

# REVIEW OF EXTENSION ONE MATHS

does not contain curve sketching & circle geometry

1) Differentiate:

a)  $\sin^{-1} \frac{x}{4}$

b)  $\cos^{-1} 2x$

c)  $\tan^{-1} 4x^2$

d)  $x^2 \sin^{-1} x$

2) Evaluate:

a)  $\int \frac{5}{4+x^2} dx$

b)  $\int \frac{dx}{\sqrt{1-4x^2}}$

c)  $\int \frac{-1}{\sqrt{4-25x^2}} dx$

3) Write the exact value of:

a)  $\cos(\cos^{-1} \frac{4}{5})$

b)  $\tan(\sin^{-1} \frac{3}{5})$

4) Write the domain and range of:

a)  $y = 3 \sin^{-1} \frac{x}{2}$

b)  $y = 4 \cos^{-1} 2x$

c)  $y = \tan^{-1} 3x$

5) Sketch each of the functions in Q4.

6) A is the point (6,4) and B is the point (2,1).

Divide:

a) AB internally in the ratio 4:5

b) BA externally in the ratio 4:5

7) Find the acute angle (nearest degree) between the lines  $x+y=6$  and  $y=3x$

8) Solve for  $x$ :

$$\frac{x-7}{x^2-9} \geq 1$$

9) Evaluate:

a)  $\int \frac{x^2}{\sqrt{x^3-1}} dx$  using the substitution  $u=x^3-1$

b)  $\int_4^5 x\sqrt{x-4} dx$  using  $u=x-4$

10) Evaluate:

$$\int_0^1 \sqrt{1-x^2} dx$$

using  $x = \sin \theta$

11) Consider the word

REVISION

How many ways can the letters of the word be arranged:

a) in total?

b) if the 'R' and 'V' must be next to each other?

12) 12 people attend a dinner and are seated around a circular table. What is the probability that a particular couple will be seated next to each other?

13) To crack the code of a certain alarm, two stages must be solved. The probability that Stage 1 is solved is  $\frac{1}{4}$  and Stage 2 is  $\frac{3}{5}$ .

CONT. next column...

a) what is the probability that the code is cracked on the first try?

b) Suppose 10 different people (all of equal code cracking ability) attempt to crack the code. What is the probability that exactly 3 crack the code?

14) Prove  $(x-3)$  is a factor of  $x^3 - 3x^2 - 4x + 12$

15) Write  $P(x)$  in terms of its linear factors if  $P(x) = x^3 + 3x^2 - 4$

16) Sketch  $P(x)$  from Q15 without the use of calculus.

17) Divide  $5x^4 - 3x^3 + 1$  by  $x^3 - 2$  and hence find the remainder  $R(x)$ .

18) Write the inverse of:

a)  $y = 4x^2 - 1$  ( $x > 0$ )

b)  $y = e^x + 2$

c)  $y = 2 + \cos 4x$  ( $0 \leq x \leq \pi$ )

19) Use the principle of Mathematical Induction to prove that for all positive integer values of  $n$ :

$$a) 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2}{4} (n+1)^2$$

b)  $5^n - 1$  is divisible by 4.

20) Use both the 'Auxillary Angle' Method and 't' Method to solve for  $0 \leq \theta \leq 2\pi$ :

$$\sin \theta + \cos \theta = -1$$

21) Evaluate:

$$\int_0^{\frac{\pi}{2}} 3 \sin^2 x \, dx$$

22) Show that  $\sin 3\theta$  equals  $3 \sin \theta - 4 \sin^3 \theta$

23) Write the exact value of:

a)  $\sin 75^\circ$

b)  $\tan 15^\circ$

c)  $\tan 112\frac{1}{2}^\circ$

24) Given  $P(x) = x^3 - 2x^2 + 9x - 11$  show a root exists between  $x = 1$  and  $x = 2$ .

a) Use the 'halving the interval' method to find the root correct to 1 dp.

b) Using  $x = 1.5$  as a first approximation, use Newton's method to find a better approximation correct to 1 dp. (1 application)

25) Find the term independent of  $x$  in the expansion of

a)  $(x^2 - \frac{4}{x})^{15}$

b)  $(x^3 - \frac{3}{2x})^8$

26) Consider the expansion of  $(2x+3)^{11}$ .

a) Write the general term of the expansion.

b) Show  $\frac{T_{k+1}}{T_k} = \frac{36-3k}{2kx}$

c) Hence find the greatest coefficient in the expansion of  $(2x-3)^{11}$ .

