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YEAR 12 – EXT. 1 MATHS

REVIEW TOPIC (SP3)

POLYNOMIALS

CEM – Yr 12 – 3U Polynomials – Review Booklet – Paper 3

1. Show that $(2x + 1)$ is a factor of $2x^3 + 7x^2 - x - 2$

2. The remainder when the polynomial x^4 is divided by $x + a$ is 16.
Find the value of a

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3. A monic polynomial $P(x)$ of degree 4 is known to have exactly two zeros at 2 and -2. It is also known that $P(x)$ is an even function.

   Further, when $x = 3$ the value of $P(x)$ is 55. Determine the polynomial function $P(x)$.

4. The polynomials $3x^3 - x + 1$ and $ax(x-1)(x+2) + bx(x-1) + cx + d$ are equal for all values of x.

Determine the values of a, b, c and d.

5. The polynomial $P(x) = x^2 + ax + b$ has a zero at $x = 2$. When $P(x)$ is divided by $x - 1$, the remainder is 2.
   Find the value of a and b

6. Find the quotient and remainder when
   $$x^4 - 2x^3 + x^2 - 5x + 7$$
   is divided by
   $$x^2 + x - 1$$

7. $(x-2)$ is a factor of the polynomial

$$P(x) = 2x^3 + x + a.$$

Find a.

8. Consider The function $f(x) = x^3 - \ln(x+1)$ has one root between 0·5 and 1.

(i) Show the root lies between 0·8 and 0·9.

(ii) Hence use the halving-the-interval method to find the value of the root, correct to one decimal place .

9. It is known that two of the roots of the equation $2x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. Find the value of k.

10. The polynomial $P(x) = x^3 - 6x^2 + kx + 14$ has a zero at $x = -2$.

i. Find the value of k.

ii. Express $P(x)$ as a product of linear factors.

iii. By sketching the graph of $y = P(x)$, hence, or otherwise, solve $P(x) < 0$.

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Answers

1. If $(2x+1)$ is a factor, $P(-\frac{1}{2})=0$

$$2(-\tfrac{1}{2})^3+7(-\tfrac{1}{2})^2-(-\tfrac{1}{2})-2$$
$$= -\tfrac{1}{4}+\tfrac{7}{4}+\tfrac{1}{2}-2$$
$$= 0$$

2.
$f(x)=x^4$
divided by $x+a$
$R=16$

$$f(-a)=(-a)^4$$
$$=a^4$$
$$a^4=16$$
$$\therefore a=\pm 2$$

3. $P(x)=x^4+bx^3+cx^2+dx+e$

Since $P(x)$ is even

then $P(x)=x^4+cx^2+e$

$P(2)=0$ $\quad \therefore 2^4+c(2)^2+e=0$
$$16+4c+e=0$$
$$4c+e=-16 \quad (1)$$

When $x=3$, $P(x)=55$,
$$3^4+c(3)^2+e=55$$
$$81+9c+e=55$$
$$9c+e=-26 \quad (2)$$

4. i) $3x^3-x+1 \equiv ax(x-1)(x+2)+bx(x-1)+cx+d$

Equating

coefficient of x^3 $\quad 3=a$

When $x=0$ $\quad 1=d$

$x=1$ $\quad 3=c+d$

so $3=c+1$

$\therefore c=2$

Try when $x=-2$, $\quad 3(-2)^3-(-2)+1 = 0+b(-2)(-2-1)+c(-2)+d$
$$-24+2+1 = 6b-2c+d$$
$$-21=6b-2\times2+1$$
$$-21=6b-4+1$$
$$-21=6b-3$$
$$6b=-18$$
$$\therefore b=-3$$

So $a=3,\ b=-3,\ c=2,\ d=1$

5. (c) $P(x)=x^2+ax+b$

Factor $(x-2)$
$$P(2)=4+2a+b$$
$$0=4+2a+b$$
$$\therefore b=-2a-4 \quad \ldots\ldots(1)$$
$$P(1)=1+a+b$$
$$2=1+a+b$$
$$b=-a+1 \quad \ldots\ldots(2)$$

(1) = (2):
$$-2a-4=-a+1$$
$$-5=a$$

Sub a in (2):
$$b=-(-5)+1$$
$$=6$$
$$\therefore a=-5,\ b=6$$

6.
$$\begin{array}{r} x^2-3x+5 \\ x^2+x-1\,\overline{)\,x^4-2x^3+x^2-5x+7} \\ x^4+x^3-x^2 \\ \hline -3x^3+2x^2-5x \\ -3x^3-3x^2+3x \\ \hline 5x^2-8x+7 \\ 5x^2+5x-5 \\ \hline -13x+12 \end{array}$$

