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YEAR 12 – EXT. 1 MATHS

REVIEW TOPIC (SP2)

INVERSE FUNCTIONS & INVERSE TRIGONOMETRIC FUNCTIONS

CEM – Yr 12 – 3U Inverse Functions, Inverse Trigonometric Functions – Review Paper 2

1. Differentiate the following with respect to x:

   i. $y = \cos^{-1}\left(\frac{1}{x} - 1\right)$

   ii. $y = \tan^{-1}\sqrt{x^2 - 1}$

2. Evaluate, in terms of π,

   i. $\cos^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)$

   ii. $2\tan^{-1}(1) + \tan^{-1}\left(-\sqrt{3}\right)$

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3. Find the exact values of x and y which satisfy the simultaneous equations

$$\sin^{-1} x + \frac{1}{2}\cos^{-1} y = \frac{\pi}{3} \quad \text{and}$$

$$3\sin^{-1} x - \frac{1}{2}\cos^{-1} y = \frac{2\pi}{3}.$$

4. If $y = \tan^{-1}(x^2)$, find $\dfrac{d^2y}{dx^2}$

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5. Find the exact value of:

$$\tan\left\{\sin^{-1}\left(\frac{5}{13}\right)-\cos^{-1}\left(\frac{3}{5}\right)\right\}$$

6. (i) Sketch the graph: $y=3\cos^{-1}\left(\frac{x}{2}\right)$

(ii) State its domain and range

7. Consider the function $f(x) = 2\cos^{-1}\frac{x}{3}$. Draw its graph $y = f(x)$ and state its domain and range.

8. Find $\tan[\sin^{-1}(-\frac{2}{3})]$

9. Find the equation of the tangent to $y = \sin^{-1}(x-1)$ at the point $(\frac{3}{2}, \frac{\pi}{6})$.

10. Differentiate with respect to x :

(i) $\sin^{-1}(3x)$

(ii) $x^2 \tan^{-1} x$

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Answers

1. (i) $y = \cos^{-1}\left(\frac{1}{x} - 1\right)$

let $u = \frac{1}{x} - 1$ $\therefore y = \cos^{-1} u$

$u = x^{-1} - 1$

$\frac{du}{dx} = -x^{-2} = -\frac{1}{x^2}$ ✓ $\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{-1}{\sqrt{1-u^2}} \times -\frac{1}{x^2}$

$= \frac{1}{x^2\sqrt{1-\left(\frac{1}{x}-1\right)^2}}$

$= \frac{1}{x^2\sqrt{1-\left(\frac{1}{x^2}-\frac{2}{x}+1\right)}}$

$= \frac{1}{x^2\sqrt{\frac{2}{x}-\frac{1}{x^2}}}$

$= \frac{1}{x^2\sqrt{\frac{2x-1}{x^2}}}$

$= \frac{1}{x^2\frac{\sqrt{2x-1}}{\sqrt{x^2}}}$

$= \frac{1}{x^2 \cdot \frac{\sqrt{2x-1}}{x}}$

$= \frac{1}{x\cdot\sqrt{2x-1}}$ ✓

(ii) $y = \tan^{-1}\sqrt{x^2-1}$

let $u = \sqrt{x^2-1}$ $y = \tan^{-1} u$

$\frac{du}{dx} = \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \times 2x = \frac{x}{\sqrt{x^2-1}}$ ✓ $\frac{dy}{du} = \frac{1}{1+u^2}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{x}{\sqrt{x^2-1}}$

$= \frac{x}{x^2\cdot\sqrt{x^2-1}}$

$\frac{1}{x\cdot\sqrt{x^2-1}}$ ✓

2. (a) (i) $\cos^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ ✓

note: $\cos^{-1}(-x) = \pi - \cos^{-1}x$
$\sin^{-1}(-x) = -\sin^{-1}x$

$= \pi - \frac{\pi}{3} + \frac{\pi}{6}$ ✓

$= \frac{6\pi - 2\pi + \pi}{6}$

$= \frac{5\pi}{6}$ ✓

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(ii) $2\tan^{-1}(1) + \tan^{-1}(-\sqrt{3})$

$= 2\tan^{-1}(1) - \tan^{-1}(\sqrt{3})$ ✓

$= 2\cdot\frac{\pi}{4} - \frac{\pi}{3}$ ✓ note: $\tan^{-1}(-x) = -\tan^{-1}x$

$= \frac{\pi}{2} - \frac{\pi}{3}$ ✓

$= \frac{\pi}{6}$ ✓

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3.

