1) > 0$

$\tan\theta < 1 \qquad \tan\theta > 3$ ✓

$\theta < 45° \qquad \theta > 72°$

$\theta = 45$

$\theta = 72°$



$\theta > 15°$: (given)

upper limit :

$y = 0, \; x = 40.$

$0 = 40\tan\theta - \dfrac{1600}{160}(\tan^2\theta + 1)$ ✓

$= 40\tan\theta - 10\tan^2\theta - 10.$

$\tan^2\theta - 4\tan\theta + 1 = 0.$

$\tan\theta = \dfrac{4 \pm \sqrt{16-1}}{2}$ ✓

$\theta = 75°$

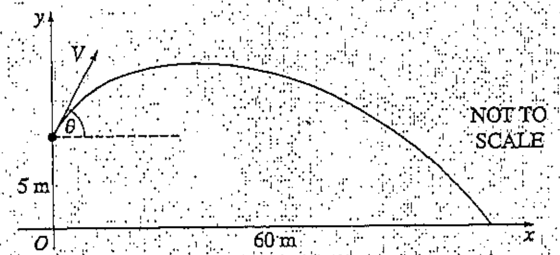
$\therefore 15 < \theta < 45$ ✓

$72 < \theta < 75$

Show that $\theta = 15°$ or $75°$; hence $15° < \theta < 45°$ or $72° < \theta < 75°$

HSC 02

(6)(a)



An angler casts a fishing line so that the sinker is projected with a speed $V\text{ m s}^{-1}$ from a point 5 metres above a flat sea. The angle of projection to the horizontal is θ, as shown.

Assume that the equations of motion of the sinker are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10,$$

referred to the coordinate axes shown.

(i) Let (x, y) be the position of the sinker at time t seconds after the cast, and before the sinker hits the water. **2**

It is known that $x = Vt\cos\theta$.

Show that $y = Vt\sin\theta - 5t^2 + 5$.

$\dot{y} = V\sin\theta - 10t + C$

$t = 0 \quad \dot{y} = V\sin\theta \quad C = 0$

$\therefore \dot{y} = V\sin\theta - 10t$

$y = Vt\sin\theta - 5t^2 + C$

$t = 0 \quad y = 5 \quad \therefore C = 5$

$\therefore y = Vt\sin\theta - 5t^2 + 5.$

(ii) Suppose the sinker hits the sea 60 metres away as shown in the diagram. **3**

Find the value of V if $\theta = \tan^{-1}\frac{3}{4}$.

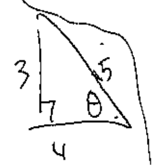
$x = 60:$

$60 = Vt\cos\theta$

$= Vt \times \frac{4}{5}$

$Vt = 75$

$t = \frac{75}{V}$



$y = 0: \quad 0 = Vt\sin\theta - 5t^2 + 5$

$5t^2 - Vt\sin\theta - 5 = 0$

$5\left(\frac{75}{V}\right)^2 - \frac{75V}{V} \times \frac{3}{5} - 5 = 0$

$\frac{28125}{V^2} - 50 = 0$

$50 = \frac{28125}{V^2}$

$V^2 = \frac{1125}{2}$

$V = \frac{15\sqrt{5}}{\sqrt{2}}$

$= \frac{15\sqrt{10}}{2}$

$V = \frac{15\sqrt{10}}{2}$

(iii) For the cast described in part (ii), find the maximum height above sea level that the sinker achieved. **2**

$y = 0 \quad 0 = V\sin\theta - 10t$

$10t = V\sin\theta$

$= \frac{15\sqrt{10}}{2} \times \frac{3}{5}$

$t = \frac{9\sqrt{10}}{20}$ s

$y = Vt\sin\theta - 5t^2 + 5$

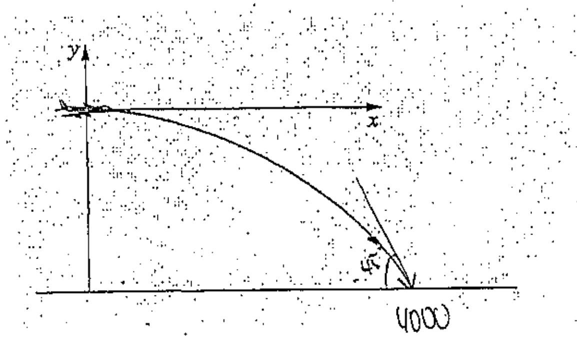
$= \frac{15\sqrt{10}}{2} \times \frac{9\sqrt{10}}{20} \times \frac{3}{5} - 5\left(\frac{9\sqrt{10}}{20}\right)^2 + 5$

$= \frac{121}{8}$ m

$15\frac{1}{8}$ m

HSC 01

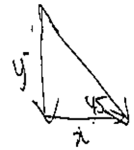
(4) (b) An aircraft flying horizontally at V ms^{-1} releases a bomb that hits the ground 4000 m away, measured horizontally. The bomb hits the ground at an angle of 45^0 to the vertical. **4**



Assume that, t seconds after release, the position of the bomb is given by

$x = Vt, \; y = -5t^2.$

Find the speed V of the aircraft.