$Q(x)=x^2-3x+5$
$R(x)=-13x+12$

7. If $(x-2)$ is a factor of $P(x)$ then

$P(2)=0$

$2\times 2^3+2+a=0$

$18+a=0$

$a=-18$

8. i) $f(x)=x^3-\ln(x+1)$

(i) $f(0\cdot8)=-0\cdot0757\ldots<0$

$f(0\cdot9)=0\cdot0871\ldots>0$

Thus as $f(0\cdot8)$ and $f(0\cdot9)$ have different sign, then for some value a, $0\cdot8<a<0\cdot9$, $f(a)=0$ and 'a' is the root of $f(x)=0$.

8. ii) (ii) $x_3=\frac{0\cdot8+0\cdot9}{2}=0\cdot85$

$f(0\cdot85)=-0\cdot00106\ldots<0$

$\therefore$ The root of $f(x)=0$ is between $0\cdot85$ and $0\cdot9$ and is closer to $0\cdot9$ than $0\cdot8$

Thus the value of the root to 1dp is $0\cdot9$.

9. $\sum\alpha=-\frac{1}{2}=\alpha+\beta+\frac{1}{\beta}$

$\sum\alpha\beta=-\frac{k}{2}$

$\alpha\beta\gamma$: $\alpha=-3$.

$-\frac{1}{2}=-3+\frac{\beta^2+1}{\beta}$

$\frac{5}{2}=\frac{\beta^2+1}{\beta}$

$5\beta=2\beta^2+2$

$2\beta^2-5\beta+2=0$

$\therefore\beta=2,\frac{1}{2}$

$\therefore$ Roots: $2,\frac{1}{2},-3$.

$\alpha\beta+\beta\gamma+\alpha\gamma=-\frac{k}{2}$

$1-6-\frac{3}{2}=-\frac{k}{2}$

$-5-\frac{3}{2}=-\frac{k}{2}$

$\frac{k}{2}=\frac{3}{2}+5$

$k=3+10$

$=13$

10. i) $p(-2)=-8-6(4)-2k+14=0$

$=-8-24-2k+14$

$=-32+14-2k$

$=-18-2k=0$

~~$2k=-18$~~

$-2k=18$

$k=-9$.

$p(-2)=-8-24-2k+14$

$0=-32+18-2k$

$18=-2k$

$k=-9$.

ii) $P(x) = x^3 - 6x^2 - 9x + 14$.

$$\begin{array}{r|l} & x^2 - 8x + 7 \\ \hline x+2 & x^3 - 6x^2 - 9x + 14 \\ & x^3 + 2x^2 \\ \hline & -8x^2 - 9x + 14 \\ & -8x^2 - 16x \\ \hline & 7x + 14 \\ & 7x + 14 \\ \hline & 0 \end{array}$$

$\Rightarrow (x+2)(x-1)(x-7)$. ✓

$P(-2) = -8 - 24 + 18 + 14$

$= 32 - 32 = 0$.

$P(1) = 1 - 6 - 9 + 14 = 0$.

$P(7) = 7^3 - 6 \times 49 - 9 \times 7 + 14$

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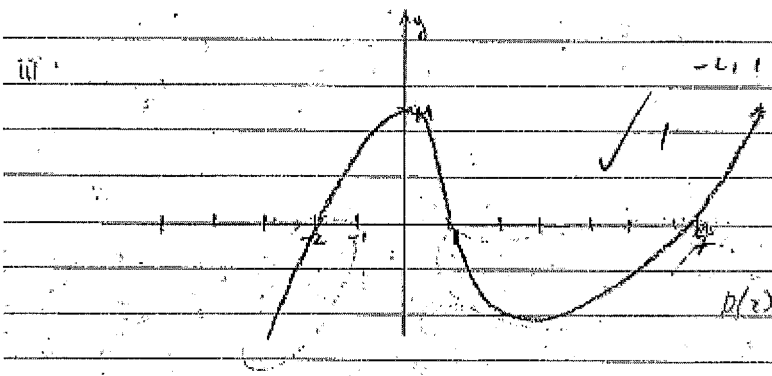
iii)



$-2, 1, 7$.

✓ 1

$P(x) < 0$.

$\therefore x < -2$

$1 < x < 7$. ✓ 1

$P(x) < 0$

$x < -2 \quad 1 < x < 7$.