27)  $P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  are variable points on the parabola  $x^2 = 4ay$ .

a) Write the equation of the locus of the midpoint of PQ if PQ is a focal chord

b) If the tangents at P and Q meet at T, and PQ passes through the point  $(0, 6a)$ , find the locus of T.

c) If the chord PQ subtends a right angle at the origin O, i) show  $pq = -4$

ii) find the eqn of the locus of the midpoint of PQ.

28) If  $\alpha, \beta, \gamma$  are the roots of  $2x^3 - 5x^2 + x - 1 = 0$ , find:

a)  $(\alpha + \beta + \gamma)^2$

b)  $(\alpha+1)(\beta+1)(\gamma+1)$

c)  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

d)  $\alpha^2 + \beta^2 + \gamma^2$

29) Two of the roots of  $x^3 + ax^2 + bx + 8 = 0$  are 4 and -2. Find the values of a and b.

30) If  $x^3 - 5x^2 + 7x + k = 0$  has a double integral root, find k.

31) A particle moves in SHM according to  $x = 2 \sin(4t - \frac{\pi}{4})$ . Write down its:

- a) period
- b) amplitude
- c) centre of motion
- d) initial phase

32) A particle moves according to:

$$x = 2 \sin 3t + 3 \cos 3t$$

a) Show that the particle is moving in SHM.

b) Find its period, amplitude, centre of motion

33) A particle, initially at rest 2 metres to the right of O, moves according to the rule  $\ddot{x} = -4x$ .

Find where it next rests and determine its maximum speed.

34) A particle moving according to  $\ddot{x} = -4x$  is initially stationary at  $x=6$ . Find its speed and acceleration at  $x=3$ .

35) A particle's motion satisfies the equation  $v^2 = 10 + 3x - x^2$ .

a) Show that the motion is simple harmonic.

b) Find the centre, period and amplitude of the motion.

36) A particle's motion is given by  $v = \frac{1}{2}(x-1) \text{ ms}^{-1}$ .

If the particle is initially 2 metres to the right of O, find the exact position of the particle, 2 secs. at

37) The acceleration of a particle is given by

$$\frac{d^2x}{dt^2} = 6x^2 - 4x - 3.$$

Find the exact velocity of the particle when it is 2cm to the right of O, if initially the particle is at the origin with velocity  $3 \text{ cms}^{-1}$ .

38) Evaluate:

$$\int_{3/5}^4 \frac{dx}{1+x^2}$$

39) The rate of change of volume in a dam is of water

given by:

$$\frac{dv}{dt} = k(v - 5000)$$

a) Show that a solution of this differential equation is  $v = 5000 + Ae^{kt}$

b) If the initial volume is 87000 KL and at 10 hours the volume is 129000 KL, find the values of A and k.

40) A piece of metal is heated to  $80^\circ\text{C}$  and placed in a room where the temperature is a constant  $18^\circ\text{C}$ .

Assuming the metal cools according to Newton's law of cooling and cools to  $68^\circ\text{C}$  at 15 mins, find when the metal cools to  $30^\circ\text{C}$ ?

41) The surface area of a spherical bubble is increasing at a constant rate of  $1.49 \text{ mm}^2/\text{s}$ . Find the rate of increase in its volume when its radius is  $0.6 \text{ mm}$ .

42) An observer sees a plane  $x \text{ km}$  away flying at a speed of  $500 \text{ km/h}$  and at a height of  $2000 \text{ m}$ .

At what rate is the angle of elevation from the observer to the plane increasing when the plane is  $2 \text{ km}$  away?

43) A projectile is launched at an angle of  $\theta$  to the horizontal at an initial velocity of  $V \text{ m/s}$ .

a) Derive the horizontal and vertical components of motion.

b) Derive the cartesian equation of motion.

c) Find the time of flight for the projectile to hit the ground.

d) Find the range of the projectile.

e) Calculate the maximum height of the projectile.

44) A particle is projected at an initial velocity of  $40 \text{ ms}^{-1}$ . What is its maximum range if its angle of projection is  $30^\circ$ ?

45) A particle is projected at  $80 \text{ ms}^{-1}$  to strike an object  $50 \text{ m}$  away and  $2 \text{ m}$  above the ground. Find, nearest degree, the two angles of projection possible.

46) A particle is projected at an initial velocity of  $100 \text{ ms}^{-1}$  and so that it just clears a wall  $40 \text{ metres}$  away and  $2 \text{ metres}$  high. What is the furthest past the wall the particle can land?

