C.E.M. TUITION

Name : ______________________________

Review Topic : Angle between lines, parametric equations and circle geometry

Year 12 - Maths Ext. 1

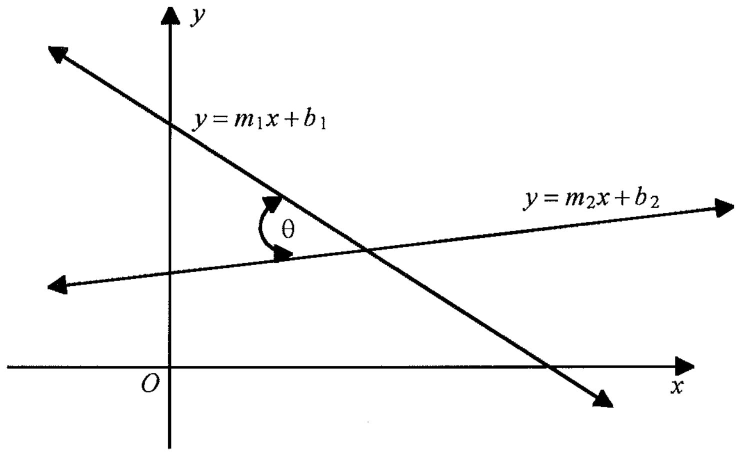
PHONE : 9666-3331 **FAX : 9316-4996**

MOBILE: 0412 880 475

For corrections refer to pages:

1. Angle between two lines :

If m_1 and m_2 are the gradients of the lines : $L_1 : y = m_1 x + b_1$ and $L_2 : y = m_2 x + b_2$ and θ is the angle between the lines, then



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Examples :

(1) Find the acute angle between the lines :
$L_1 : y = 3x + 1$ and $L_2 : x + 2y + 7 = 0$

$81^0 52'$

(2) Find ***two*** possible values of p if the lines

$L_1 : px - y - 7 = 0$ and $L_2 : 3x + y - 2 = 0$ intersect at 45^0.

$p = 2$ or $-\frac{1}{2}$

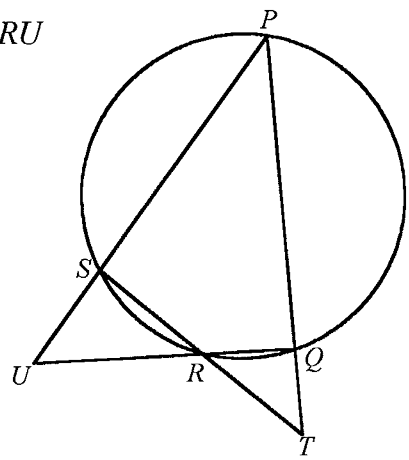
(3) What are the ***two*** possible gradients of lines that make an angle of 45^0 with the line $2x - y + 5 = 0$?

-3 or $\frac{1}{3}$

2. Circle Geometry :

Referring to your 3 Unit Rules and formulae booklet, try these examples.

(1) In the diagram shown, PQT, PSU, SRT and QRU are straight lines and $\angle PTS = \angle PUQ$.



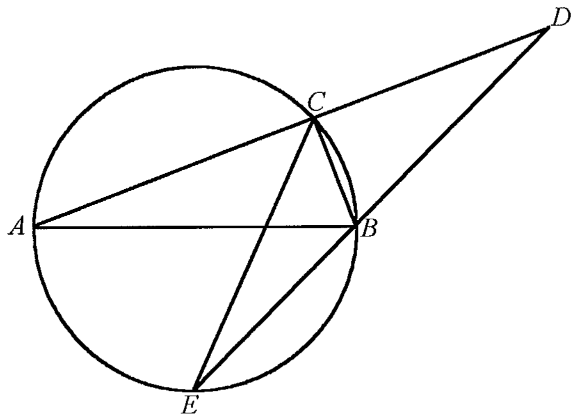
(a) Show that $\angle PQR = \angle PSR$.

(b) Prove that PR is the diameter of the circle.

(2) AB is the diameter of the circle $ACBE$.
ACD and EBD are straight lines.
AB, BC and EC are joined.

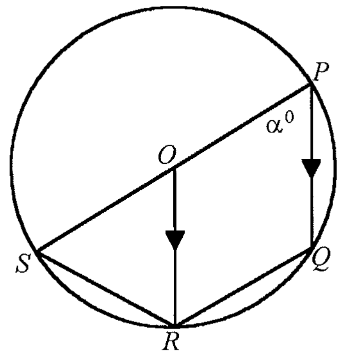
$\angle BAC = 28^0$ and $\angle CDB = 30^0$,

find the size of $\angle BCE$ giving reasons.

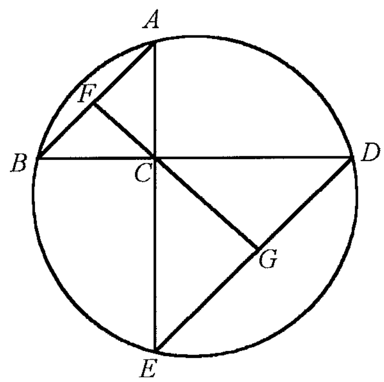


32^0

(3) PS is the diameter of the circle, with centre O.
OR and PQ are parallel, and $\angle OPQ = \alpha^0$.
If SR and QR are joined, prove that
OR bisects $\angle SRQ$.



