NAME :

Centre of Excellence in Mathematics
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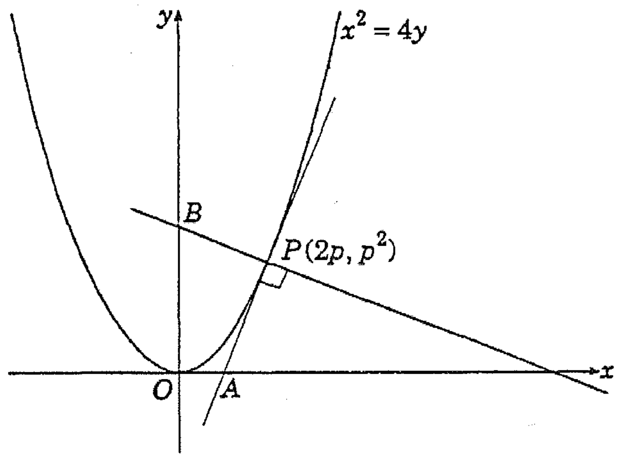
YEAR 12 – EXT.1 MATHS

REVIEW TOPIC (SP3)
PARAMETRIC EQUATIONS OF THE PARABOLA

EXERCISES:

HSC '99

(4) (b) **6**



The diagram shows the graph of the parabola $x^2 = 4y$. The tangent to the parabola at $P(2p, p^2)$, $p > 0$, cuts the x axis at A. The normal to the parabola at P cuts the y axis at B.

(i) Derive the equation of the tangent AP.

$y = px - p^2$

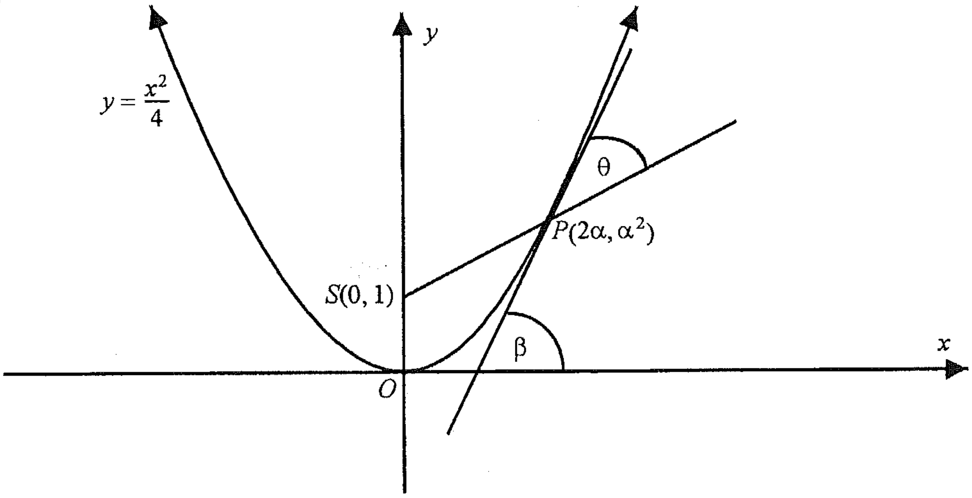
(ii) Show that B has coordinates $(0, p^2 + 2)$.

(iii) Let C be the midpoint of AB. Find the Cartesian equation of the locus of C.

$y = 2x^2 + 1,\ x > 0$

HSC 95

(3)(c)



Let $P(2\alpha, \alpha^2)$ be a point on the parabola $y = \frac{x^2}{4}$, and let S be the point (0, 1). The tangent to the parabola at P makes an angle of β with the x axis. The angle between SP and the tangent is θ. Assume that $\alpha > 0$, as indicated.

(i) Show that $\tan\beta = \alpha$.

(ii) Show that the gradient of SP is $\frac{1}{2}\left(\alpha - \frac{1}{\alpha}\right)$.

(iii) Show that $\tan\theta = \frac{1}{\alpha}$.

(iv) Hence find the value of $\theta + \beta$.