Inverse functions

1999	- 5b)
2000	- 5b
2001	- 5a
2002	- 5c
2002	- 7a
2003	- 5b
2004	- 5b
1998	- 4, b

3d Trig

1999	- 6Cb)
2000	- 3cc)
2003	- 7ca)
2004	- 3cd)

Newtons

2001	, 3a
2003	, 4b

physics

1999	6a
2002	4c
1996	6
2001	7a
2003	6a
1996	5b
2003	5c

SOLUTIONS

1) a) Using table of standard integrals:

$$\frac{d}{dx} = \frac{1}{\sqrt{16-x^2}}$$

b) let  $y = \cos^{-1} 2x$  and  $u = 2x$

$$\therefore y = \cos^{-1} u \quad \therefore \frac{du}{dx} = 2$$

$$\therefore \frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{-1}{\sqrt{1-u^2}} \cdot 2$$

$$= \frac{-2}{\sqrt{1-4x^2}}$$

c) let  $y = \tan^{-1} 4x^2$  and  $u = 4x^2$

$$\therefore y = \tan^{-1} u \quad \therefore \frac{du}{dx} = 8x$$

$$\therefore \frac{dy}{du} = \frac{1}{1+u^2}$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{1+u^2} \cdot 8x$$

$$= \frac{8x}{1+16x^4}$$

d) let  $u = x^2$  and  $v = \sin^{-1} x$

$$\therefore \frac{du}{dx} = 2x \quad \text{and} \quad \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\text{Now } \frac{d}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$= x^2 \cdot \frac{1}{\sqrt{1-x^2}} + 2x \sin^{-1} x$$

$$= x \left( \frac{x}{\sqrt{1-x^2}} + 2 \sin^{-1} x \right)$$

2) a)  $5 \int \frac{1}{4+x^2} dx$

$$= 5 \left[ \frac{1}{2} \tan^{-1} \frac{x}{2} \right] + C$$

$$= \frac{5}{2} \tan^{-1} \frac{x}{2} + C$$

b)  $\int \frac{1}{\sqrt{1-4x^2}} dx$

$$= \int \frac{1}{\sqrt{4(\frac{1}{4}-x^2)}} dx$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{\frac{1}{4}-x^2}} dx$$

$$= \frac{1}{2} \sin^{-1} \frac{x}{\frac{1}{2}} + C$$

$$= \frac{1}{2} \sin^{-1} 2x + C$$

2) c)  $\int \frac{-1}{\sqrt{4-25x^2}} dx$

$$= \int \frac{-1}{\sqrt{25(\frac{4}{25}-x^2)}} dx$$

$$= -\frac{1}{5} \int \frac{1}{\sqrt{\frac{4}{25}-x^2}} dx$$

$$= -\frac{1}{5} \sin^{-1} \frac{x}{\frac{2}{5}} + C$$

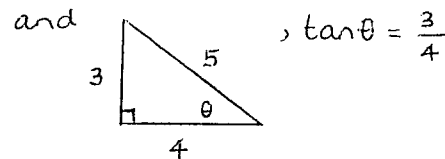
$$= -\frac{1}{5} \sin^{-1} \frac{5x}{2} + C$$

3) a)  $\frac{4}{5}$

b) let  $\theta = \sin^{-1} \frac{3}{5}$

$$\therefore \sin \theta = \frac{3}{5}$$

Using Pythagoras Theorem



$$\therefore \tan(\sin^{-1} \frac{3}{5})$$

$$= \tan \theta$$

$$= \frac{3}{4}$$

4) a) For  $y = \sin^{-1} x$ ,

$$D: -1 \leq x \leq 1$$

$$R: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$\therefore$  for  $y = 3 \sin^{-1} \frac{x}{2}$ ,

$$\text{ie. } \frac{y}{3} = \sin^{-1} \frac{x}{2}$$

$$D: -1 \leq \frac{x}{2} \leq 1$$

$$\therefore D: -2 \leq x \leq 2$$

$$R: -\frac{\pi}{2} \leq \frac{y}{3} \leq \frac{\pi}{2}$$

$$\therefore R: -\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$$

b) For  $y = \cos^{-1} x$ ,

$$D: -1 \leq x \leq 1$$

$$R: 0 \leq y \leq \pi$$

$\therefore$  for  $y = 4 \cos^{-1} 2x$ ,

$$\text{ie. } \frac{y}{4} = \cos^{-1} 2x$$

$$D: -1 \leq 2x \leq 1$$

$$\therefore D: -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$R: 0 \leq \frac{y}{4} \leq \pi$$

