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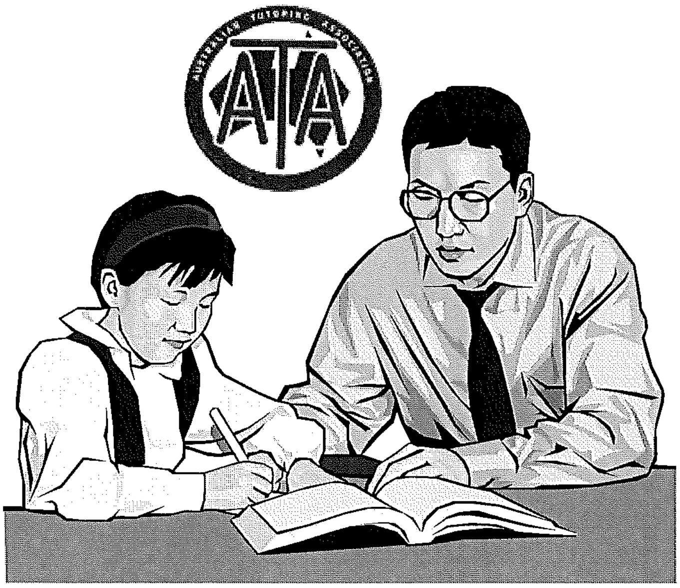

YEAR 12 – MATHS EXT.1

REVIEW TOPIC (PAPER 2): BINOMIAL THEOREM

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GRAMMAR 2004 Q5

(b) (i) Write down the expansion of $(1+x)^n$ in ascending powers of x. Then differentiate both sides of your identity. [1]

(ii) Make an appropriate substitution for x to show that [1]

$$\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+4\binom{n}{4}+\cdots+n\binom{n}{n}=n(2^{n-1}).$$

(iii) Hence find an expression for [1]

$$2\binom{n}{1}+3\binom{n}{2}+4\binom{n}{3}+5\binom{n}{4}+\cdots+(n+1)\binom{n}{n}.$$

JAMES RUSE 2002 Q7

(b) (i) Show that $(1+x)^m(1-\frac{1}{x})^m = (x-\frac{1}{x})^m$ **1**

(ii) By considering the term(s) independent of x in the expansion of the result from part (b) (i), justify the result: **3**

$$\binom{2002}{0}^2 - \binom{2002}{1}^2 + \binom{2002}{2}^2 - \dots + \binom{2002}{2002}^2 = -1\binom{2002}{1001}$$

(iii) Hence, or otherwise, show that: **3**

$$\sum_{k=0}^{1001}(-1)^k\binom{2002}{k}^2=-\frac{1}{2}\binom{2002}{1001}\left[1+\binom{2002}{1001}\right].$$

SBHS 2004 Q7

(a) Using the expansion of $(1+x)^n$

(i) Find an expression for $\sum_{r=1}^{n} r\binom{n}{r}$ **2**

(ii) Hence, or otherwise, prove that $\sum_{r=0}^{n}(r+1)\binom{n}{r} = 2^{n-1}(n+2)$ **2**

ST IGNATIUS 2002 Q4

(c) Prove that $\binom{n+1}{2}+\binom{n+2}{2}$ is the square of an integer. 3

SOLUTIONS

GRAMMAR 2004 Q5

(b)(i) $(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots \binom{n}{k}x^k + \dots \binom{n}{n}x^n$

$\frac{d}{dx}$: $n(1+x)^{n-1} = \binom{n}{1} + 2x\binom{n}{2} + 3x^2\binom{n}{3} + \dots kx^{k-1}\binom{n}{k} + \dots nx^{n-1}\binom{n}{n}$

(ii) Let $x = 1$

$n(2)^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + 4\binom{n}{4} + \dots n\binom{n}{n}$

(iii) Add $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots \binom{n}{n}$ RHS $= 2\binom{n}{1} + 3\binom{n}{2} + \dots (n+1)$