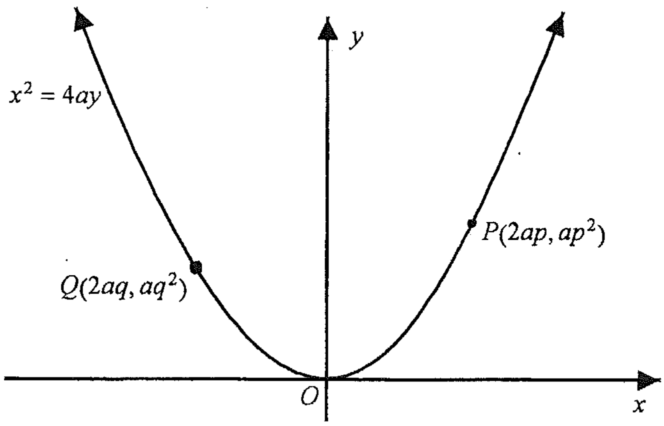
$\boxed{\theta + \beta = 90^0}$

(v) Find the coordinates of P if $\theta = \beta$.

$\boxed{P(2,1)}$

HSC 94

(3) (d) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.



(i) Show the equation of the tangent to the parabola at P is $y = px - ap^2$.

(ii) The tangent at P and the line through Q parallel to the y-axis intersect at T. Find the coordinates of T.

$(2aq, 2apq - ap^2)$

(iii) Write down the coordinates of M, the midpoint of PT.

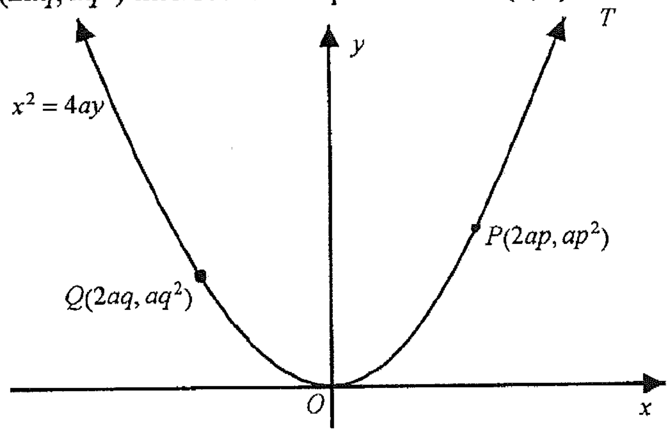
$\boxed{\left(a\left(p+q\right), apq\right)}$

(iv) Determine the locus of M when $pq=-1$.

$\boxed{y=-a}$

HSC '93

(7) (a) Consider the parabola $4ay = x^2$ where $a > 0$, and suppose the tangents at $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ intersect at the point T. Let $S(0, a)$ be the focus of the parabola.



(i) Find the coordinates of T. (You may assume the equation of the tangent at P is $y = px - ap^2$.

$(a(p+q), apq)$

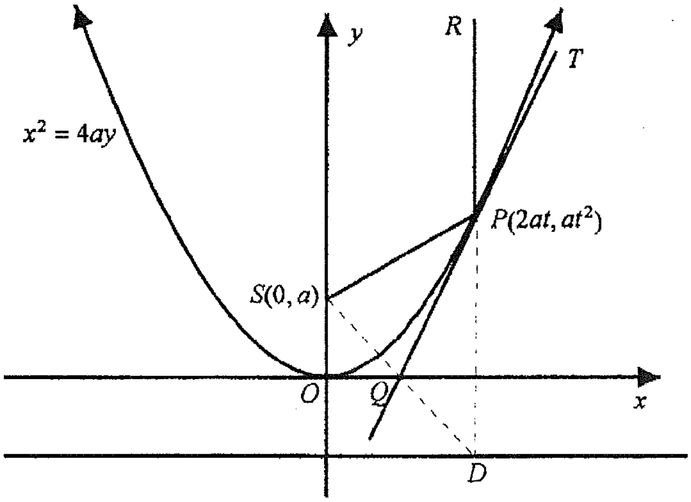
(ii) Show that $SP = a\left(p^2 + 1\right)$

(iii) Suppose P and Q move on the parabola in such a way that $SP + SQ = 4a$.
Show that T is constrained to move on a parabola.

Locus is $x^2 = 2a^2 + 2ay$

HSC '92

(5) (a)



The diagram shows the parabola $x^2 = 4ay$ with focus $S(0, a)$ and directrix $y = -a$. The point $P(2at, at^2)$ is an arbitrary point on the parabola and the line PR is drawn parallel to the y-axis, meeting the directrix at D. The tangent QPT to the parabola at P intersects SD at Q.

(i) Explain why $SP = PD$.

(ii) Find the gradient m_1 of the tangent at P.

$m_1 = t$

(iii) Find the gradient m_2 of the line SD.

$m_2 = -\frac{1}{t}$

(iv) Prove that PQ is perpendicular to SD.

(v) Prove that $\angle RPT = \angle SPQ$.