$$\therefore R: 0 \leq y \leq 4\pi$$

c) For  $y = \tan^{-1} x$ ,

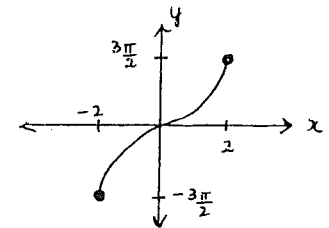
D: all real x

$$R: -\frac{\pi}{2} < y < \frac{\pi}{2}$$

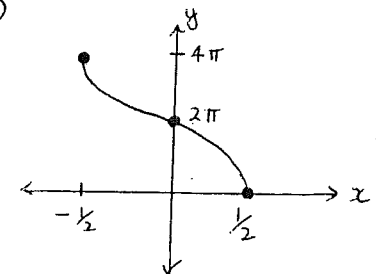
$\therefore$  for  $y = \tan^{-1} 3x$ ,

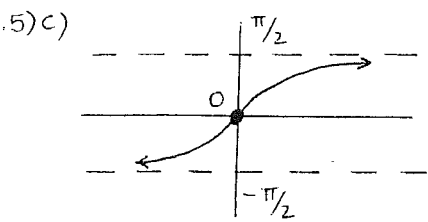
D and R same as above

5) a)



b)





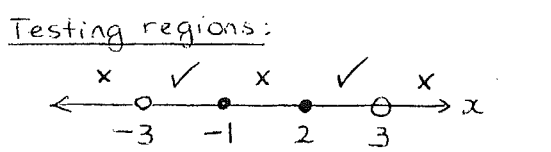
6) Using:  $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

a)  $(6, 4) (2, 1) \quad 4:5$   
 $(x_1, y_1) (x_2, y_2) \quad m:n$   
 $\left(\frac{4(2) + 5(6)}{4+5}, \frac{4(1) + 5(4)}{4+5}\right)$   
 $= \left(\frac{38}{9}, \frac{8}{3}\right)$

b)  $(2, 1) (6, 4) \quad 4:-5$   
 $(x_1, y_1) (x_2, y_2) \quad m:n$   
 $\left(\frac{4(6) - 5(2)}{4-5}, \frac{4(4) - 5(1)}{4-5}\right)$   
 $= (-14, -11)$

7)  $m_1$  of  $x+y=b$  is  $-1$   
 $m_2$  of  $y=3x$  is  $3$   
 Now  $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$   
 $= \left| \frac{-1 - 3}{1 + (-1)(3)} \right|$   
 $= \left| \frac{-4}{-2} \right|$   
 $= 2$   
 $\therefore \theta = 63^\circ$   
 (nearest degree)

8)  $\frac{x-7}{(x-3)(x+3)} > 1$   
 $\therefore x \neq \pm 3$   
 Let  $\frac{x-7}{x^2-9} = 1$   
 $\therefore x-7 = x^2-9$   
 $\therefore x^2-x-2=0$   
 $(x-2)(x+1)=0$   
 $\therefore x=2, -1$



$\therefore -3 < x < -1$  and  $2 \leq x < 3$

9) a)  $\int \frac{x^2}{\sqrt{x^3-1}} dx$   
 $u = x^3 - 1$   
 $\frac{du}{dx} = 3x^2$   
 $\therefore dx = \frac{du}{3x^2}$   
 $= \int \frac{x^2}{\sqrt{u}} \frac{du}{3x^2}$   
 $= \frac{1}{3} \int u^{-1/2} du$   
 $= \frac{1}{3} [2u^{1/2}] + C$   
 $= \frac{2}{3} \sqrt{x^3-1} + C$

9) b)  $\int_4^5 x\sqrt{x-4} dx$   
 $= \int_0^1 (u+4)u^{1/2} du$   
 $= \int_0^1 u^{3/2} + 4u^{1/2} du$   
 $= \left[ \frac{2u^{5/2}}{5} + \frac{8u^{3/2}}{3} \right]_0^1$   
 $= \frac{2}{5} + \frac{8}{3}$   
 $= \frac{46}{15}$

$u = x-4$   
 $\frac{du}{dx} = 1$   
 when  $x=5$ ,  
 $u=1$   
 when  $x=4$   
 $u=0$   
 Also:  
 $x = u+4$

