

C.E.M. TUITION

Student Name : _____

Review : Integration

(HSC - PAPER 2)

Year 12 - 3 Unit

Integrate the following using the substitution provided. (If the substitution is not provided, use any suitable method.)
Leave your answer in the exact terms.

6. $\int_0^b \frac{1}{\sqrt{4-9x^2}} dx$, $b = \frac{1}{\sqrt{3}}$ $x = \frac{2}{3} \sin \theta$

7. $\int_0^3 \frac{x}{\sqrt{4-x}} dx, u = \sqrt{4-x}$

8. $\int_0^2 \frac{2 dx}{4+x^2}, x = 2 \tan \theta$

9. $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$, (Hint: $\sec^2 x = 1 + \tan^2 x$)

10. $\int_1^3 \frac{dx}{\sqrt{x+x\sqrt{x}}}, u = \sqrt{x}$

$$6. \quad 4 - 9x^2 = 9\left(\frac{4}{9} - x^2\right)$$

$$I = \int_0^b \frac{dx}{\sqrt{4 - 9x^2}}$$

$$= \frac{1}{3} \int_0^b \frac{dx}{\sqrt{\frac{4}{9} - x^2}}$$

$$I = \frac{1}{3} \left[\sin^{-1} \frac{3x}{2} \right]_0^b, \quad b = \frac{1}{\sqrt{3}}$$

$$= \frac{1}{3} \frac{\pi}{3} \text{ or } \frac{\pi}{9}$$

$$7. \quad u = 4 - x, \quad x = 4 - u, \quad \frac{du}{dx} = -1$$

$$x = 0, 3 \Rightarrow u = 4, 1$$

$$I = \int_0^3 \frac{x \, dx}{\sqrt{4 - x}} = \int_4^1 \frac{-(4 - u) \, du}{\sqrt{u}}$$

$$I = \int_1^4 (4u^{-\frac{1}{2}} - u^{\frac{1}{2}}) \, du$$

$$I = \left[8\sqrt{u} - \frac{2}{3} u\sqrt{u} \right]_1^4 = \frac{10}{3}$$

$$8. \quad x = 2 \tan \theta, \quad \frac{dx}{d\theta} = 2 \sec^2 \theta$$

$$x = 0, 2 \Rightarrow \theta = 0, \frac{\pi}{4}$$

$$\int_0^2 \frac{2 \, dx}{4 + x^2} = \int_0^{\frac{\pi}{4}} \frac{4 \sec^2 \theta \, d\theta}{4(1 + \tan^2 \theta)}$$

$$= \int_0^{\frac{\pi}{4}} d\theta = \frac{\pi}{4}$$

$$9. \quad \tan^2 x = \sec^2 x - 1$$

$$\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \left[\tan x - x \right]_0^{\frac{\pi}{4}}$$

$$= 1 - \frac{\pi}{4}$$

$$10. \quad u = \sqrt{x}, \quad x = u^2, \quad \frac{dx}{du} = 2u$$

$$x = 1, 3 \Rightarrow u = 1, \sqrt{3}$$

$$I = \int_1^{\sqrt{3}} \frac{dx}{\sqrt{x(1+x)}}$$

$$= \int_1^{\sqrt{3}} \frac{2 \, du}{1 + u^2}$$

$$= 2 \left[\tan^{-1} u \right]_1^{\sqrt{3}}$$

$$= \frac{\pi}{6}$$