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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP1)
HARDER 2U PROBLEMS

HSC '98

(1) (c) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{5x}$

$\boxed{\frac{3}{5}}$

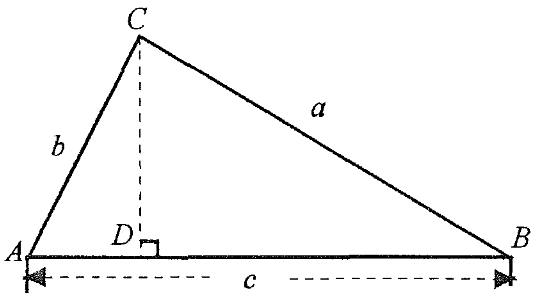
(d) Given that $\log_2 7 = 2.807$ (to three decimal places), find $\log_2 14$.

$\boxed{3.807}$

(2)(b) Paul plans to contribute to a retirement fund. He will invest \$500 on each birthday from age 25 to 64 inclusive. That is, he will make 40 contributions to the fund. The retirement fund pays interest on the investments at the rate of 8% per annum, compounded annually. How much money will be in Paul's fund on his 65th birthday? **4**

$\boxed{\$139\,890.52}$

(c) **5**



The triangle ABC has sides of length a, b and c, as shown in the diagram.
The point D lies on AB, and CD is perpendicular to AB.

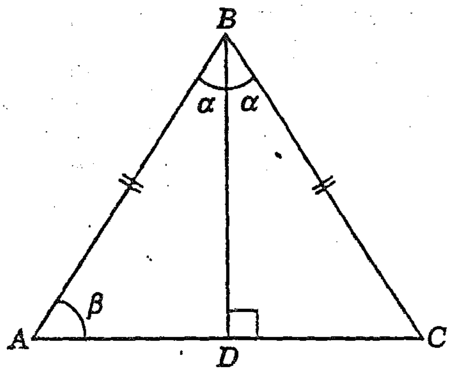
(i) Show that $a\sin B = b\sin A$

(ii) Show that $c = a\cos B + b\cos A$

(iii) If $c^2 = 4ab\cos A\cos B$, show that $a = b$.

HSC '97

(4) **(a)** 5



The triangle ABC is isosceles, with $AB = BC$, and BD is perpendicular to AC.
Let $\angle ABD = \angle CBD = \alpha$ and $\angle BAD = \beta$, as shown in the diagram.

(i) Show that $\sin\beta = \cos\alpha$.

(ii) By applying the sine rule in ΔABC, show that $\sin 2\alpha = 2\sin\alpha\cos\alpha$.

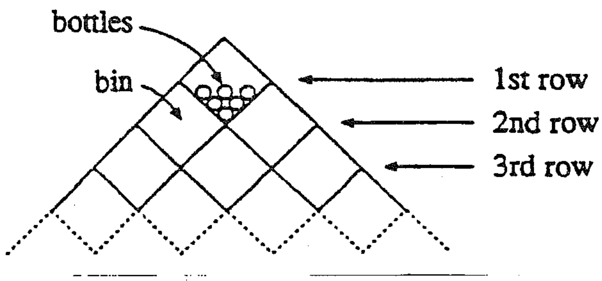
(iii) Given that $0 < \alpha < \dfrac{\pi}{2}$, show that the limiting sum of geometric series

$$\sin 2\alpha + \sin 2\alpha \cos^2 \alpha + \sin 2\alpha \cos^4 \alpha + \sin 2\alpha \cos^6 + \ldots$$

is equal to $2\cot\alpha$.

HSC '94

(6) **(b)**



The figure shows a bottle-storage rack. It consists of n rows of 'bins' stacked in such a way that the number of bins in the *r*th row is *r*, counting from the top.

(i) Show that the total number of bins in the storage rack is $\dfrac{n(n+1)}{2}$.