10)  $\int_0^1 \sqrt{1-x^2} dx$   
 $= \int_0^{\pi/2} \sqrt{1-\sin^2\theta} \cdot \cos\theta d\theta$   
 $= \int_0^{\pi/2} \cos^2\theta \cdot \cos\theta d\theta$   
 $= \int_0^{\pi/2} \cos^3\theta d\theta$   
 $= \frac{1}{2} \int_0^{\pi/2} (1 + \cos 2\theta) d\theta$   
 $= \frac{1}{2} \left[ \theta + \frac{1}{2} \sin 2\theta \right]_0^{\pi/2}$   
 $= \frac{1}{2} \left[ \frac{\pi}{2} + 0 \right]$   
 $= \frac{\pi}{4}$

$x = \sin\theta$   
 $\frac{dx}{d\theta} = \cos\theta$   
 $\therefore dx = \cos\theta d\theta$   
 when  $x=1$   
 $\theta = \sin^{-1} 1$   
 $\therefore \theta = \frac{\pi}{2}$   
 when  $x=0$   
 $\theta = 0$

since  
 $\cos 2\theta = 2\cos^2\theta - 1$

11) a) Total no. of ways =  $\frac{8!}{2!} = 20160$   
 b) Treat R and V as 1 letter  
 $\therefore$  No. of ways =  $\frac{7!}{2!} = 2520$   
 But R and V can be in the reverse order  
 $\therefore$  Total no. of ways =  $2520 \times 2 = 5040$   
 12) No. of ways of arranging 12 people =  $12!$   
 Treat couple as one  $\therefore 10!$  ways  
 But couple could sit in reverse order ie.  $2 \times 10!$  ways  
 $\therefore$  Prob (next to each other) =  $\frac{2 \times 10!}{12!} = \frac{2}{11}$   
 13) a)  $P(\text{1st time}) = \frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$   
 b)  $P(\text{exactly 3}) = {}^{10}C_3 \left(\frac{3}{20}\right)^3 \left(\frac{17}{20}\right)^7$

14) Let  $P(x) = x^3 - 3x^2 - 4x + 12$

Now  $P(3) = 0$

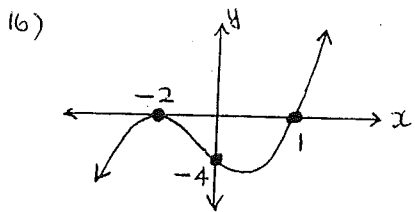
$\therefore x-3$  is a factor of  $P(x)$

15)  $P(x) = x^3 + 3x^2 - 4$

$P(1) = 0 \therefore x-1$  is a factor of  $P(x)$

$$\begin{array}{ccc|c} 1 & 3 & 0 & -4 \\ 1 & & 1 & 4 \\ \hline 1 & 4 & 4 & 0 \end{array}$$

$\therefore P(x) = (x-1)(x^2 + 4x + 4)$   
 $= (x-1)(x+2)^2$



17) 
$$\begin{array}{r} x^3 - 2 \overline{) 5x^4 - 3x^3 + 10x^2 + 1} \\ \underline{5x^4} \phantom{+ 10x^2} \phantom{+ 1} \\ -3x^3 + 10x^2 + 1 \\ \underline{-3x^3} \phantom{+ 10x^2} \phantom{+ 1} \\ 10x^2 - 5 \end{array}$$

$\therefore R(x) = 10x - 5$

18) a) I:  $x = 4y^2 - 1$

$\therefore 4y^2 = x + 1$

$\therefore y^2 = \frac{x+1}{4}$

$\therefore y^{-1} = \pm \sqrt{\frac{x+1}{4}}$

New Domain of  $y$  is  $x > 0$

$\therefore$  Range of  $y^{-1}$  is  $y > 0$

$\therefore y^{-1} = \sqrt{\frac{x+1}{4}}$

b) I:  $x = e^y + 2$

$\therefore e^y = x - 2$

$\therefore y^{-1} = \ln(x-2) \quad [x > 2]$

c) I:  $x = 2 + \cos 4y$

$\therefore x - 2 = \cos 4y$

$\therefore 4y = \cos^{-1}(x-2)$

$\therefore y^{-1} = \frac{1}{4} \cos^{-1}(x-2) \quad [1 \leq x \leq 3]$

19) let  $n=1$

a) i.e.  $1^3 = \frac{1^2}{4} (1+1)^2 \quad | \leq x \leq 3$

$\therefore 1 = 1$

$\therefore$  true for  $n=1 \quad 0 \leq y \leq \frac{\pi}{4}$

Assume true for  $n=k$

i.e.  $1^3 + 2^3 + \dots + k^3 = \frac{k^2}{4} (k+1)^2$

let  $n=k+1$

$\therefore 1^3 + 2^3 + \dots + k^3 + (k+1)^3$   
 $= \frac{k^2}{4} (k+1)^2 + (k+1)^3$