$\sin^{-1}x + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$ — ①

$3\sin^{-1}x - \frac{1}{2}\cos^{-1}y = \frac{2\pi}{3}$ — ②

① + ②

$4\sin^{-1}x = \frac{\pi}{3} + \frac{2\pi}{3}$ ✓

$4\sin^{-1}x = \pi$

$\sin^{-1}x = \frac{\pi}{4}$ ✓

$x = \frac{1}{\sqrt{2}}$ ✓

sub x in ① to solve for y: $\frac{\pi}{4} + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$

$\frac{1}{2}\cos^{-1}y = \frac{\pi}{3} - \frac{\pi}{4}$ ✓

$\cos^{-1}y = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\cos^{-1}y = \frac{\pi}{6}$ ✓

$\therefore y = \frac{\sqrt{3}}{2}$ ✓

4.) $y = \tan^{-1}(x^2)$

$\frac{dy}{dx} = \frac{2x}{1+x^4}$

$\frac{d^2y}{dx^2} = \frac{(1+x^4)(2) - 2x(4x^3)}{(1+x^4)^2}$

$= \frac{2(1+x^4-4x^4)}{(1+x^4)^2}$

$= \frac{2(1-3x^4)}{(1+x^4)^2}$

5. (b) $\tan\left\{\sin^{-1}\left(\frac{5}{13}\right) - \cos^{-1}\left(\frac{3}{5}\right)\right\}$

Let $\theta = \sin^{-1}(5/13)$ & $\alpha = \cos^{-1}(3/5)$

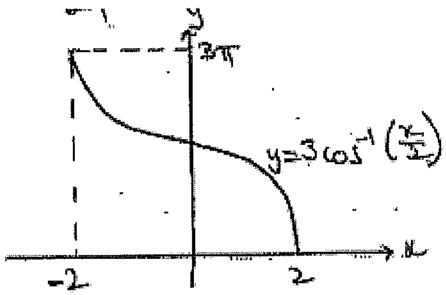
$\tan(\theta - \alpha) = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha}$

$= \frac{5/12 - 4/3}{1 + (5/12)(4/3)}$

$= \frac{-11/12}{14/9}$

$= \frac{-33}{56}$

6. (i)



(ii) Domain: $-2 \le x \le 2$

Range: $0 \le y \le 3\pi$

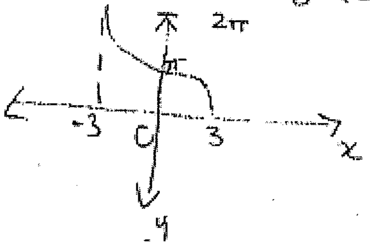
7. $f(x) = 2\cos^{-1}\left(\frac{x}{3}\right)$

Domain: $-1 \le \frac{x}{3} \le 1$

$-3 \le x \le 3$

Range: $0 \le \cos^{-1} x \le \pi$

$0 \le 2\cos^{-1}\frac{x}{3} \le 2\pi$



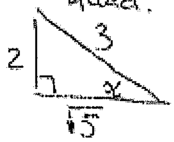
8. $\tan\left[\sin^{-1}\left(-\frac{2}{3}\right)\right]$

Let $x = \sin^{-1}(-2/3)$

$\sin x = -2/3$ where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\Rightarrow$ 4th quad.

$\therefore \tan x = -\frac{2}{\sqrt{5}}$



9. $y = \sin^{-1}(x-1)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x-1)^2}} \times \frac{d}{dx}(x-1)$

$= \frac{1}{\sqrt{2x - x^2}}$

At $x = 3/2$ $\quad m = \frac{1}{\sqrt{3 - 9/4}}$

$= \frac{1}{\sqrt{3/4}}$

$= 2/\sqrt{3}$

$\therefore$ Tangent:

$y - \frac{\pi}{6} = \frac{2}{\sqrt{3}}\left(x - \frac{3}{2}\right)$

$6\sqrt{3}y - \sqrt{3}\pi = 12x - 6$

$0 = 12x - 6\sqrt{3}y + \sqrt{3}\pi - 6$

$n\ 3/\sqrt{2}$

10. (b) (i) $\frac{d\left(\sin^{-1}(3x)\right)}{dx} = \frac{1}{\sqrt{1-(3x)^2}} \times 3$

$= \frac{3}{\sqrt{1-9x^2}}$

(b) (ii) $\frac{d\left(x^2\tan^{-1}x\right)}{dx} = x^2 \times \frac{1}{1+x^2} + 2x \times \tan^{-1}x$

$= \frac{x^2}{1+x^2} + 2x\tan^{-1}x$