NAME :

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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC : INTEGRATION BY SUBSTITUTION – SP1

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EXERCISES:

(1)

Use the substitution $x = 1 - 2u$ to evaluate

$$\int_0^{\frac{1}{2}} 2u(1-2u)^4 \, du.$$

(2) (a)

Evaluate $\int_0^{\frac{\pi}{2}} 2\sin x\cos x \, dx$.

(b)

The region under the graph $y = 2\sin x$, above the x axis, and between $x = 0$ and $x = \frac{\pi}{2}$, revolves about the x axis to form a solid. Find its volume.

(3)

Use the substitution $u = 1 - x$ to prove that, for $n > 0$,

$$\int_0^1 x(1-x)^n \, dx = \frac{1}{(n+1)(n+2)}.$$

(4)

Use the substitution $u = x^2+1$ to evaluate

$$\int_1^7 \frac{x}{\sqrt{x^2+1}}\,dx,$$

giving your answer in simplest surd form.

(5)

(i) Show that $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx = \frac{\pi+2}{8}$.

(ii) The region under the curve $y = \cos x + \sec x$, above the x axis and between $x = 0$ and $x = \frac{\pi}{4}$, makes a revolution about the x axis.

Show that the volume of the solid traced out is $\frac{5\pi(\pi+2)}{8}$ cubic units.

SOLUTIONS:

(1)

$$x = 1-2u, \qquad u = 0,\quad x = 1$$
$$\frac{dx}{du} = -2, \qquad u = \frac{1}{2},\quad x = 0$$
$$\frac{du}{dx} = -\frac{1}{2}$$

$$\begin{aligned}\int_0^{\frac{1}{2}} 2u(1-2u)^4\,du &= \int_1^0 (1-x)x^4\left(-\frac{1}{2}\right)dx \\ &= \frac{1}{2}\int_0^1 \left(x^4 - x^5\right)dx \\ &= \frac{1}{2}\left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1 \\ &= \frac{1}{2}\left(\frac{1}{5} - \frac{1}{6}\right) \\ &= \frac{1}{60}\end{aligned}$$

(2) (a)

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin 2x\,dx &= -\frac{1}{2}\Big[\cos 2x\Big]_0^{\frac{\pi}{2}} \\ &= -\frac{1}{2}(-1-1) \\ &= 1\end{aligned}$$

(b)

$$\begin{aligned}V &= \pi\int_0^{\frac{\pi}{2}} 4\sin^2 x\,dx \\ &= 2\pi\int_0^{\frac{\pi}{2}} (1-\cos 2x)\,dx \\ &= 2\pi\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} \\ &= 2\pi\left(\frac{\pi}{2} - 0\right) \\ &= \pi^2 \text{ cubic units}\end{aligned}$$

(3)

$$u = 1-x \qquad x = 0,\quad u = 1$$
$$\frac{du}{dx} = -1 \qquad x = 1,\quad u = 0$$
$$\frac{dx}{du} = -1$$

$$\begin{aligned}\int_0^1 x(1-x)^n\,dx &= \int_1^0 (1-u)\,u^n(-1)\,du \\ &= \int_0^1 \left(u^n - u^{n+1}\right)du\end{aligned}$$

$$= \left[\frac{u^{n+1}}{n+1} - \frac{u^{n+2}}{n+2}\right]_0^1$$
$$= \frac{1}{n+1} - \frac{1}{n+2}$$
$$= \frac{1}{(n+1)(n+2)}$$

(4)

$u = x^2+1, \quad x=1, \quad u=2$

$\frac{du}{dx} = 2x, \quad x=7, \quad u=50$

$$\int_1^7 \frac{x}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\int_2^{50} u^{-\frac{1}{2}}\,du$$
$$= \left[u^{\frac{1}{2}}\right]_2^{50}$$
$$= \sqrt{50} - \sqrt{2}$$
$$= 5\sqrt{2} - \sqrt{2}$$
$$= 4\sqrt{2}$$

(5)

(i) $$\int_0^{\frac{\pi}{4}} \cos^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1+\cos 2x)\,dx$$
$$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$$
$$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2} - 0\right)$$
$$= \frac{\pi+2}{8}$$

(ii) $$V = \pi\int_0^{\frac{\pi}{4}} (\cos x + \sec x)^2\,dx$$
$$= \pi\int_0^{\frac{\pi}{4}} \left(\cos^2 x + 2 + \sec^2 x\right)\,dx$$

Now $\int_0^{\frac{\pi}{4}} \cos^2 x\,dx = \frac{\pi+2}{8}$, from part (i),

and $\int_0^{\frac{\pi}{4}} \left(2+\sec^2 x\right)\,dx = \left[2x + \tan x\right]_0^{\frac{\pi}{4}}$
$$= \frac{\pi}{2} + 1$$

$$V = \pi\left(\frac{\pi+2}{8} + \frac{\pi}{2} + 1\right)$$
$$= \pi\left(\frac{5\pi+10}{8}\right)$$
$$= \frac{5\pi(\pi+2)}{8} \text{ cubic units.}$$