$\therefore \text{RHS} = \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$

$= \frac{1}{4} (k+1)^2 (k^2 + 4(k+1))$

$= \frac{1}{4} (k+1)^2 (k^2 + 4k + 4)$

$= \frac{(k+1)^2 (k+2)^2}{4}$

$\therefore$  true for  $n=k+1$

If true for  $n=k$  then true for  $n=k+1$ . Since true for  $n=1$ , then true for  $n=2$ , and hence true for all values of  $n$  positive

19) b) let  $n=1$

i.e.  $5^1 - 1 = 4$ , which is divisible by 4  $\therefore$  true for  $n=1$

Assume true for  $n=k$

i.e.  $5^k - 1 = 4M$  (where  $M$  is a positive integer)

let  $n=k+1$

i.e.  $5^{k+1} - 1 = 5 \cdot 5^k - 1$   
 $= 5 \cdot 5^k - 5 + 4$   
 $= 5(5^k - 1) + 4$   
 $= 5(4M) + 4$   
 $= 20M + 4$   
 $= 4(5M + 1)$

$\therefore$  divisible by 4.

If true for  $n=k$ , then true for  $n=k+1$ . Since true for  $n=1$ , then true for  $n=2$ , and hence true for all positive values of  $n$ .

20) Auxillary Angle Method:

let  $\sin \theta + \cos \theta = R \sin(\theta + \phi)$

$\therefore \text{RHS} = R \sin \theta \cos \phi + R \sin \phi \cos \theta$

Equating like terms:

$R \cos \phi = 1 \quad (1)$

$d \ R \sin \phi = 1 \quad (2)$

Squaring (1) and (2):

$R^2 \cos^2 \phi = 1 \quad (3)$

$R^2 \sin^2 \phi = 1 \quad (4)$

(3) + (4):

$R^2 \sin^2 \phi + R^2 \cos^2 \phi = 2$

$\therefore R^2 (\sin^2 \phi + \cos^2 \phi) = 2$

$\therefore R^2 = 2$

$\therefore R = \pm \sqrt{2}$

$\therefore R = \sqrt{2}$  (since  $R > 0$ )

$\therefore \sin \phi = \frac{1}{\sqrt{2}}$  and  $\cos \phi = \frac{1}{\sqrt{2}}$

$\therefore \phi = \frac{\pi}{4}$

$\therefore \sin \theta + \cos \theta = \sqrt{2} \sin(\theta + \frac{\pi}{4})$

If  $\sin \theta + \cos \theta = -1$ , then

$\sqrt{2} \sin(\theta + \frac{\pi}{4}) = -1$

$\therefore \sin(\theta + \frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$

$\therefore \theta + \frac{\pi}{4} = \frac{5\pi}{4}, \frac{7\pi}{4}$

for  $0 \leq \theta + \frac{\pi}{4} \leq \frac{9\pi}{4}$

$\therefore \theta = \pi, \frac{3\pi}{2}$  for  $0 \leq \theta \leq 2\pi$

't' method:

If  $\tan \frac{\theta}{2} = t$ ,  $\sin \theta = \frac{2t}{1+t^2}$

and  $\cos \theta = \frac{1-t^2}{1+t^2}$

$\therefore \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$

$\therefore 2t + 1 - t^2 = -1 - t^2$

$\therefore 2t = -2$

$\therefore t = -1$

$\therefore \tan \frac{\theta}{2} = -1$

$\therefore \frac{\theta}{2} = \frac{3\pi}{4}$  for  $0 \leq \frac{\theta}{2} \leq \pi$

$\therefore \theta = \frac{3\pi}{2}$  for  $0 \leq \theta \leq 2\pi$

Cont. next page...

