Review of Binomial Theorem: Specimen paper 1

Answer in the space provided. Show all working.

1) Yr12-3U\binomial.cat Qn1) 3U85-7i

a. Write the expansion for $(1 + x)^3$.

b. Given that $\binom{n}{r}$ and ${}^nC_r$ are different notations for the same idea, show that

$$\binom{n}{r} : \binom{n}{r-1} = (n - r + 1) : r$$

c. Hence find the sum of $\dfrac{\binom{n}{1}}{\binom{n}{0}} + \dfrac{2\binom{n}{2}}{\binom{n}{1}} + \dfrac{3\binom{n}{3}}{\binom{n}{2}} + \ldots + \dfrac{n\binom{n}{n}}{\binom{n}{n-1}}$.†

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2) Yr12-3U\binomial.cat Qn2) 3U86-7ii

a. Find the greatest co-efficient in the expansion of $(\frac{1}{3}+2x)^{18}$.

b. Given that $x = \frac{2}{7}$ in the above expansion, show that there are two consecutive terms which are equal in value and greater than all other terms.†

3) Yr12-3U\binomial.cat Qn3) 3U87-5a

In the expansion of $(x^2 + \frac{1}{2x})^{14}$ in powers of x, show that the terms involving x^{13} and x^{16} have the same numerical coefficient and state the value of this coefficient as a rational number.†

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4)	Yr12-3U\binomial.cat Qn4) 3U88-3c Find, as a rational number, the coefficient of x in the expansion of $(x^2 + \frac{1}{2x})^8$.†
5)	Yr12-3U\binomial.cat Qn5) 3U89-4a Find, expressed as a rational number, the term independent of x in the expansion of $(5x^2 + \frac{3}{x^3})^{10}$ and show also that it is the greatest coefficient in the expansion.†

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6)	Yr12-3U\binomial.cat Qn6) 3U90-4c Find the value of the term independent of x in the expansion of $(2x - \frac{1}{x^2})^{12}$.†

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[Answers]

1«→ a) $1 + 3x + 3x^2 + x^3$ b) Proof c) $\frac{n(n+1)}{2}$ »

2«→ a) 17×2^{16} b) Proof »

3«→ Proof, $\frac{1001}{16}$ »

4«→ $\frac{7}{4}$ »

5«→ *265 781 250* »

6«→ *126 720* »

Answer in the space provided. Show all working.

1) Yr12-3U\binomial.cat Qn1) 3U85-7i

a. Write the expansion for $(1+x)^3$.

b. Given that $\binom{n}{r}$ and ${}^nC_r$ are different notations for the same idea, show that

$$\binom{n}{r}:\binom{n}{r-1}=(n-r+1):r$$

c. Hence find the sum of $\dfrac{\binom{n}{1}}{\binom{n}{0}}+\dfrac{2\binom{n}{2}}{\binom{n}{1}}+\dfrac{3\binom{n}{3}}{\binom{n}{2}}+\ldots+\dfrac{n\binom{n}{n}}{\binom{n}{n-1}}$.†

a) $\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^2+\binom{3}{3}x^3$ ✓

b) $\binom{n}{r}:\binom{n}{r-1}=\dfrac{\left(\frac{n!}{(n-r)!\,r!}\right)}{\left(\frac{n!}{(n-r+1)!\,(r-1)!}\right)}=\dfrac{(n-r+1)!\,(r-1)!}{(n-r)!\,r!}=\dfrac{(n-r)!(n-r+1)(r-1)!}{(n-r)!\,(r)(r-1)!}=\dfrac{n-r+1}{r}$ ✓

$(n-1)$ times

c) $\Sigma=\dfrac{\binom{n}{1}}{n}+\dfrac{2(n-2+1)}{2}+\dfrac{3(n-3+1)}{3}+\ldots+\dfrac{n(n-n+1)}{n}$

$=(n)+n-1+n-2+n-3+\ldots+1$

$=\dfrac{1+n}{2}\times n$ ✓

$\dfrac{(n-1)\binom{n}{n-1}}{\binom{n}{n-2}}=\dfrac{(n-1)(n-(n-1)+1)}{(n-1)}=2$

2) Yr12-3U\binomial.cat Qn2) 3U86-7ii

a. Find the greatest co-efficient in the expansion of $(\frac{1}{3}+2x)^{18}$.

b. Given that $x=\frac{2}{7}$ in the above expansion, show that there are two consecutive terms which are equal in value and greater than all other terms.†

a) $\binom{18}{R}\left(\frac{1}{3}\right)^{18-R}(2x)^R$ ~~$=\binom{18}{R}\left(\frac{1}{3}\right)^{18-R}(2)^R x^R$~~

