Review of Permutations, Combinations & Probability

Answer in the space provided. Show all working.

1) Yr12-3U\furprob.hsc Qn1) 3U84-3ii

A box contains ten tennis balls of which four have never been used. For the first game two balls are selected at random and, after play, are returned to the box. For the second game two balls are also selected at random from the box. Find the probability of each of the following events:

a. precisely one of the balls selected for the first game has been used before;

b. neither ball selected for the first game has been used before, but both balls selected for the second game have been used before the second game.¤

2) Yr12-3U\furprob.hsc Qn3) 3U86-6ii

A given school in a certain State has *3* Mathematics teachers. The probability in that State that a Mathematics teacher is female is *0·4*.

a. What is the probability that in the given school there is at least one female Mathematics teacher?

b. In the same State the probability that a Mathematics teacher (male or female) is a graduate is *0·7*. What is the probability that in the given school none of the three Mathematics teachers is a female graduate? (Give your answer correct to 3 decimal places.)¤

Review of Permutations, Combinations & Probability

3) Yr12-3U\furprob.hsc Qn4) 3U87-3iii

One fifth of all jellybeans are black. A random sample of ten jellybeans is chosen.

a. What is the probability that this sample contains exactly two black jellybeans? Give your answer correct to 3 decimal places.

b. What is the probability that the sample contains fewer than two black jellybeans? Give your answer correct to 3 decimal places.

c. Which is more likely: the sample contains fewer than *2* black jellybeans, or the sample contains more than *2* black jellybeans? Give reasons for your answer.¤

4) Yr12-3U\furprob.hsc Qn5) 3U88-5b

A meeting room contains a round table surrounded by ten chairs. These chairs are indistinguishable and equally spaced around the table.

i. A committee of ten people includes three teenagers. How many seating arrangements are there in which all three sit together? Give brief reasons for your answer.

ii. Elections are held for the position of Chairperson and Secretary in a second committee of ten people seated around this table. What is the probability that the two people elected are sitting directly opposite each other? Give brief reasons for your answer.¤

5) Yr12-3U\furprob.hsc Qn6) 3U89-3a

A committee of *3* is to be elected from a club of *8* members.

i. How many different committees can be formed?

ii. If there are *4* Queenslanders in the club, what is the probability that a randomly selected committee of *3* contains only Queenslanders?¤

6) Yr12-3U\furprob.hsc Qn8) 3U91-2b

When Mendel crossed a tall strain of pea with a dwarf strain of pea, he found that $\frac{3}{4}$ of the offspring were tall and $\frac{1}{4}$ were dwarf. Suppose five such offspring were selected at random. Find the probability that:

i. all of these offspring were tall

ii. at least three of these offspring were tall.

Leave your answers in index form.¤

7) Yr12-3U\furprob.hsc Qn10) 3U92-6b

A total of five players is selected at random from four sporting teams. Each of the teams consists of ten players numbered from *1* to *10*.

i. What is the probability that of the five selected players, three are numbered '*6*' and two are numbered '*8*'?

ii. What is the probability that the five selected players contain at least four players from the same team?¤

8) Yr12-3U\furprob.hsc Qn12) 3U95-5b

In a Jackpot Lottery, *1500* numbers are drawn from a barrel containing the *100 000* ticket numbers available. After all the *1500* prize-winning numbers are drawn, they are returned to the barrel and a jackpot number is drawn. If the jackpot number is the same as one of the *1500* numbers that have already been selected, then the additional jackpot prize is won.

The probability that the jackpot prize is won in a given game is thus $p = \dfrac{1500}{100\,000} = 0{\cdot}015.$

i. Calculate the probability that the jackpot prize will be won exactly once in *10* independent lottery games.

ii. Calculate the probability that the jackpot prize will be won at least once in *10* independent lottery games.

iii. The jackpot prize is initially *$8000*, and it increases by *$8000* each time the prize in NOT won. Calculate the probability that the jackpot prize will exceed *$200 000* when it is finally won.¤

9) Yr12-3U\furprob.hsc Qn13) 3U96-5c

Mice are placed in the centre of a maze which has five exits. Each mouse is equally likely to leave the maze through any one of the five exits. Thus, the probability of any given mouse leaving by a particular exit is $\frac{1}{5}$. Four mice, *A, B, C* and *D*, are put into the maze and behave independently.

i. What is the probability that *A, B, C* and *D* all come out the same exit?

ii. What is the probability that *A, B* and *C* come out the same exit, and *D* comes out a different exit?

iii. What is the probability that any three of the four mice come out the same exit, and the other comes out a different exit?

iv. What is the probability that no more than two mice come out the same exit?¤

[End Of Qns]

Review of Permutations, Combinations & Probability

[Answers]

«1→ a) $\frac{8}{15}$ b) $\frac{56}{675}$ »

«2→ a) $0{\cdot}784$ b) $0{\cdot}373$ »