Test  $\theta = \pi$

$\sin \pi + \cos \pi = -1$

$\therefore$  Solns are:  $\pi, 3\pi/2$

21)  $\int_0^{\pi/2} 3 \sin^2 x \, dx$

$= 3 \int_0^{\pi/2} \sin^2 x \, dx$

$= 3 \int_0^{\pi/2} \frac{1}{2} (1 - \cos 2x) \, dx$

$= \frac{3}{2} \int_0^{\pi/2} 1 - \cos 2x \, dx$

$= \frac{3}{2} \left[ x - \frac{1}{2} \sin 2x \right]_0^{\pi/2}$

$= \frac{3}{2} \left[ \left( \frac{\pi}{2} \right) - 0 \right]$

$= 3\pi/4$

22)  $\sin 3\theta = \sin (2\theta + \theta)$

$= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$

$= 2 \sin \theta \cos \theta \cos \theta +$

$\sin \theta (1 - 2 \sin^2 \theta)$

$= 2 \sin \theta \cos^2 \theta + \sin \theta$

$- 2 \sin^3 \theta$

$= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta$

$- 2 \sin^3 \theta$

$= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta$

$- 2 \sin^3 \theta$

$= 3 \sin \theta - 4 \sin^3 \theta$

23)  $\sin 75 = \sin (45 + 30)$

$= \sin 45 \cos 30 + \sin 30 \cos 45$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$

b)  $\tan 15 = \tan (45 - 30)$

$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$= \frac{3 - 2\sqrt{3} + 1}{2}$

$= \frac{4 - 2\sqrt{3}}{2}$

$= 2 - \sqrt{3}$

23) c) Let  $t = \tan \frac{\theta}{2}$

and  $\theta = 225^\circ$

Now  $\tan \theta = \frac{2t}{1-t^2}$

$\therefore \tan 225 = \frac{2t}{1-t^2}$

$\therefore 1 = \frac{2t}{1-t^2}$

$\therefore 1 - t^2 = 2t$

$\therefore t^2 + 2t - 1 = 0$

$\therefore t = \frac{-2 \pm \sqrt{8}}{2}$

$\therefore t = \frac{-2 \pm 2\sqrt{2}}{2}$

$\therefore t = -1 \pm \sqrt{2}$

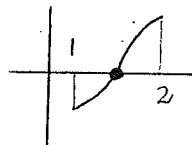
But  $\tan 112\frac{1}{2}^\circ < 0$

$\therefore t = -1 - \sqrt{2}$

$\therefore \tan 112\frac{1}{2} = -1 - \sqrt{2}$

24)  $P(1) < 0$  and  $P(2) > 0$

$\therefore$  a root of  $P(x)$  exists between  $x=1$  and  $x=2$



$P(1.5) < 0 \therefore$  root lies  $1.5 < x < 2$

$P(1.8) < 0 \therefore$  root lies  $1.8 < x < 2$

$P(1.9) > 0 \therefore$  root lies  $1.8 < x < 1.9$

$P(1.85) > 0 \therefore$  root is 1.8 (1dp)

b)  $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1.5 - \frac{-3.625}{9.75}$

$= 1.9$  (1dp)

25) a)

$T_{k+1} = {}^{15}C_k (x^2)^{15-k} (-4/x)^k$

$= {}^{15}C_k (x^2)^{15-k} (-4)^k (x^{-1})^k$

$= {}^{15}C_k x^{30-2k} (-4)^k x^{-k}$

$= {}^{15}C_k x^{30-3k} (-4)^k$

Term indep. of  $x$  when  $30-3k=0$

$\therefore k=10$

$\therefore$  Term indep. of  $x$ :

$T_{11} = {}^{15}C_{10} (-4)^{10}$

b)  $T_{k+1} = {}^8C_k (x^3)^{8-k} (-3/2x)^k$

$= {}^8C_k (x^3)^{8-k} (-3)^k (2^{-1})^k (x^{-1})^k$

$= {}^8C_k x^{24-3k} (-3)^k (2^{-k}) x^{-k}$

$= {}^8C_k x^{24-4k} (-3)^k (2)^{-k}$

Term indep. of  $x$  when  $24-4k=0$

$\therefore k=6$

$\therefore$  Term indep. of  $x$ :

$T_7 = {}^8C_6 (-3)^6 (2)^{-6}$

$= \frac{5103}{16}$

General term of  $(a+b)^n$

is:  $T_{k+1} = {}^nC_k a^{n-k} b^k$

26) a) General term:

