C.E.M.TUITION

Student Name :_____

Review : Integration

(HSC - PAPER 1)

Year 12 - 3 Unit

Integrate the following using the substitution provided. (If the substitution is not provided, use any suitable method.) Leave your answer in the exact terms.

1. $\int_0^{\frac{1}{2}} x\sqrt{1-x^2}\,dx,\ u=1-x^2$

2. $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^3}}\,dx, \quad u = 1-x^3$

3. Show that $\frac{2}{x^2+2x} = \frac{1}{x} - \frac{1}{x+2}$ and hence evaluate

$$\int_1^4 \frac{2}{x^2+2x}\,dx$$

4. $\int_1^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x}\,dx,\ u=\sin x$

5. $\int_0^{\log_e 2} \frac{e^x}{4+e^{2x}}\,dx,\ u=e^x$

1. $I = \int_0^{\frac{1}{2}} x\sqrt{1-x^2}\,dx$

$u = 1-x^2, \frac{du}{dx} = -2x,$

$x\,dx = -\frac{du}{2}$

$x = 0, \frac{1}{2} \Rightarrow u = 1, \frac{3}{4}$

$I = -\frac{1}{2}\int_1^{\frac{3}{4}} \sqrt{u}\,du = -\frac{1}{3}\left[u^{\frac{3}{2}}\right]_1^{\frac{3}{4}}$

$I = \frac{8-3\sqrt{3}}{24}$

2. $I = \int \frac{x^2\,dx}{\sqrt{1-x^2}}, \quad u = 1-x^3$

$\frac{dx}{du} = -3x^2, \quad x^2\,dx = -\frac{du}{3}$

$I = -\frac{1}{3}\int \frac{du}{\sqrt{u}} = -\frac{2}{3}\sqrt{u} + c$

$I = -\frac{2}{3}\sqrt{1-x^3} + c$

3. $\frac{1}{x} - \frac{1}{x+2} = \frac{x+2-x}{x(x+2)}$

$= \frac{2}{x(x+2)}$

$\therefore \int_1^4 \frac{2\,dx}{x(x+2)}$

$= \left[\log_e x - \log_e(x+2)\right]_1^4$

$= \log_e 2$

4. $u = \sin x, \quad \frac{du}{dx} = \cos x$

$x = 0, \frac{\pi}{2} \Rightarrow u = 0, 1$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos x\,dx}{1+\sin x} = \int_0^1 \frac{du}{1+u}$

$I = \left[\log_e(1+u)\right]_0^1 = \log_e 2$

5. $u = e^x, \quad \frac{du}{dx} = e^x, \quad e^x\,dx = du$

$x = 0, \log_e 2 \Rightarrow u = 1, 2$

$I = \int_0^{\log 2} \frac{e^x}{4+e^{2x}}\,dx = \int_1^2 \frac{du}{4+u^2}$

$I = \frac{1}{2}\left[\tan^{-1}\frac{u}{2}\right]_1^2$

$= \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\frac{1}{2}$