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YEAR 12 – EXT. 1 MATHS

REVIEW TOPIC (SP1)

POLYNOMIALS

CEM – Yr 12 – 3 Unit Polynomials – Review Booklet – Paper 1

1. Let α, β and γ are the roots of the polynomial $2x^3-5x-1=0$.
   Find $\alpha^{-1}\beta^{-1}\gamma^{-1}$.

2. Show that $x+4$ is a factor of $P(x)=x^3+2x^2-23x-60$ and hence factorise $P(x)$.

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3. Consider the polynomial $P(x) = x^5 + 2x - 4$.

   i. Show that $P(x) = 0$ has only one real root and that it lies in the interval $1 < x < 1.5$.

   ii. Taking $x_1 = 1.2$ as a first approximation to this root, use one step of Newton's Method to find a better approximation x_2, correct to two decimal places.

4. The cubic polynomial $P(x) = x^3 + rx^2 + sx + t$, where r, s and t are real numbers, has three real zeros, 1, α and $-\alpha$.

   i. Find the value of r.

   ii. Find the value of $s + t$.

5. Suppose $x^3 - 2x^2 + a \equiv (x+2)Q(x) + 3$ where $Q(x)$ is a polynomial.

Find the value of a.

6. If α, β, and γ are the roots of the equation $2x^3 - 3x^2 + 4x - 6 = 0$ find the value of:

i) $\alpha + \beta + \gamma$
ii) $(\alpha + 1)(\beta + 1)(\gamma + 1)$.

7. Use the remainder theorem to find a factor of: $P(x) = x^3 - 7x - 6$, then factorise completely.

8. Solve the equation $x^3+6x^2+3x-10=0$ if the roots form consecutive terms of an arithmetic series.

9. If α,β,γ are the zeros of the polynomial $P(x)=2x^3+8x^2-x+6$ evaluate:
$\alpha^2+\beta^2+\gamma^2$

10. When $Q(x) = ax^3 + bx^2 + c$ is divided by $(x + 2)$ the remainder is 3 and, when $Q(x)$ is divided by $(x^2 - 1)$ the remainder is $(2x + 4)$.

Find a, b and c.

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Answers

1. $2x^3 - 5x - 1 = 0$

$\alpha^{-1}\beta^{-1}\gamma^{-1} = \frac{1}{\alpha}\cdot\frac{1}{\beta}\cdot\frac{1}{\gamma}$

$= \frac{1}{\alpha\beta\gamma}$

where $\alpha\beta\gamma = -\frac{d}{a}$

$= -\frac{-1}{2}$

$= \frac{1}{2}$ ✓

$\therefore \alpha^{-1}\beta^{-1}\gamma^{-1} = \frac{1}{\frac{1}{2}}$ ✓

$= 2$ ✓

2

2. $P(-4) = (-4)^3 + 2(-4)^2 - 23(-4) - 60$

$= -64 + 32 + 92 - 60$

$= 0$

$\therefore x+4$ is a factor. ✓

$$\begin{array}{r|l} & x^2 - 2x - 15 \\ \hline x+4 & x^3 + 2x^2 - 23x - 60 \\ & \underline{x^3 + 4x^2} \\ & \quad -2x^2 - 23x - 60 \\ & \quad \underline{-2x^2 - 8x} \\ & \qquad -15x - 60 \\ & \qquad \underline{-15x - 60} \\ & \qquad\qquad 0 \end{array}$$

✓

$\therefore P(x) = (x+4)(x^2 - 2x - 15)$

$= (x+4)(x-5)(x+3)$ ✓

3

3. i) $P(x) = x^3 + 2x - 4$

$P'(x) = 3x^2 + 2 > 0$ for all real x,

$\therefore$ the curve $y = P(x)$ has no turning point. ✓

The graph of $P(x)$ intersects the x-axis only once; $\therefore P(x) = 0$ has only one real root! ✓

$P(1) = 1 + 2 - 4 = -1 < 0$ ✓

$P(1.5) = 3.375 + 3 - 4 = 2.375 > 0$ ✓

Since $P(1)$ & $P(1.5)$ are of opposite signs, the root lies between 1 and 1.5.