$\dfrac{T_{R+1}}{T_R}=\dfrac{n-r+1}{r}\cdot\dfrac{b}{a}>1$

$=\dfrac{18-r+1}{r}\cdot\dfrac{2}{(\frac{1}{3})}>1$ ✓

$\therefore 18-r+1>\dfrac{r}{6}$

$108-6r+6>r$

$\therefore 108-7r+6>0$

$\therefore -7r>-114$

$r<16.28$ $\therefore r=16.$

$\therefore$ coeff $=\binom{18}{16}\left(\frac{1}{3}\right)^2(2)^{16}$

$=1{,}114{,}112$ ✓

b) $\dfrac{T_{R+1}}{T_R}\geq 1;$ $\dfrac{18-r+1}{r}\cdot\dfrac{2(\frac{2}{7})}{\frac{1}{3}}\geq 1$

$\therefore 19r\leq 228$

$\therefore r\leq 12$

$\therefore T_{R+1}=\binom{18}{12}\left(\frac{1}{3}\right)^6\left(2\left(\frac{2}{7}\right)\right)^{12}=0.0302$

$T_R=\binom{18}{11}\left(\frac{1}{3}\right)^7\left(2\left(\frac{2}{7}\right)\right)^{11}=0.0302$

$T_{R+1}=T_R.$

3) Yr12-3U\binomial.cat Qn3) 3U87-5a

In the expansion of $(x^2+\frac{1}{2x})^{14}$ in powers of x, show that the terms involving x^{13} and x^{16} have the same numerical coefficient and state the value of this coefficient as a rational number.†

$T_{R+1}=\binom{14}{R}(x^2)^{14-R}\left(\frac{1}{2x}\right)^R$

$\therefore\binom{14}{R}(2)^{-R}(x)^{28-2R}(x)^{-R}$

$\therefore x^{28-3R}=x^{13}$ ✓ or x^{16} ✓

$\therefore R=5$ $\quad R=4$

coeff T_6: $\binom{14}{5}(2)^{-5}=62.5625$

" " T_5: $\binom{14}{4}(2)^{-4}=62.5625$

coeff $=\dfrac{1001}{16}.$

4) Yr12-3U\binomial.cat Qn4) 3U88-3c

Find, as a rational number, the coefficient of x in the expansion of $(x^2 + \frac{1}{2x})^8$.†

$T_{r+1} = \binom{8}{r}(x^2)^{8-r}(\frac{1}{2x})^r$

$= \binom{8}{r}(x)^{16-2r}(2)^{-r}(x)^{-r}$

$= \binom{8}{r}(2)^{-r}(x)^{16-3r}$

$\therefore 16-3r = 1$

$r = 5$

$\therefore$ coeff $T_6 = \binom{8}{5}(2)^{-5}$

$= \frac{7}{4}$

5) Yr12-3U\binomial.cat Qn5) 3U89-4a

Find, expressed as a rational number, the term independent of x in the expansion of $(5x^2 + \frac{3}{x^3})^{10}$ and show also that it is the greatest coefficient in the expansion.†

$T_{r+1} = \binom{10}{r}(5x^2)^{10-r}(\frac{3}{x^3})^r$

$= \binom{10}{r}(5)^{10-r}(x)^{20-2r}(3)^r(x)^{-3r}$

$= \binom{10}{r}(5)^{10-r}(3)^r(x)^{20-5r}$

$20-5r=0$

$\therefore r=4$

Coeff $T_5 = \binom{10}{4}(5)^6(3)^4$

$= 265,781,250$

$\frac{T_{r+1}}{T_r} > 1 \quad \therefore \frac{n-r+1}{r} \times \frac{b}{a} > 1$

$\therefore \frac{10-r+1}{r} \cdot \frac{3}{5} > 1$

$\therefore 30-3r+3 > 5r$

$\therefore -8r > -33$

$\therefore r < 4.125$

$\therefore r = 4.$

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6) Yr12-3U\binomial.cat Qn6) 3U90-4c

Find the value of the term independent of x in the expansion of $(2x - \frac{1}{x^2})^{12}$.†

$T_{r+1} = \binom{12}{r}(2x)^{12-r}(-\frac{1}{x^2})^r$

$= \binom{12}{r}(2)^{12-r}(x)^{12-r}(-1)^r(x)^{-2r}$

$= \binom{12}{r}(2)^{12-r}(-1)^r(x)^{12-3r}$

$\therefore 12-3r=0$

$\therefore r=4$

$\therefore$ coeff $T_5 = \binom{12}{4}(2)^8(-1)^4$

$= 126,720$