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YEAR 12 – EXT. 1 MATHS

REVIEW TOPIC (SP1)

INVERSE FUNCTIONS & INVERSE TRIGONOMETRIC FUNCTIONS

CEM – Yr 12 – 3U Inverse Functions, Inverse Trigonometric Functions – Review Paper 1

1. Sketch the graph of $y = 2\sin^{-1} 3x$ showing clearly the domain and range of the function as well as any intercepts.

2. Find $\frac{d}{dx}\left(3x^2 \cos^{-1} x\right)$

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3. Evaluate $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)+\cos^{-1}\left(\cos\frac{3\pi}{4}\right)$.

4. (i) Find the domain and range of the function $y=3\sin^{-1}\left(\frac{x}{2}\right)$.

(ii) Draw a neat sketch of $y=3\sin^{-1}\left(\frac{x}{2}\right)$.

5. (i) Differentiate $x\cos^{-1}x-\sqrt{1-x^2}$.

(ii) Hence $\int_0^1 \cos^{-1}x+1\,dx$.

6. Evaluate $tan^{-1}(-\sqrt{3})$

7. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

8. Given $y = a\cos^{-1}(bx)$ is a function. (where $a, b > 0$)

   i) State the domain and range

   ii) Sketch this function

9. Differentiate, $e^{(\cos^{-1}x)}$

10. Find the area enclosed between the curve $y = \cos^{-1}x$, the y-axis and the lines $y = 0$ and $y = \frac{\pi}{4}$

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Answers

1.) $y = 2\sin^{-1}3x$

$-1 \le 3x \le 1$

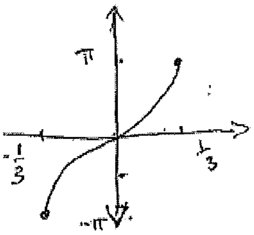
$-\frac{1}{3} \le x \le \frac{1}{3}$

$y = \sin^{-1}3x$ Range of

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\therefore$ $y = 2\sin^{-1}3x$ has a range of

$-\pi \le y \le \pi$



2. $\frac{d}{dx}\left(3x^2\cos^{-1}x\right)$

$= 6x\cos^{-1}x + 3x^2 \times \frac{-1}{\sqrt{1-x^2}}$ (1)

$= 6x\cos^{-1}x - \frac{3x^2}{\sqrt{1-x^2}}$ (1)

3. $\sin^{-1}\left(\cos\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right)$

$= \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$= -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \left[\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

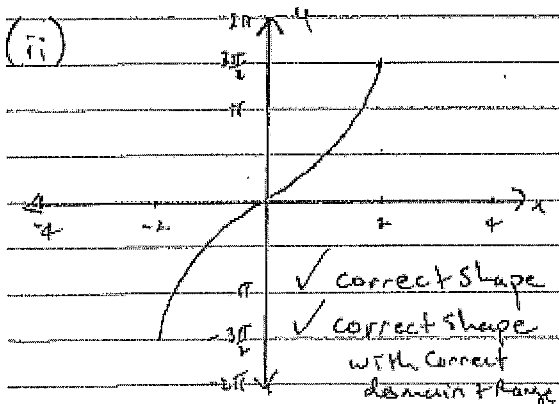
$= -\frac{\pi}{4} + \left[\pi - \frac{\pi}{4}\right]$

$= \frac{\pi}{2}$ ✓

4. (i) Domain $= \{x : -2 \le x \le 2\}$ ✓

Range $= \left\{y : -\frac{3\pi}{2} \le y \le \frac{3\pi}{2}\right\}$ ✓

(ii)



✓ correct shape

✓ correct shape with correct domain + range

2 marks

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5. $\frac{d}{dx}\left(x\cos^{-1}x - \sqrt{1-x^2}\right)$ ✓

$= \left(\cos^{-1}x\right)\times 1 + x \times \left(\frac{-1}{\sqrt{1-x^2}}\right) - \left[\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}} \times -2x\right]$

$= \cos^{-1}x - \frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$

$= \cos^{-1}x$ ✓

(ii)

$\int_0^1 \cos^{-1}x + 1 \, dx = \left[x\cos^{-1}x - \sqrt{1-x^2} + x\right]_0^1$ ✓

$= \left[(0 - 0 + 1) - (0 - 1 + 0)\right]$

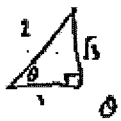
$= 2$ ✓

6. $\tan^{-1}(-\sqrt{3})$

$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

Note: Domain of $\tan^{-1}x$;

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

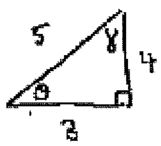


$\theta = 60$

$\theta = \frac{\pi}{3}$

7. $\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) =$

$\theta + \gamma =$



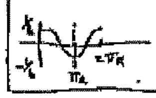
$90° = \gamma + \theta = \frac{\pi}{2}$

∴ By inspection

$\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

8. d) i) $y = a\cos^{-1}(bx)$ $(a, b > 0)$

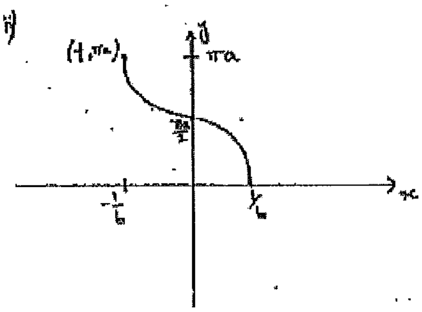
$x = \frac{1}{b}\cos\left(\frac{y}{a}\right)$



D: $-\frac{1}{b} \le x \le \frac{1}{b}$

R: $0 \le y \le \pi a$

ii)



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9. a) $y = e^{\cos^{-1}(x)}$

$y = e^{u}$ let $u = \cos^{-1}(x)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{du} = e^{u}$

$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = e^{u} \times \frac{-1}{\sqrt{1-x^2}}$

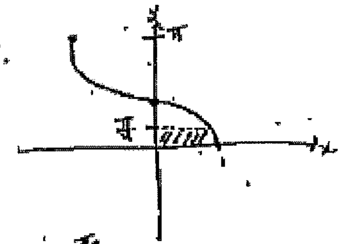
$= \frac{-e^{\cos^{-1}(x)}}{\sqrt{1-x^2}}$

10. $y = \cos^{-1}(x)$

$y = 0$

$y = \frac{\pi}{4}$

$x = 0$



$\text{Area} = \int_0^{\frac{\pi}{4}} \cos(x)\, dx$

$= [\sin(x)]_0^{\frac{\pi}{4}}$

$= \sin(\frac{\pi}{4}) - \sin(0)$

$= \frac{1}{\sqrt{2}} - 0$

$\text{Area} = \frac{1}{\sqrt{2}}$ sq units.

$= \frac{\sqrt{2}}{2}$ sq units