C.E.M. TUITION

Student Name : ______________________________

Review Topic : Permutation & Combination

(Preliminary)

Year 12 - 3 Unit

1. (a) A man can travel from Sydney to Tokyo by air in 3 ways and by sea in 2 ways. In how many ways can he travel from Sydney to Tokyo?

(b) A man can travel from Sydney to Hong Kong by air in 3 ways and from Hong Kong to Tokyo by sea in 2 ways. In how many ways can he travel from Sydney to Tokyo, via Hong Kong?

2. In a certain library, the books are labelled by using 24 of the 26 letters (excluding O and I) of the English alphabet, followed by a two-digit number. (Note that O1, O2, ... , O9) are not 2-digit numbers). How many books in the library can be labelled under the system?

3. (i) How many distinct 11-letter words can be formed from the letters of the word INDEPENDENT?

(ii) How many distinct 11-letter words have D's at the ends?

4. A bag contains 4 white, and 3 red balls. 5 balls are taken from the bag and arranged in a row. How many distinct arrangements are possible?

5. In how many ways can 4 men and 2 women sit in a row if the

(a) two women are to sit together?

(b) two women are not to sit together?

(c) two women are to be separated by 2 men?

6. A table has 7 seats, 4 being on one side facing the window and 3 being on the opposite side. In how many ways can 7 people be seated at the table if

(a) 2 people, A and B, must sit on the same side?

(b) A and B must sit on the opposite sides?

(c) 3 people A, B, C must sit facing the window?

7. In how many ways can a committee of 5 can be selected from 10 people consisting of 4 men, 4 women and a husband-wife pair, if

(i) there is no restriction in the selection

(ii) the committee must not include the wife-husband pair

(iii) one man C refuses to work in the same committee with a certain woman D?

8. 4 men, 2 women and a child sit at a round table. In how many ways can these 7 people be arranged

(i) without restriction

(ii) if the child is seated between two women

(iii) if the child is seated between two men

(iv) What is the probability that the child is seated between the 2 women?

9. In how many ways can 7 people sit in a row of 7 seats if 4 people A, B, C, D must sit side by side?

10. In how many of the arrangements of the letters of the word DOMINUS do the

(i) vowels occupy alternate places?

(ii) 3 vowels occur together?

1. (a) 5 ways

(b) $2 \times 3 = 6$ ways

Students note that the counting principle applies when there are 2 successive operations.

2. 

The first position can be filled in 24 ways, the second in 24 ways, the third in 9 ways and the fourth in 10 ways. Hence by counting principle, the number of labelling the books is $24 \times 24 \times 9 \times 10 = 51\,840$.

3. (i) There are 11 letters in the word INDEPENDENCE, I, 3 N's, 2 D's, 3 E's, P, T.

Using the formula $\frac{n!}{p!q!r!}$, the required number of arrangements is

$$\frac{11!}{3!2!3!} = 554\,400.$$

(ii) With 2 D's at the two ends, there are 9 letters, I, 3 N's, 3 E's, P, T. Hence the number of arrangements is

$$\frac{9!}{3!3!} = 10\,080.$$

4. (4 white + 3 red) balls. W = a white ball, R = a red ball. 5 balls can be selected in the following ways:

(i) 4W + 1R

(ii) 3W + 2R

(iii) 2W + 3R

Using the formula $\frac{n!}{p!q!}$ for each type of selection, the required number of arrangements is:

$$\frac{5!}{4!} + \frac{5!}{3!2!} + \frac{5!}{2!3!}$$
$$= 5 + 10 + 10$$
$$= 25$$

5. 4 men and 2 women

(a) Consider 2 women A we have 5 persons to arrange in a row, which can be done in $5! = 120$ ways. But 2 women can be arranged as AB or BA in each of these arrangements. Hence the required number of arrangements is $2! \times 5! = 240$.

(b) There are $6! = 720$ unrestricted sitting arrangements. Using the result of part (a), the number of arrangements in which 2 women are not together is $720 - 240 = 480$.

