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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP1)
BINOMIAL THEOREM II

***HSC '99**

(7) (b) By considering $(1-x)^n\left(1+\frac{1}{x}\right)^n$, or otherwise, express

$\binom{n}{2}\binom{n}{0}-\binom{n}{3}\binom{n}{1}+\ldots+(-1)^n\binom{n}{n}\binom{n}{n-2}$ in the simplest form.

$\binom{n}{\frac{n+2}{2}}(-1)^{\frac{n+2}{2}}$ if n is even; 0 if n is odd.

HSC '98

(7) (a) (i) Use the binomial theorem to obtain an expansion for

$(1+x)^{2n}+(1-x)^{2n}$, where n is a positive integer.

$$\boxed{2\left(1+{}^{2n}C_2x^2+{}^{2n}C_4x^4+....+{}^{2n}C_{2n}x^{2n}\right)}$$

(ii) Hence evaluate $1+{}^{20}C_2+{}^{20}C_4+....+{}^{20}C_{20}$.

$$\boxed{2^{19}}$$

HSC '97

(7) (b) (i) Simplify $n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}$. **3**

$\boxed{n\left(2^{n-1}-2\right)}$

(ii) Find the smallest positive integer n such that

$$n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}>20\ 000.$$

$\boxed{n=12}$

HSC '96

(7) (a) Using the fact that $(1+x)^4(1+x)^9=(1+x)^{13}$, show that **2**

$${}^4C_0\,{}^9C_4+{}^4C_1\,{}^9C_3+{}^4C_2\,{}^9C_2+{}^4C_3\,{}^9C_1+{}^4C_4\,{}^9C_0={}^{13}C_4 .$$

HSC '95

(3) (b) Find the value of the term that does not depend on x in the expansion of

$$\left(x^2+\frac{3}{x}\right)^6$$

1215

HSC '92

(5) (c) Consider the binomial expansion $1+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{n}x^n=(1+x)^n$.

(i) Show that $1-\binom{n}{1}+\binom{n}{2}-\ldots+(-1)^n\binom{n}{n}=0$.

(ii) Show that $1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\ldots+(-1)^n\frac{1}{n+1}\binom{n}{n}=\frac{1}{n+1}$.

HSC '90

(6) (c) (i) Show that $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n=(1+x)^{2n}$.

(ii) Hence prove that $1+\binom{n}{1}^2+\binom{n}{2}^2+\ldots+\binom{n}{n}^2=\binom{2n}{n}$.

MISCELLANEOUS EXERCISES:

(1)

Statistics show that, of motorists tested for drink-driving, 3% are found to be over the limit. Find, as decimals to three places, the probability that, in a group of thirty drivers tested:

(i) none will be over the limit

0.401

(ii) exactly one will be over the limit

0.372

(iii) at least two will be over the limit.

0.227

(2) A die is loaded in such a way that in 8 throws of the die, the probability of getting 3 even numbers is four times the probability of getting 2 even numbers.

Find the probability that a single throw of the die results in an even number.

$\frac{2}{3}$

(3) An unbiased die is thrown six times. Find the probabilities that the six scores obtained will: (i) be 1, 2, 3, 4, 5, 6 in some order,

$\boxed{\frac{5}{324}}$

(ii) have a product which is an even number

$\boxed{\frac{63}{64}}$

(iii) consists of exactly two 6's and four odd numbers

$\boxed{\frac{5}{192}}$

(iv) be such that a 6 occurs only on the last throw and exactly three of the first five throws result in odd numbers.

$\boxed{\frac{5}{216}}$

(4) A given school in a certain State has 3 mathematics teachers. The probability in that State that a mathematics teacher is female is 0.4.

(a) What is the probability that in the given school there is at least one female mathematics teacher?

0.784

(b) In the same State the probability that a mathematics teacher (male or female) is a graduate is 0.7. What is the probability that in the given school none of the three mathematics teachers is a female graduate?

0.373

SOLUTIONS:

(1)

(i) **Probability** $= (0\cdot97)^{30} = 0\cdot401$

(ii) **Probability** $= {}^{30}C_1(0\cdot97)^{29}(0\cdot03)$
$= 0\cdot372$

(iii) **Probability** $= 1-(0\cdot401+0\cdot372)$
$= 0\cdot227$

(2) Let p be the probability of throwing an even number and q be the probability of throwing an odd number.

In 8 throws of a die: $P(3 \text{ even numbers}) = \text{term in } p^3$ in the expansion of

$$(q+p)^8 = {}^8C_3q^5p^3 = 56q^5p^3.$$

$P(2 \text{ even numbers}) = \text{term in } p^2$ in the expansion of

$(q+p)^8 = {}^8C_2q^6p^2 = 28q^6p^2$ and since

$$P(3 \text{ even nos}) = 4\times P(2 \text{ even nos})$$
$$56q^5p^3 = 4\times 28q^6p^2$$

Assuming $p,q \neq 0$ then $p = 2q = 2(1-p)$

$$\therefore p = P(1 \text{ even number}) = \frac{2}{3}$$

(3) (i) P(scores will be 1,2,3,4,5,6 in some order) $= \dfrac{6!}{6^6} = \dfrac{5}{324}$

(ii) P(scores will have a product which is an even number)
$= 1-$ P(scores will have a product which is an odd number)
$= 1-$ P(all 6 scores are odd numbers)
$$= 1-\left(\frac{1}{2}\right)^6 = \frac{63}{64}$$

(iii) P(scores will consist of exactly two 6's and four odd numbers)
$$= {}^6C_2\left(\frac{1}{6}\right)^2\left(\frac{1}{2}\right)^4 = \frac{5}{192}$$

(iv) P(scores will be such that a 6 occurs only on the last throw and exactly three of the first five throws result in odd numbers)
$$= \frac{1}{6}\times {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{3}\right)^2 = \frac{5}{216}$$

(4) (a) P(at least one female mathematics teacher)

$= 1 -$ P(no female mathematics teacher)

$= 1 -$ P(3 male mathematics teacher)

$= 1 - (0.6)^3 = 0.784$

(b) Let $p =$ Probability of any teacher being a female graduate

$= 0.4 \times 0.7 = 0.28$

$q = 0.72$

Using the binomial expansion of $(p+q)^3$

Probability of no female graduates $= {}^3C_0 p^0 q^3 = (0.72)^3 = 0.373$