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YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP1)
INVERSE TRIG FUNCTIONS

HSC 99

(1) (a) Evaluate $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$ **2**

$\boxed{\dfrac{\pi}{3}}$

HSC 98

(7) (b) (i) Use the substitution $y=\sqrt{x}$ to find **7**

$$\int \frac{dx}{\sqrt{x(1-x)}}$$

$\boxed{2\sin^{-1}\sqrt{x}+c}$

(ii) Use the substitution $z = x - \frac{1}{2}$ to find another expression for

$$\int \frac{dx}{\sqrt{x(1-x)}}.$$

$$\boxed{\sin^{-1}(2x-1)+c}$$

(iii) Use the results of parts (i) and (ii) to express $\sin^{-1}(2x-1)$ in terms of $\sin^{-1}\left(\sqrt{x}\right)$ for $0 < x < 1$.

$$\boxed{\sin^{-1}(2x-1) = 2\sin^{-1}\sqrt{x} - \frac{\pi}{2}}$$

HSC 97

(6) (a) The function $f(x) = \sec x$ for $0 \le x < \dfrac{\pi}{2}$, is not defined for other values of x. **4**

(i) State the domain of the inverse function $f^{-1}(x)$.

$$\boxed{D:\{x : x \ge 1\}}$$

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right)$.

(iii) Hence find $\dfrac{d}{dx}\left[f^{-1}(x)\right]$.

$$\boxed{\frac{1}{x\sqrt{x^2-1}}}$$

HSC 96

(1) (f) Using the substitution $u = e^x$, find $\int \frac{e^x}{1+e^{2x}}\,dx$ **3**

$\boxed{\tan^{-1}\left(e^x\right)+c}$

(3) (d) The function $h(x)$ is given by

$$h(x) = \sin^{-1} x + \cos^{-1} x, \quad \text{for } 0 \le x \le 1.$$

(i) Find $h'(x)$.

(ii) Sketch the graph of $y = h(x)$.

HSC 93

(3) (a) Consider the function $f(x) = 2\cos^{-1}\left(\frac{x}{3}\right)$.

(i) Evaluate $f(0)$.

$\boxed{\pi}$

(ii) Draw the graph of $y = f(x)$.

(iii) State the domain and range of $y = f(x)$.

$D:\{x:-3 \le x \le 3\};\ R:\{y:0 \le y \le 2\pi\}$

HSC 92

(3) (b) Consider the function $f(x) = 2\tan^{-1} x$.

(i) Evaluate $f(\sqrt{3})$.

$\frac{2\pi}{3}$

(ii) Draw the graph of $y = f(x)$, labelling any key features.

(iii) Find the slope of the curve at the point where it cuts the y-axis.

$y' = 2$

HSC 91

(5) (a) Consider the function $f(x) = 3\sin^{-1}\left(\frac{x}{2}\right)$.

(i) Evaluate $f(2)$.

$\frac{3\pi}{2}$

(ii) Draw the graph of $y = f(x)$.

(iii) State the domain and range of $y = f(x)$.

$$\boxed{D:\{x:-2 \le x \le 2\};\ R:\left\{y:-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}\right\}}$$

HSC 90

(1) (a) (i) Find $\int_0^1 \frac{dx}{1+x^2}$.

$$\boxed{\frac{\pi}{4}}$$

(4) (c) (i) State the domain and range of the function given by

$y = \cos^{-1} 2x$.

$$\boxed{D:\left\{x:-\frac{1}{2} \le x \le \frac{1}{2}\right\};\ R:\{y:0 \le y \le \pi\}}$$

(ii) Sketch the graph of the function $y = \cos^{-1} 2x$.

(iii) Find the slope of the curve at the point where it cuts the y-axis.

$\boxed{-2}$

Miscellaneous Questions:

(1)

A function and its inverse is said to be mutually inverse functions if

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

(i) Show that the function $y = \dfrac{1}{x+1}$ and its inverse **2**

are mutually inverse functions.

(ii) Sketch the above function and its inverse on the same axes. **3**

(2)

Consider the function $f(x) = 2\cos^{-1}x$

(i) Find the exact value of $f\left(\dfrac{1}{\sqrt{2}}\right)$. **1**

(ii) Sketch the graph of $y = f(x)$ showing its domain and range. **2**

(iii) Find the equation of the normal to the curve at the point where **2**

$x = \frac{1}{\sqrt{2}}$.

(3) (a)

Find $\int \frac{4}{4+9x^2}\,dx$ **2**

(b)

Find $\frac{d}{dx}\left(x\sin^{-1}x\right)$. Hence or otherwise, evaluate $\int_0^1 \sin^{-1}x\,dx$. **4**