(4) BD and AE are two perpendicular chords intersecting at C. $CG \perp DE$ and GC is produced to meet AB at F.



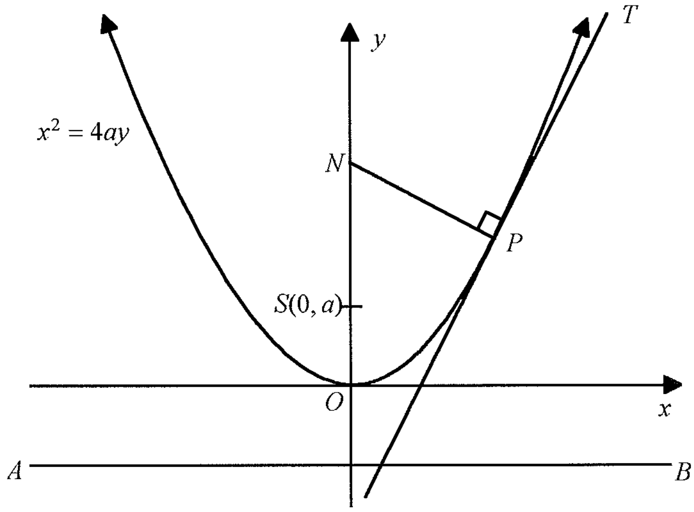
(a) Show that ΔBFC is isosceles.

(b) Show that GF bisects AB.

PARAMETRIC FORM OF THE PARABOLA

Review :

For the parabola $x^2 = 4ay$, complete the following :



(a) the point $S(0, a)$ is called the ______________________________ .

(b) the equation of the directrix AB is ___________________________ .

(c) PT is called the ______________ .

(d) PN is called the _______________ .

(e) the parametric equations of P (using p as a parameter) is :

$x =$ __________ $y =$ ___________

(f) draw the line representing the ***"latus rectum"***

(g) the length of the latus rectum is ___________________ .

(h) draw a focal chord passing through P.

Derivation of equations of the chord, tangent and normal

Students are required to be able to derive the following equations:
(A clear concise diagram is always helpful).

(1)(a) Show that the equation of the chord joining $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ is given by

$$(p+q)x - 2y - 2apq = 0.$$

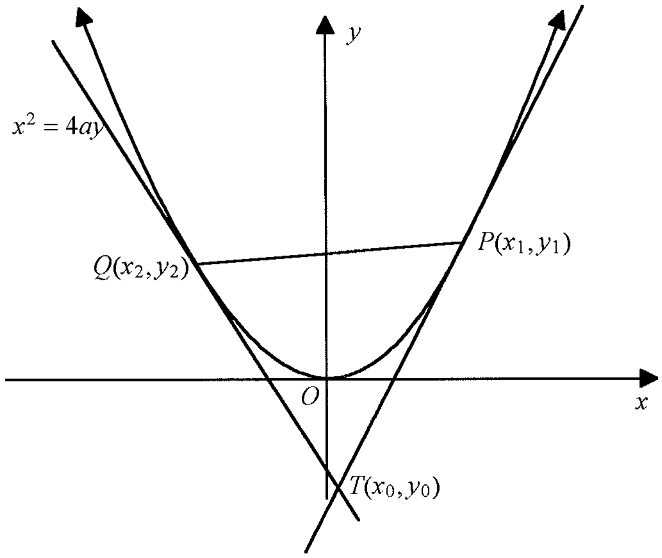
(b) If this chord passes through the focus, show that $pq = -1$.

(2) Show that the equation of the

(a) tangent to the parabola $x^2 = 4ay$ at $P(2ap, ap^2)$ is $px - y - ap^2 = 0$

(b) normal at $P(2ap, ap^2)$ is $x + py = 2ap + ap^3$.

(3)



(a) Show that the equation of the tangent at $P(x_1, y_1)$ is $xx_1 = 2a(y + y_1)$.

(ii) Hence, or otherwise, show that the equation of chord of contact drawn from an external point $T(x_0, y_0)$ is given by :

$$xx_0 = 2a(y + y_0)$$

Examples :

(1) If the point $P(2ap, ap^2)$ is one end of a focal chord of the parabola $x^2 = 4ay$, show that the other end is the point $Q\left(-\frac{2a}{p}, \frac{a}{p^2}\right)$.

(2) Show that the tangents at the extremities of a focal chord of a parabola $x^2 = 4ay$ meet at right angles on the directrix.

(3) A chord of the parabola $x^2 = 4ay$ subtends a right angle at the vertex.
Find the locus of the midpoint of the chord.

$x^2 = 2a(y - 4a)$

(4) Normals at the extremities of a focal chord of the parabola $x^2 = 4ay$ intersects at Q. Show that the locus of Q as the chord varies is given by $x^2 = a(y - 3a)$.

(5) (a) If P and Q are the points $(2ap, ap^2)$ and $(2aq, aq^2)$ on the parabola $x^2 = 4ay$, show that the coordinates of the point of intersection T of the tangents at P and Q are $(a(p+q), apq)$.

(b) Hence, show that the area of ΔTPQ is $\frac{1}{2}a^2(p-q)^3$.

(6) If the line $px + qy + r = 0$ is a tangent to the parabola $x^2 = 4ay$, show that :

$$ap^2 = qr.$$