«3→ a) $0{\cdot}302$ b) $0{\cdot}376$ c) The probability that the sample contains more than 2 black jellybeans is $0{\cdot}322$. ∴ It is more likely the sample contains less than 2 black jellybeans. »

«4→ i) $30\,240$ ii) $\frac{1}{9}$ »

«5→ i) 56 ii) $\frac{1}{14}$ »

«6→ i) $(\frac{3}{4})^5$ ii) $34 \times \frac{3^3}{4^5}$ »

«7→ i) $\frac{1}{27\,417}$ ii) $\frac{28}{703}$ »

«8→ i) $0{\cdot}131$ (to 3 d.p.) ii) $0{\cdot}140$ (to 3 d.p.) iii) $0{\cdot}685$ (to 3 d.p.) »

«9→ i) $\frac{1}{125}$ ii) $\frac{4}{125}$ iii) $\frac{16}{125}$ iv) $\frac{108}{125}$ »

Answer in the space provided. Show all working.

1) Yr12-3U\furprob.hsc Qn1) 3U84-3ii

A box contains ten tennis balls of which four have never been used. For the first game two balls are selected at random and, after play, are returned to the box. For the second game two balls are also selected at random from the box. Find the probability of each of the following events:

a. precisely one of the balls selected for the first game has been used before;

b. neither ball selected for the first game has been used before, but both balls selected for the second game have been used before the second game.¤

Total = $^{10}C_2 \times {}^{10}C_2$

a) $^{10}C_2 \times 2 \times 8$ ∴ P() = $\frac{^{10}C_2 \times 2 \times 8}{^{10}C_2 \times {}^{10}C_2} = \frac{16}{45}$

(a) 6 used, 4 not used

$P(\text{exactly one used}) = \frac{^6C_1 \times {}^4C_1}{^{10}C_2} = \frac{24}{45} = \frac{8}{15}$

b) No of selections of 2 from 4 not used balls

$P() = \frac{^4C_2 \times \text{[illegible]}}{^{10}C_2 \times {}^{10}C_2} = \frac{2}{45}$

After game one: 8 used, 2 not used

$\frac{^8C_2}{^{10}C_2} = \frac{28}{45}$

Total prob = $\frac{2}{15} \times \frac{28}{45} = \frac{56}{675}$

2) Yr12-3U\furprob.hsc Qn3) 3U86-6ii

A given school in a certain State has 3 Mathematics teachers. The probability in that State that a Mathematics teacher is female is 0·4.

a. What is the probability that in the given school there is at least one female Mathematics teacher?

b. In the same State the probability that a Mathematics teacher (male or female) is a graduate is 0·7. What is the probability that in the given school none of the three Mathematics teachers is a female graduate? (Give your answer correct to 3 decimal places.)¤

a) $P() = \binom{3}{0}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^0 + \binom{3}{1}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^1 + \binom{3}{2}\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^2 = \frac{8}{125} + \frac{36}{125} + \frac{54}{125} = \frac{98}{125}$

(a) P(at least 1 female) = 1 − P(all males) = $1 - (0.6)^3 = 0.784$

a) $p = \frac{2}{5}$, $q = \frac{3}{5}$

b) $p = \frac{7}{25}$, $q = \frac{18}{25}$

b) $\binom{3}{0}\left(\frac{7}{25}\right)^0\left(\frac{18}{25}\right)^3$ = 0,373 (3 dp)

p = prob. of female teacher = 0.4

q = $\bar{p}$ = 0.6

0.4 × 0.7 = $\frac{7}{25}$

(b) p = prob. of female graduate = 0.4 × 0.7 = 0.28

q = 0.72

∴ $P(q^3) = (0.72)^3$ = 0.373 (to 3 d.p.).

$p = \frac{1}{5}$ $q = \frac{4}{5}$

3) Yr12-3U\furprob.hsc Qn4) 3U87-3iii

One fifth of all jellybeans are black. A random sample of ten jellybeans is chosen.

a. What is the probability that this sample contains exactly two black jellybeans? Give your answer correct to 3 decimal places.

b. What is the probability that the sample contains fewer than two black jellybeans? Give your answer correct to 3 decimal places.

c. Which is more likely: the sample contains fewer than 2 black jellybeans, or the sample contains more than 2 black jellybeans? Give reasons for your answer.¤

a) $\binom{10}{8}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8$ = 0.302 ✓

b) $\binom{10}{9}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^9 + \binom{10}{10}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10}$ = 0.376 ✓

c) More than 2 = 1 − 0.376 = 0.302

= 0.322

∴ fewer than 2 = more likely.

