NAME :

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YEAR 12 – MATHS EXT.1

REVIEW TOPIC (PAPER 2): PARAMETRIC EQUATIONS

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CARINGBAH 2004 Q2

d) A parabola is defined by the parametric equations

$$x = 12t$$
$$y = 6t^2 + 3$$

i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. 1

ii) If Q is the point where $t = -1$, find

α) the coordinates of Q. 1

β) the slope of the tangent to the curve at Q. 1

γ) the equation of the tangent at Q. 1

CARINGBAH 2004 Q6

b) The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$.

[You are given that the equation of the chord PQ is $2y = (p+q)x - 2apq$]

i) If the chord PQ passes through $(2a, 0)$ show that $pq = p + q$. 1

ii) Hence, find the locus of M, the midpoint of PQ. 4

CRANBROOK 2004 Q1

(c) P $(2t, t^2)$ is a point on the parabola $x^2 = 4y$ with focus S. The point M divides the interval SP externally in the ratio 3:1.

(i) Find the coordinates of M

(ii) Show that the locus of M is $x^2 = 6y + 3$
Hence find the coordinates of the focus and the equation of the directrix of M.

JAMES RUSE 2002 Q5

(b) A chord joining the points $P(2p, p^2)$ and $Q(2q, q^2)$ on the parabola $x^2 = 4y$ passes through the point (0,-1)

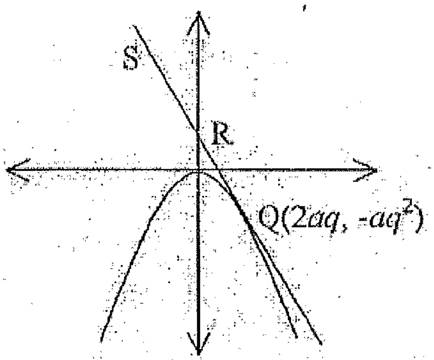
(i) Find the coordinates of M, the midpoint of PQ, as a function of m, the gradient of the chord

(ii) Show that the cartesian equation of the locus of M is

$$x^2 = 2(y+1) \text{ for } |x| \geq 2.$$

MANLY 2002 Q4

b) The point $Q(2aq, -aq^2)$ is a variable point on the parabola $x^2 = -4ay$. The tangent at Q meets the y axis at R. The point S lies on the tangent and divides QR externally in the ratio 3:1.



i) Show that the equation of the tangent at Q is $qx + y = aq^2$ (2)

ii) Find the coordinates of the points R. (1)

iii) Show the coordinates of S are $(-aq, 2aq^2)$ (2)

iv) Show that the locus of S is a parabola. (1)

SOLUTIONS

CARINGBAH 2004 Q2

d) i) $\frac{dx}{dt} = 12$ and $\frac{dy}{dt} = 12t$

ii) α) $Q(-12,9)$.

β) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ using the chain rule

$= 12t \times \frac{1}{12} = -1$ when $t = -1$.

γ) $y - 9 = -1(x + 12)$

$\therefore$ $y = -x - 3$.

CARINGBAH 2004 Q6

6b) i) Substitute the point $(2a, 0)$ into the equation

$2y = (p+q)x - 2apq$ to obtain

$0 = (p+q) \times 2a - 2apq$

$\therefore 0 = 2ap + 2aq - 2apq$

$0 = p + q - pq \quad \Rightarrow \quad pq = p + q$

ii) The midpoint M of the chord PQ has

coordinates $M\left(a(p+q), a\left(\frac{p^2+q^2}{2}\right)\right)$

$$\therefore x^2 = a^2(p+q)^2$$

$$= a^2\left(p^2+q^2\right)+2a^2pq$$

$$= 2a \times a\left(\frac{p^2+q^2}{2}\right)+2a^2pq$$

$$= 2ay + 2a \times a(p+q) \text{ -------- using (i)}$$

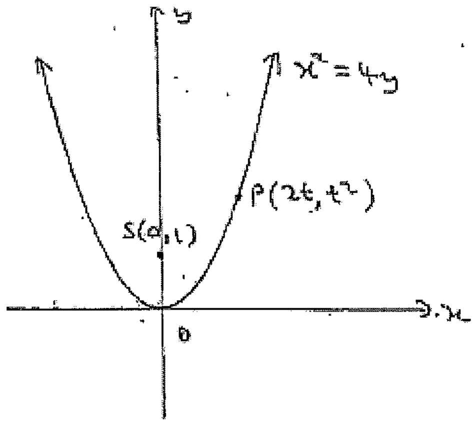
$$= 2ay + 2ax$$

Hence the locus of M is given by the equation

$$x^2 = 2a(y+x)$$

CRANBROOK 2004 Q1

c)



