C.E.M.TUITION

Name : ______________________________

Topic : Further Trigonometry

(Specimen Paper)

Year 12 - Extension 1 Maths

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(1) Given that $\sin A = \frac{8}{17}$ and $\cos B = \frac{3}{5}$ where both A and B are acute angles. Find the exact values of

(a) $\sin 2A$ [2]

(b) $\cos(A - B)$ [2]

(2) Express the term $\tan(x - 45°)$ in terms of $\tan x$.

Hence solve the equation

$$\tan(x - 45°) = 6\tan x,$$

giving all solutions between $0°$ and $360°$ [6]

(3) Prove the identity

$$\frac{\sin(\theta+60)-\sin(\theta+30)}{\cos(\theta+60)+\cos(\theta+30)}=\tan 15$$ [4]

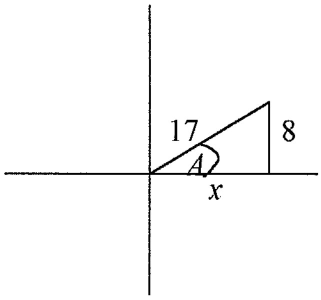
(4) Use the formulae for $\cos(A+B)$ and $\cos(A-B)$ to prove that

(a) $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

(b) $\sin^2 A = \frac{1}{2}(1-\cos 2A)$ [4]

Solutions:

(1)



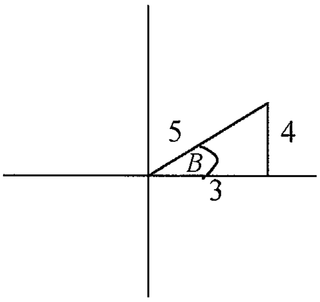
By Pythagoras' theorem

$$178^2 =+ \quad x$$

$$\Rightarrow \quad x^2 = 225$$

$$\Rightarrow \quad x = 15$$

$$\Rightarrow \quad \cos A = \frac{15}{17}$$



$$\sin B = \frac{4}{5}$$

(a)

$$\sin 2A = 2 \sin A \cos A$$

$$= 2 \times \frac{8}{17} \times \frac{15}{17}$$

$$\sin 2A = \frac{240}{289}$$

(b)

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{15}{17} \times \frac{3}{5} + \frac{8}{17} \times \frac{4}{5}$$

$$\cos (A - B) = \frac{77}{85}$$

(2)

Using

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(x - 45°) = \frac{\tan x - \tan 45°}{1 + \tan x \tan 45°}$$

$$\tan(x - 45°) = \frac{\tan x - 1}{1 + \tan x}$$

$$\tan(x - 45°) = 6 \tan x$$

$\Rightarrow$ $$\frac{\tan x - 1}{1 + \tan x} = 6 \tan x$$

$\times(1 + \tan x)$ $$\tan x - 1 = 6 \tan x(1 + \tan x)$$

$$\tan x - 1 = 6 \tan x + 6 \tan^2 x$$

$\Rightarrow$ $$6 \tan^2 x + 5 \tan x + 1 = 0$$

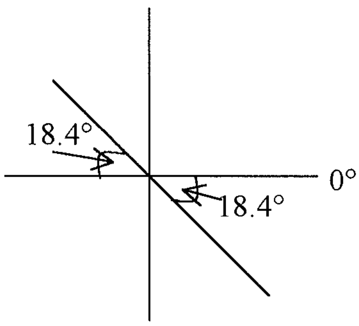
$\Rightarrow$ $$(3 \tan x + 1)(2 \tan x + 1) = 0$$

$\Rightarrow$ $$3 \tan x + 1 = 0 \quad \text{or} \quad 2 \tan x + 1 = 0$$

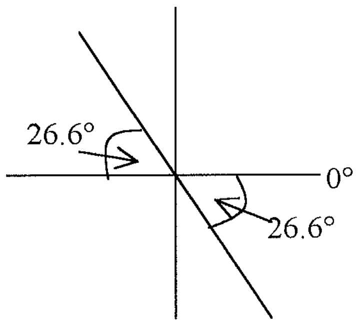
$$\tan x = -\frac{1}{3}, \qquad \tan x = -\frac{1}{2}$$

$$\tan^{-1}\frac{1}{3} = 18.4° \qquad \tan^{-1}\frac{1}{2} = 26.6°$$

tangent is negative in 2nd and 4th quadrants



$x = 161.1°, 341.6°$



$x = 153.4°, 333.4°$

(3) Using the factor formula for the difference of two sines and the sum of two cosines,

$$\frac{\sin(\theta+60)-\sin(\theta+30)}{\cos(\theta+60)+\cos(\theta+30)} =$$

$$\frac{\not{2}\sin\frac{1}{2}\left[(\theta+60)-(\theta+30)\right]\cos\frac{1}{2}\left[(\theta+60)+(\theta+30)\right]}{\not{2}\cos\frac{1}{2}\left[(\theta+60)+(\theta+30)\right]\cos\frac{1}{2}\left[(\theta+60)-(\theta+30)\right]}$$

$$= \frac{\sin\frac{1}{2}(30)\cos\frac{1}{2}(2\theta+90)}{\cos\frac{1}{2}(2\theta+90)\cos\frac{1}{2}(30)}$$

$$= \frac{\sin 15\,\cancel{\cos(\theta+45)}}{\cancel{\cos(\theta+45)}\,\cos 15}$$

$$= \frac{\sin 15}{\cos 15}$$

$$\Rightarrow \quad \frac{\sin(\theta+60)-\sin(\theta+30)}{\cos(\theta+60)+\cos(\theta+30)} = \tan 15$$

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(4) (a)

$$\cos(A-B) = \cos A\cos B + \sin A\sin B \quad -\{1\}$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B \quad -\{2\}$$

$\{1\} - \{2\}$

$$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$$

(b) Using solution from (a)

Let $B = A$

$$\Rightarrow \quad 2\sin A\sin A = \cos(A-A) - \cos(A+A)$$
$$2\sin^2 A = 1 - \cos 2A$$
$$\sin^2 A = \frac{1}{2}(1-\cos 2A)$$