NAME:



# Centre of Excellence in Mathematics S201 / 414 GARDENERS RD. ROSEBERY 2018 www.cemtuition.com.au



### YEAR 12 - EXT.1 MATHS

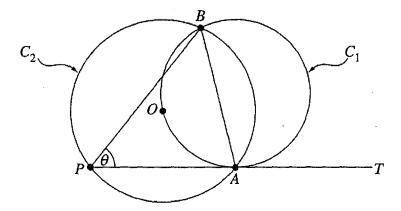
REVIEW TOPIC (SP3) CIRCLE GEOMETRY

1

### **PAST HSC EXAMINATION QUESTIONS:**

### **HSC 01**

(3) (b)



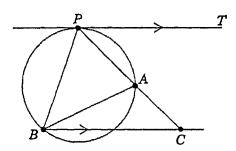
Two circles,  $C_1$  and  $C_2$ , intersect at points A and B. Circle  $C_1$  passes through the centre O of circle  $C_2$ . The point P lies on circle  $C_2$  so that the line PAT is tangent to circle  $C_1$  at point A. Let  $\angle APB = \theta$ .

Copy or trace the diagram into your writing booklet.

- (i) Find  $\angle AOB$  in terms of  $\theta$ . Give reasons for your answer. 1
- (ii) Explain why  $\angle TAB = 2\theta$ .
- (iii) Deduce that PA = BA.

## HSC 2000 (5) (a)

4



In the diagram, A, P and B are points on the circle. The line PT is tangent to the circle at P, and PA is produced to C so that BC is parallel to PT. Copy the diagram.

(i) Show that  $\angle PBA = \angle PCB$ .

Deduce that  $PB^2 = PA \times PC$ . (ii)

HSC '99 (3)(c) P

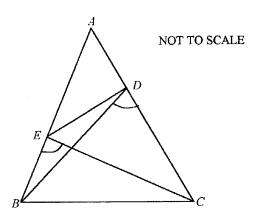
Two circles intersect at two points A and B as shown in the diagram. The diameter of one circle is CA and this line intersects the other circle at A and P. The line CB intersects the second circle at B and Q.

Copy or trace the diagram into your Writing Booklet.

Prove that  $\angle CPQ$  is a right angle.

HSC '98 (4) (c)

2



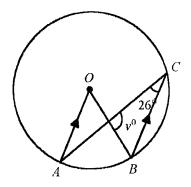
ABC is an acute-angled triangle. D is a point on AC, E is a point on AB, and  $\angle BEC = \angle BDC$ , as shown in the diagram.

Sonya was asked to prove that  $\angle AED = \angle ACB$ . She provided a two-step proof but did not give reasons.

(i) State a reason for her correct statement that *EDCB* is a cyclic quadrilateral.

(ii) State a reason why she could correctly conclude that  $\angle AED = \angle ACB$ .

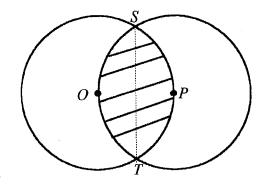
- (2) (a) The points A, B, and C lie on a circle with centre O. The lines AO and BC are parallel, and OB and AC intersect at D. Also,  $\angle ACB = 26^{\circ}$  and  $\angle BDC = y^{\circ}$ , as shown in the diagram. Copy and trace the diagram into your writing booklet.
  - (i) State why  $\angle AOB = 52^{\circ}$ .



(ii) Find y. Justify your answer.

(2) (c) The points O and P in the plane are d cm apart. A circle centre O is drawn to pass through P, and another circle centre P is drawn to pass through O. The two circles meet at S and T, as in the diagram.

5



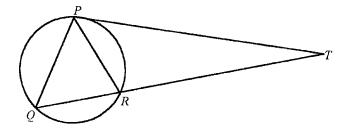
(i) Show that triangle SOP is equilateral.

(ii) Show that the size of angle SOT is  $120^{\circ}$ .

(iii) Hence find the area of the shaded region in terms of d.

$$d^2 \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{ units}^2$$

### HSC '95 (6) (a)



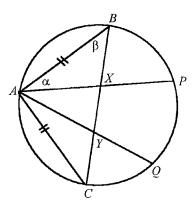
PT is a tangent to the circle PRQ, and QR is a secant intersecting the circle in Q and R. The line  $\overrightarrow{QR}$  intersects PT at T.

Copy or trace the diagram into your Writing Booklet.

Prove that the triangles *PRT* and *QPT* are similar.

#### Hence prove that $PT^2 = QT \times RT$ (ii)

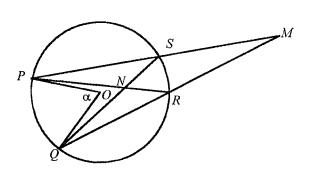
- (2) (b) Let ABPQC be a circle such that AB = AC, AP meets BC at X, and AQ meets BC at Y, as in the diagram. Let  $\angle BAP = \alpha$  and  $\angle ABC = \beta$ .
- (i) Copy the diagram into your Writing Booklet and state why  $\angle AXC = \alpha + \beta$ .



- (ii) Prove that  $\angle BQP = \alpha$ .
- (iii) Prove that  $\angle BQA = \beta$ .

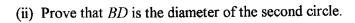
(iv) Prove that PQYX is a cyclic quadrilateral.

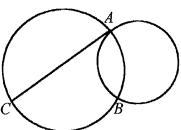
- (3) (c) In the diagram P, Q, R, and S are points on a circle centre O, and  $\angle POQ = \alpha$ . The lines PS and QR intersects at M and the lines QS and PR intersect at N.
- (i) Explain why  $\angle PRM = 180^{\circ} \frac{1}{2}\alpha$ .



(ii) Show that  $\angle PNQ + \angle PMQ = \alpha$ .

- (2) (c) The diagram shows two circles intersecting at A and B. The diameter of one circle is AC. Copy this diagram into your examination booklet.
- (i) On your diagram draw a straight line through A, parallel to CB, to meet the second circle in D.

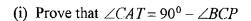


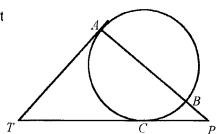


(iii) Suppose that BD is parallel to CA. Prove that the circles have equal radii.

(3) (a) AB is a diameter of a circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P.

Copy the diagram into your examination booklet. Join AC and CB.





(ii) Hence, or otherwise, prove that  $\angle ATC = 2 \angle BCP$ .

- (5) (a) AB and CD are intersecting chords of a circle and CD is parallel to the tangent to the circle at B.
  - (i) Draw a neat sketch of the above information in your writing booklet.
  - (ii) Prove that AB bisects  $\angle CAD$ .