(i) $S(0,1)$ $P(2t, t^2)$ 3:1

$M = \left(\frac{3(2t) - 1(0)}{2}, \frac{3(t^2) - 1(1)}{2}\right)$

$\therefore x = 3t$ ✓ $y = \frac{3t^2 - 1}{2}$ ✓

(ii) $t = \frac{x}{3}$ $2y = 3\left(\frac{x^2}{9}\right) - 1$

$2y = \frac{x^2}{3} - 1$

$6y = x^2 - 3$

$\therefore x^2 = 6y + 3$

$x^2 = 6\left(y + \frac{1}{2}\right)$

vertex is $\left(0, -\frac{1}{2}\right)$

$4a = 6$

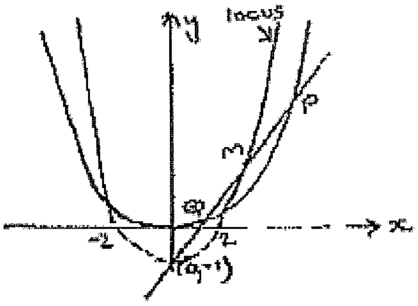
$a = \frac{3}{2}$

focus $(0, 1)$ ✓

directrix $y = -2$ ✓

JAMES RUSE 2002 Q5

(b)



(i) $y+1=mx$ .... (1)

$x^2=4y$ .... (2)

$\therefore x^2=4(mx-1)$ on subst.

$x^2-4mx+4=0$

$\frac{x_p+x_q}{2}=x_m$ where x_p, x_q are roots

$2x_m=x_p+x_q=\alpha+\beta=4m$

$\therefore x_m=2m$ .... (3)

$y_m=m(2m)-1$ subst. in (1)

$M(2m, 2m^2-1)$ ... (3)

(ii) For locus $x^2-4mx+4=0$

Δ : $16m^2-16\geq 0$

$|m|\geq 1$ (1)

Subst. in (3)

$|x|\geq 2$

$x=2m$ .... (4)

$y=2m^2-1$ .... (5)

Square (4) Sub in (5) i.e. $x^2=2(2m^2)$

$\therefore y+1=\frac{x^2}{2}$

$x^2=2(y+1)$ (1)

MANLY 2004 Q5

(b). (i) $x^2 = -4ay$

$y = -\frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{-2x}{4a}$

$= \frac{-x}{2a}$

at $x = 2aq$ ✓

$m = \frac{-4aq}{4a}$

$= -q$

Eqn: $y + aq^2 = -q(x - 2aq)$

$y + aq^2 = -qx + 2aq^2$

$\underline{y + qx = aq^2}$

(ii). ~~m n~~

R: when $x = 0$.

$y = aq^2$ ✓

$\underline{R = (0, aq^2)}$

(iii). $\overset{m\ n}{3:1}$ $Q(2aq, -aq^2)$ $R(0, aq^2)$

External

$x = \frac{mx_2 - nx_1}{m - n}$ $y = \frac{my_2 - ny_1}{m - n}$

$= \frac{0 - 2aq}{2}$ ✓ $= \frac{3aq^2 + aq^2}{2}$

$= -aq$ $= 2aq^2$

$\therefore \underline{S(-aq, 2aq^2)}$ ✓

(iv). $x = -aq \qquad y = 2aq^2$

$-\frac{x}{a} = q$

$\therefore y = 2a\left(\frac{x}{a}\right)^2$

$= \frac{2ax^2}{a^2}$ ✓

$y = \frac{2x^2}{a}$

$\underline{ay = 2x^2}$ ✓