4) Yr12-3U\furprob.hsc Qn5) 3U88-5b

A meeting room contains a round table surrounded by ten chairs. These chairs are indistinguishable and equally spaced around the table.

i. A committee of ten people includes three teenagers. How many seating arrangements are there in which all three sit together? Give brief reasons for your answer.

ii. Elections are held for the position of Chairperson and Secretary in a second committee of ten people seated around this table. What is the probability that the two people elected are sitting directly opposite each other? Give brief reasons for your answer.¤

i) 7! (circle) × 3! (permutable 3 teenagers) = 36,240 ✓

ii) $\frac{8!}{9!} = \frac{1}{9}$ ✓ (keep 2 people still; 9! ← total)

5) Yr12-3U\furprob.hsc Qn6) 3U89-3a

A committee of 3 is to be elected from a club of 8 members.

i. How many different committees can be formed?

ii. If there are 4 Queenslanders in the club, what is the probability that a randomly selected committee of 3 contains only Queenslanders?¤

i) $^8C_3$ = 56 ✓

ii) $\frac{^4C_3}{^8C_3} = \frac{4}{56} = \frac{1}{14}$ ✓

6) Yr12-3U\furprob.hsc Qn8) 3U91-2b

When Mendel crossed a tall strain of pea with a dwarf strain of pea, he found that $\frac{3}{4}$ of the offspring were tall and $\frac{1}{4}$ were dwarf. Suppose five such offspring were selected at random. Find the probability that:

i. all of these offspring were tall

ii. at least three of these offspring were tall.

Leave your answers in index form.

i) $\binom{5}{5}\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$ ✓ $\frac{243}{1024}$

ii) $\binom{5}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + \binom{5}{4}\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1 + \binom{5}{5}\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$

Need to evaluate & leave answers in index form.

7) Yr12-3U\furprob.hsc Qn10) 3U92-6b

A total of five players is selected at random from four sporting teams. Each of the teams consists of ten players numbered from *1* to *10*.

i. What is the probability that of the five selected players, three are numbered '6' and two are numbered '8'?

ii. What is the probability that the five selected players contain at least four players from the same team?

~~$\binom{3}{3}\left(\frac{4}{10}\right)^3\left(\frac{6}{10}\right)^0 \times \binom{2}{2}$~~

i) $\frac{^4C_3 \times {^4C_2}}{^{40}C_5}$ ✓ $= \frac{1}{27417}$

ii) $p = \frac{1}{4}, q = \frac{3}{4}$

$\binom{5}{1}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + \binom{5}{0}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$ ✓

40. − 4 6

$\frac{1}{10}$

$\frac{200\,000}{8000} = 25$ 25 times

8) Yr12-3U\furprob.hsc Qn12) 3U95-5b

In a Jackpot Lottery, *1500* numbers are drawn from a barrel containing the *100 000* ticket numbers available. After all the *1500* prize-winning numbers are drawn, they are returned to the barrel and a jackpot number is drawn. If the jackpot number is the same as one of the *1500* numbers that have already been selected, then the additional jackpot prize is won.

The probability that the jackpot prize is won in a given game is thus $p = \frac{1500}{100\,000} = 0{\cdot}015$. $q = 0.985$

i. Calculate the probability that the jackpot prize will be won exactly once in *10* independent lottery games.

ii. Calculate the probability that the jackpot prize will be won at least once in *10* independent lottery games.

iii. The jackpot prize is initially *$8000*, and it increases by *$8000* each time the prize in NOT won. Calculate the probability that the jackpot prize will exceed *$200 000* when it is finally won.

i) $\binom{10}{1}(0.015)^1(0.985)^9$ ✓

ii) $1 - \binom{10}{0}(0.015)^0(0.985)^{10}$ ✓

iii) ~~$\binom{1}{1}(0.015)^1(0.985)^0 + \binom{2}{1}(0.015)^1(0.985)^1 + \binom{3}{1}(0.015)^1(0.985)^2 \ldots + \binom{25}{1}(0.015)^1(0.985)^{25}$~~ $= P() = (0.985)^{25} = 25$ no wins

9) Yr12-3U\furprob.hsc Qn13) 3U96-5c

Mice are placed in the centre of a maze which has five exits. Each mouse is equally likely to leave the maze through any one of the five exits. Thus, the probability of any given mouse leaving by a particular exit is $\frac{1}{5}$. Four mice, *A*, *B*, *C* and *D*, are put into the maze and behave independently.

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iv. What is the probability that no more than two mice come out the same exit?

i) ~~$\binom{4}{0}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^0$~~ $1 \times \left(\frac{1}{5}\right)^3$ others to follow; Any exit for first mouse

ii) $\binom{4}{1}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^1$ $1 \times \left(\frac{1}{5}\right)^2 \times \frac{4}{5} = \frac{4}{125}$

iii) $\binom{4}{1}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^1 \times 3$ $\left(\frac{4}{125}\right) \times 4$ ✓ 4 diff. ways of choosing a diff. exit.

iv) $1 - \left[\binom{4}{1}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right) \times 3 + \binom{4}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^0\right]$

[End Of Qns]

P(no more than 2 out of same exit) = 1 − (P(all same exit) + P(3 same, 1 diff))