(ii) Each bin in the rth row contains $c + r$ bottles, where c is a constant.
(For example, each bin in the third row contains c+3 bottles.)

Find an expression for the total number of bottles in the storage rack.
[You may assume that $1^2 + 2^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$.]

$\boxed{\frac{1}{6}n(n+1)\{3c+2n+1\}}$

(iii) Enzo notices that $c = 5$ and that the average number of bottles per bin in the storage rack is 10.
Calculate the number of rows in the storage rack.

7 rows

HSC '93

(b) (i) Prove that the graph of $y = \log_e x$ is concave down for all $x > 0$.

(ii) Sketch the graph of $y = \log_e x$.

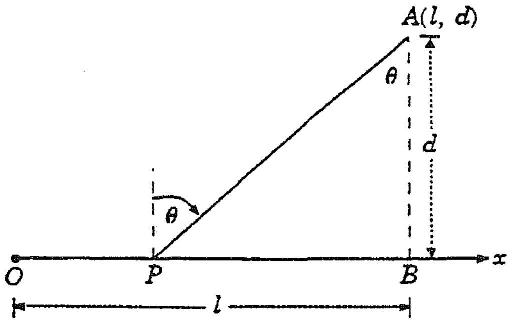
(iii) Suppose $0 < a < b$ and consider the points $A(a, \ln a)$ and $B(b, \ln b)$ on the graph of $y = \log_e x$. Find the coordinates of the point P that divides the line segment AB in the ratio 2 :1.

$x = \dfrac{a+2b}{3}$; $y = \dfrac{\ln a + 2\ln b}{3}$

(iv) By using (ii) and (iii) deduce that

$$\frac{1}{3}\ln a+\frac{2}{3}\ln b<\ln\left(\frac{1}{3}a+\frac{2}{3}b\right).$$

(b)



In the diagram, the x axis represents a major blood vessel, whilst the line PA represents a minor blood vessel that joins the major blood vessel at P. The point A has coordinates (l, d) and PA makes an angle $\theta < \frac{\pi}{2}$ with the normal to the x axis at P, as shown in the diagram.

It is known that the resistance to flow in a blood vessel is proportional to its length, where the constant of proportionality depends upon the particular blood vessel.

Let R be the sum of the resistances to flow in OP and PA.

(i) Show that $R = c_1(l - d\tan\theta) + c_2 d\sec\theta$, where c_1 and c_2 are constants.

(ii) The blood vessel PA is joined to the blood vessel Ox in such a way that R is minimized.

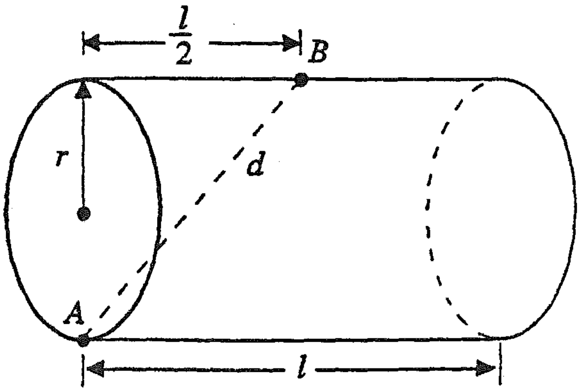
If $\frac{c_2}{c_1} = 2$, find the angle θ that minimizes R. (You may assume that l is large compared to d.)

$$\boxed{\theta = \frac{\pi}{6}}$$

HSC '92

(4)

(c)



The diagram shows a cylindrical barrel of length l and radius r. The point A is at one end of the barrel, at the very bottom of the rim. The point B is at the very top of the barrel, half-way along its length.
The length of AB is d.

(i) Show that the volume of the barrel is $V = \frac{\pi l}{4}\left(d^2 - \frac{l^2}{4}\right)$.

(ii) Find l in terms of d if the barrel has maximum volume for the given d.

$$\boxed{l = \frac{2d}{\sqrt{3}}}$$