LHS $= \binom{n}{1} + \binom{n}{2} + \dots \binom{n}{n} + n(2)^{n-1}$

$= 2^n - 1 + n(2)^{n-1}$

$= 2^{n-1}[n+2] - 1$

JAMES RUSE 2002 Q7

(b)(i) $(1+x)^m\left(1-\frac{1}{x}\right)^m = \left[(1+x)\left(1-\frac{1}{x}\right)\right]^m$

$= \left[1 - \frac{1}{x} + x - 1\right]^m$

$= \left[x - \frac{1}{x}\right]^m$ (1)

(ii) Letting $m = 2002$

LHS $= (1+x)^{2002}\left(1-\frac{1}{x}\right)^{2002}$

$= \left[\binom{2002}{0} + \binom{2002}{1}x + \dots \binom{2002}{r}x^r + \dots \binom{2002}{2002}x^{2002}\right]$

$\times \left[\binom{2002}{0} - \binom{2002}{1}\frac{1}{x} + \dots + (-1)^r\binom{2002}{r}\frac{1}{x^r} + \dots + \binom{2002}{2002}\frac{1}{x^{2002}}\right]$ (1)

(iii) L.H.S. $= \binom{2002}{0}^2 - \binom{2002}{1}^2 + \binom{2002}{2}^2 - \dots + \binom{2002}{1000}^2 - \binom{2002}{1001}^2 + \binom{2002}{1002}^2 - \dots + \binom{2002}{2000}^2 - \binom{2002}{2001}^2 + \binom{2002}{2002}^2$

$= 2\left[\binom{2002}{0}^2 - \binom{2002}{1}^2 + \binom{2002}{2}^2 - \dots + \binom{2002}{1000}^2\right] - \binom{2002}{1001}^2$ as ${}^nC_r = {}^nC_{n-r}$ (1)

$= 2\left[\binom{2002}{0}^2 - \binom{2002}{1}^2 + \binom{2002}{2}^2 - \dots + \binom{2002}{1000}^2 - \binom{2002}{1001}^2\right] + \binom{2002}{1001}^2$ (1)

$\therefore -1\binom{2002}{1001} = 2\Sigma + \binom{2002}{1001}^2$

$2\Sigma = -1\binom{2002}{1001} - \binom{2002}{1001}^2$

$\therefore \sum_{k=0}^{1001}(-1)^k\binom{2002}{k}^2 = -\frac{1}{2}\binom{2002}{1001}\left[1 + \binom{2002}{1001}\right]$ (1)

SBHS 2004 Q7

(a) now

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + \binom{n}{r}x^r + \cdots + \binom{n}{n}x^n \quad \text{— (A)}$$

(i) differentiate both sides of (A) above.

$$n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + \cdots + r\binom{n}{r}x^{r-1} + \cdots + n\binom{n}{n}x^{n-1}.$$

Let $x = 1$

$$n \cdot 2^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + r\binom{n}{r} + \cdots + n\binom{n}{n}$$

ie. $\boxed{\sum_{r=1}^{n} r\binom{n}{r} = n2^{n-1}}$ ✓✓ (NB this is equivalent to $\sum_{r=0}^{n} r\binom{n}{r} = n2^{n-1}$)

(ii) R.T.P. $\sum_{r=0}^{n} (r+1)\binom{n}{r} = 2^{n-1}(n+2)$

$$LHS = \sum_{r=0}^{n} r\binom{n}{r} + \sum_{r=0}^{n}\binom{n}{r}$$

$= n2^{n-1} + 2^n$ (If we let $x=1$ in (A) $2^n = \sum_{r=0}^{n}\binom{n}{r}$)

$= \boxed{2^{n-1}(n+2)}$ ✓✓

$= RHS.$

ST IGNATIUS 2002 Q4

(c) $\binom{n+1}{2} + \binom{n+2}{2} = \frac{(n+1)n}{2} + \frac{(n+2)(n+1)}{2}$

$= \frac{n+1}{2}[n + n+2]$

$= \frac{n+1}{2} \times 2(n+1)$

$= (n+1)^2$

which is the square of an integer.