(c) The possible arrangements are identified below: M = a man, W = a woman

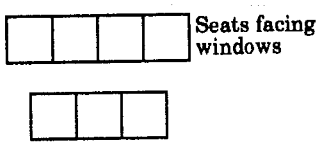
(i) W M M W M M

(ii) M W M M W M

(iii) M M W M M W

Hence the required number of arrangements is

$$4!\cdot 2! + 4!\cdot 2! + 4!\cdot 2!$$
$$= 3(4!\cdot 2!)$$
$$= 144.$$

6. 

(a) When A and B are on the side with 4 seats, the number of arrangements of 7 people is $^4P_2 \times {}^5P_5 = 12 \times 120 = 1440$.

When A and B are on the side with 3 seats, the number of arrangements is $^3P_2 \times {}^5P_5 = 6 \times 120 = 720$.

$\therefore$ The required number of arrangements $= 1440 + 720 = 2160$.

(b) A and B must sit on the opposite sides. When A is on the side with 4 seats and B with 3 seats, the number of arrangements is $^4P_1 \cdot {}^3P_1 \cdot {}^5P_5 = 1440$. Again, when A is on the side with 3 seats and B with 4 seats, the number of arrangements is $^3P_1 \cdot {}^4P_1 \cdot {}^5P_5 = 1440$. Hence the required number of arrangements is $2 \times 1440 = 2880$.

(c) A, B, C can be arranged in $^4P_3$ ways. Then the remaining 4 people can be arranged in $^4P_4$ ways. The required number of arrangements

$$= {}^4P_3 \cdot {}^4P_4$$
$$= 576.$$

7. 10 people
A committee of 5 from 4 men + 4 women + A husband and a wife.
Let x = the required number of committees

(i) When there is no restriction,

$$x = {}^{10}C_5 = 252$$

(ii) When the wife-husband pair is excluded, we have 8 people from which we can select 5 people in

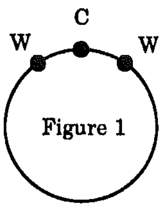
$$x = {}^8C_5 = 56 \text{ ways}.$$

(iii) The number of committees in which C and D are together is ${}^8C_5 = 56$.
Using the result of part (i)
$x = 252 - 56 = 196$
committees in which C and D are not included.

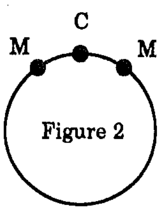
8. 4 men + 2 women + 1 child = 7 people.
x = the required number of arrangements

(a) $x = 6! = 720$

(b) C = a child. Let C occupy a position on the circle (Figure 1).



The two women can occupy 2 seats surrounding the child in 2 ways and the four men can then be seated in the remaining 4 seats in



${}^4P_4 = 24$ ways.
$\therefore\ x = 2 \times 24 = 48$

(c) Two seats on either side of C (Figure 2) can be occupied by 4 men in
${}^4P_2 = 24$ ways. The remaining 4 seats can be occupied by the remaining 4 people in ${}^4P_4 = 24$ ways
$\therefore\ x = 12 \times 24 = 288$

(d) Using the results of part (i) and (ii), the required probability is $\frac{48}{288} = \frac{1}{15}$.

9. Consider A, B, C and D as one unit (ABCD). We now have four units to arrange in a row. This can be done in ${}^4P_4 = 24$ ways, but A, B, C, D among themselves can be arranged in ${}^4P_4 = 24$ ways. Hence the required number of arrangements is $24 \times 24 = 576$.

10. × × × × × × ×
2 4 6

(i) The 3 vowels O, I, U can occupy 3 even positions in ${}^3P_3 = 6$ ways and the 4 consonants D, M, N, S then occupy the other 4 positions in ${}^4P_4 = 24$ ways. Hence the required number of arrangements
$= 6 \times 24 = 144$.

(ii) Consider 3 vowels (OIU) as one unit, so we have 5 units D, M, N, S, (OIU) to arrange in a row. This can be done in ways ${}^5P_5 = 120$. (OIU) can be arranged in ${}^3P_3$ ways. By counting method, the required numbers of arrangements
$= 120 \times 6 = 720$.