$x = 4000 = Vt$

$t = \frac{4000}{V}$

$\dot{x} = V \qquad \dot{y} = -10t$

$\tan 45 = \frac{\dot{y}}{\dot{x}}$

$|\dot{y}| = |\dot{x}|$

$|V| = \left|-10\left(\frac{4000}{V}\right)\right|$

$V^2 = 40000$

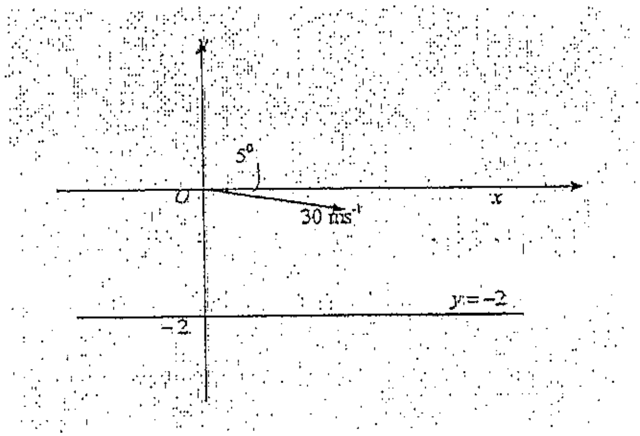
$V = 200$ ms^{-1}

200 ms^{-1} (to the nearest integer)

HSC '99

(7) (a) A cricket ball leaves the bowler's hand 2 metres above the ground with a velocity of 30 ms^{-1} at angle of 5^0 below the horizontal. The equations of motion for the ball are **8**

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -10.$$



Take the origin to be the point where the ball leaves the bowler's hand.

(i) Using calculus, prove that the coordinates of the ball at time t are given by

$$x = 30t\cos(5^0), \text{ and } \quad y = -30t\sin(5^0) - 5t^2.$$

$\dot{x} = 30\cos 5$ ✓

$x = 30t\cos 5 + C$.

$x = 0, \ t = 0$

$\therefore x = 30t\cos(5)$ ✓

$\dot{y} = -30\sin 5 - 10t$. ✓

$y = -30t\sin 5 - 5t^2 + C$.

$y = 0 \quad t = 0$

$y = -30t\sin(5) - 5t^2$ ✓

(ii) Find the time at which the ball strikes the ground.

$y = -2$:

$-2 = -30t\sin(5) - 5t^2$

$5t^2 + 30t\sin(5) = 2$ ✓

$5t^2 + 30t\sin(5) - 2 = 0$.

$t = \dfrac{-30\sin 5 \pm \sqrt{900\sin^2 5 - 4(5)(-2)}}{10}$ ✓

$= 0.42s$ or $-0.95s$.

$\therefore t = 0.42s$. ✓

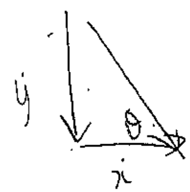
0.42 sec (to 2 d.p.)

(iii) Calculate the angle at which the ball strikes the ground.

$\dot{x} = 30\cos 5$

$\dot{y} = -30\sin 5 - 10(0.42)$

$= -30\sin 5 - 4.2$. ✓



$\tan\theta = \left|\dfrac{\dot{y}}{\dot{x}}\right|$

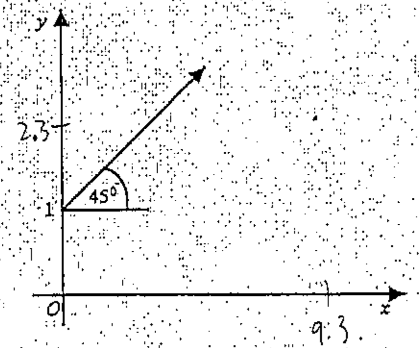
$= \dfrac{30\sin 5 + 4.2}{30\cos 5}$

$\theta = 12° 51'$

13⁰

HSC '98 **Marks**

(6) (a) 8



A particle is projected from the point (0, 1) at an angle of 45° with a velocity of V metres per second. The equations of motion of the particle are

$\ddot{x} = 0$ and $\ddot{y} = -g$.