2

ii) $P(1.2) = 1.2^3 - 2(1.2) - 4$

$= 0.128$ ✓

$P'(1.2) = 3(1.2)^2 + 2$ ✓

$= 6.32$

$x_2 = x_1 - \frac{P(1.2)}{P'(1.2)}$ ✓

$= 1.2 - \frac{0.128}{6.32}$ ✓

$= 1.1797\ldots$

$= 1.18$ (to 2 dp)

2

4. i) $P(x) = x^3 + rx^2 + 5x + t$

the zeros are: $1, \alpha, -\alpha$

(i) Sum of roots $= 1 + \alpha - \alpha = -\frac{b}{a}$ ✓

$-\frac{b}{a} = 1$

$-r = 1$

$\therefore r = -1$ ✓

1

ii) (ii) If 1 is a zero and $r = -1$ ✓

then $P(1) = 0$ ✓

i.e. $1^3 + r(1)^2 + 1s + t = 0$

$1 + r + s + t = 0$

$1 - 1 + s + t = 0$

$\therefore s + t = 0$ ✓

2

5. $x^3-2x^2+a \equiv (x+2)\,Q(x)+3$

Since $(x+2)$ is a factor

$\therefore (-2)^3-2(-2)^2+a \equiv (-2+2)Q(x)+3$ ✓ 2

✓

$-8-8+a \equiv 0.Q(x)+3$

$-16+a \equiv 0+3$ ✓

$\therefore a = 19$

6. i) $\alpha+\beta+\gamma = \frac{3}{2}$ 0.5

ii) $(\alpha+1)(\beta+1)(\gamma+1)$

$= \alpha\beta\gamma + \Sigma\alpha\beta + \Sigma\alpha + 1$ 1

$= 3 + 2 + \frac{3}{2} + 1$ 1

$= \frac{15}{2}$

7. $P(x) = x^3-7x-6$

$P(-1) = -1+7-6$

$= 0$ 1M

$(x+1)$ is a factor. 1M

$$x+1\overline{)\,x^3-7x-6\,}\quad x^2-x-6$$

$\therefore P(x) = (x+1)(x^2-x-6)$ 1M

$= (x+1)\ (x-3)(x+2)$

8. $a^3+6x^2+3x-10=0$

Let the roots be

$a-d,\ a,\ a+d.$

$a-d+a+a+d=-6$

$3a=-6$

$a=-2$ 1M

$(a-d)\,a\,(a+d)=10$

$a(a^2-d^2)=10$

$-2(4-d^2)=10$

$4-d^2=-5$

$d^2=9$

$d=\pm 3$ 1M

The roots are

$-5,\ -2,\ 1$ 1M.

9. $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= (-4)^2-2(-\frac{1}{2})$

$= 16+1$

$= 17$

10. c) $Q(x) = ax^3 + bx^2 + c$

$Q(-2) = 3$

$a(-2)^3 + b(-2)^2 + c = 3$

$\underline{-8a + 4b + c = 3}$ ①

$Q(1) = 2x+4$

$a(1)^3 + b(1)^2 + c = 2+4$

$\underline{a + b + c = 6}$ ②

$Q(-1) = 2x+4$

$a(-1)^3 + b(-1)^2 + c = 2$

$\underline{-a + b + c = 2}$ ③

② - ③ we get

$a + b + c = 6$ ②

$-a + b + c = 2$ ③

$2a = 4$

$\therefore a = 2$

Subst. $a = 2$ into ① we get

$-8(2) + 4b + c = 3$

$-16 + 4b + c = 3$

$\underline{4b + c = 19}$ ④

Subst $a = 2$ into ② we get

$2 + b + c = 6$

$\therefore \underline{b + c = 4}$ ⑤

④ - ⑤ we get

$4b + c = 19$

$b + c = 4$

$3b = 15$

$\therefore b = 5.$

Subst. $b = 5$ into ⑤ we obtain

$5 + c = 4$

$\therefore c = -1$

So $a = 2$, $b = 5$, $c = -1$