(i) Using calculus, derive the expressions for the position of the particle at time t. Hence show that the path of the particle is given by

$$y = 1 + x - g\frac{x^2}{V^2}.$$

$\dot{x} = V\cos 45 = \frac{V}{\sqrt{2}}$

$x = \frac{Vt}{\sqrt{2}}$

$t = \frac{\sqrt{2}x}{V}$

$\dot{y} = V\sin 45 - gt = \frac{V}{\sqrt{2}} - gt$

$y = \frac{Vt}{\sqrt{2}} - \frac{gt^2}{2} + 1$

$y = \frac{V}{\sqrt{2}}\left(\frac{\sqrt{2}x}{V}\right) - \frac{g}{2}\left(\frac{\sqrt{2}x}{V}\right)^2 + 1$

$= x - \frac{gx^2}{V^2} + 1$

A volleyball player serves a ball with initial speed V metres per second and angle of projection 45°. At the moment the bottom of the ball is 1 metre above the ground and its horizontal distance from the net is 9.3 metres. The ball just clears the net, which is 2.3 m high.

(ii) Show that the initial speed of the ball is approximately 10.3 metres per second. (Take $g = 9.8\ ms^{-2}$)

$y = x + 1 - \frac{gx^2}{V^2}$ $x = 9.3$ $y = 2.3$ $g = 9.8$

$2.3 = 9.3 + 1 - \frac{9.8(9.3)^2}{V^2}$

$\frac{9.8(9.3)^2}{V^2} = 8$

$V^2 \doteq 105.95$

$V \doteq 10.293\ ms^{-1}$

$\doteq 10.3\ ms^{-1}$ (3sf)

(iii) What is the horizontal distance from the net to the point where the ball lands?

$y = 0$: $0 = x + 1 - \frac{9.8x^2}{105.95}$

$\frac{9.8x^2}{105.95} - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(-1)\left(\frac{9.8}{105.95}\right)}}{2 \times \frac{9.8}{105.95}}$

$= 11.73$

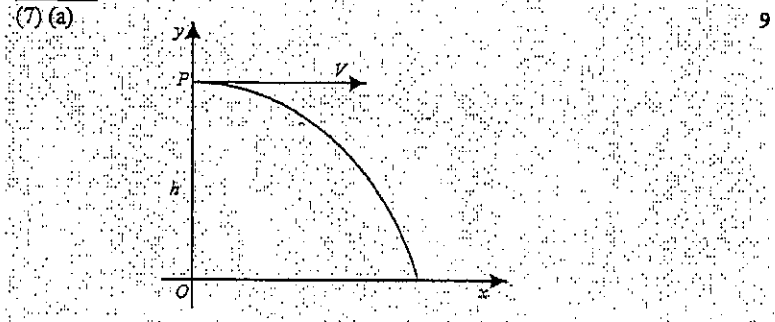
d to net $= x - 9.3$

$\doteq 2.43$ (2dp)

2.4 m (1 d.p.)

HSC '97

(7) (a) 9



A particle is projected horizontally from a point P, h metres above O, with a velocity of V metres per second.

The equations of motion of the particle are $\ddot{x} = 0$ and $\ddot{y} = -g$.

(i) Using calculus, show that the position of the particle at time t is given by

$x = Vt$ and $y = h - \frac{1}{2}gt^2$.

$\dot{x} = V$. $\dot{y} = -gt$.

$x = Vt$ ✓ $y = -\frac{gt^2}{2} + h$. ✓

A canister containing a life raft is dropped from a plane to a stranded sailor. The plane is traveling at a constant velocity of 216 km/hr, at a height of 120 metres above sea level, along a path that passes above the sailor.

(ii) How long will the canister take to hit the water? (Take $g = 10$ m/s^2).

$y = -5t^2 + 120$.

$y = 0$ ∴

$5t^2 = 120$ ✓

$t^2 = 24$. ✓

$t = 2\sqrt{6}$ s.

$t = 2\sqrt{6}$ s

(iii) A current is causing the sailor to drift at a speed of 3.6 km/h in the direction as the plane is travelling. The canister is dropped from the plane when the horizontal distance from the plane to the sailor is D metres. What values can D take if the canister lands at most 50 metres from the stranded sailor?

216 km/hr.

216 000 m/hr ✓

60m/hr.

3600 m/h

1 m/s.

$x = Vt$. $- 1(2\sqrt{6})$

$D = 60 \times 2\sqrt{6}$ $- 2\sqrt{6}$

$D \doteq 289$ m. ✓

→ V

← 1

±50m: 239m ≤ D ≤ 339m. ✓

50m ← D → 50m

239 m ≤ D ≤ 339 m