$$T_{k+1} = {}^{11}C_k (2x)^{11-k} (3)^k$$

$$= {}^{11}C_k 2^{11-k} x^{11-k} (3)^k$$

b)  $T_k = {}^{11}C_{k-1} 2^{12-k} x^{12-k} (3)^{k-1}$

Now:  ${}^{11}C_k \div {}^{11}C_{k-1}$

$$= \frac{11!}{k!(11-k)!} \times \frac{(k-1)!(12-k)!}{11!}$$

$$= \frac{12-k}{k}$$

and  $2^{11-k} x^{11-k} 3^k \div 2^{12-k} x^{12-k} 3^{k-1}$

$$= 2^{-1} x^{-1} 3$$

$$= \frac{3}{2x}$$

$$\frac{T_{k+1}}{T_k} = \frac{12-k}{k} \cdot \frac{3}{2x}$$

$$= \frac{36-3k}{2kx}$$

c) Greatest coeff when  $\frac{T_{k+1}}{T_k} > 1$

$$\therefore \frac{36-3k}{2k} > 1 \quad \therefore k < 7\frac{1}{2}$$

Greatest coeff. in expansion of  $(2x+3)^{11}$  when  $k=7$

at in expansion of  $(2x-3)^{11}$ , 's (ie. when  $k=7$ )  $< 0$

∴ st  $T_7$  and  $T_9$

$$T_7 = 10\ 777\ 536 \quad \text{and}$$

$$T_9 = 8\ 660\ 520$$

Greatest coeff is 10 777 536

27) a)

$$\text{Mpt of PQ} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= \left( \frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2} \right)$$

$$= \left( a(p+q), \frac{a(p^2+q^2)}{2} \right)$$

$$\therefore x = a(p+q) \quad \text{ie. } p+q = \frac{x}{a}$$

and  $y = \frac{a(p^2+q^2)}{2}$

$$\therefore \frac{2y}{a} = p^2+q^2$$

$$= (p+q)^2 - 2pq$$

Now, given PQ is a focal chord,

$$pq = -1$$

$$\therefore \frac{2y}{a} = \left( \frac{x}{a} \right)^2 - 2(-1)$$

$$\therefore 2ay = x^2 + 2a^2$$

$$\therefore x^2 = 2ay - 2a^2$$

$$\therefore x^2 = 2a(y-a)$$

b)  $x^2 = 4ay$

$$\therefore y = \frac{x^2}{4a}$$

$$\therefore y' = \frac{x}{2a}$$

$$\therefore y'(2ap) = p$$

$$\therefore \text{m of T at P} = p$$

$$\therefore \text{m of T at Q} = q$$

Eqn of T at P:

$$y - ap^2 = p(x - 2ap)$$

$$\therefore y - ap^2 = px - 2ap^2$$

$$\therefore y = px - ap^2$$

$$\therefore \text{Eqn of T at Q: } y = qx - aq^2 \quad q.$$

Coords of T:

$$px - ap^2 = qx - aq^2$$

$$\therefore px - qx = ap^2 - aq^2$$

$$\therefore (p-q)x = a(p-q)(p+q)$$

$$\therefore x = a(p+q)$$

$$\therefore y = ap(p+q) - ap^2$$

$$\therefore y = ap^2 + apq - ap^2$$

$$\therefore y = apq$$

$$\therefore \text{Coords of T are } (a(p+q), apq)$$

Now  $x = a(p+q)$

and  $y = apq$

Also, m of PQ =  $\frac{ap^2 - aq^2}{2ap - 2aq}$

$$= \frac{a(p-q)(p+q)}{2a(p-q)}$$

$$= \frac{p+q}{2}$$

and eqn of PQ:

$$y - ap^2 = \frac{p+q}{2} (x - 2ap)$$

$$\therefore 2y - 2ap^2 = px + qx - 2ap^2 - 2apq$$

$$\therefore 2y = px + qx - 2apq$$

If PQ passes thru  $(0, 6a)$

$$12a = -2apq$$

$$\therefore pq = -6$$

Since T is  $(a(p+q), apq)$

then locus of T is:  $y = -6a$

c) i) m of OP =  $\frac{ap^2}{2ap}$

$$= \frac{p}{2}$$

and m of OQ =  $\frac{q}{2}$

Since  $OP \perp OQ$ ,  $\frac{p}{2} \cdot \frac{q}{2} = -1$

$$\therefore pq = -4$$

ii) mpt PQ =  $\left( a(p+q), \frac{a(p^2+q^2)}{2} \right)$

$$\therefore x = a(p+q) \quad \therefore p+q = \frac{x}{a}$$

and  $y = \frac{a(p^2+q^2)}{2}$

$$\therefore \frac{2y}{a} = p^2+q^2$$

$$= (p+q)^2 - 2pq$$

$$= \left( \frac{x}{a} \right)^2 - 2(-4)$$

$$\therefore \frac{2y}{a} = \frac{x^2}{a^2} + 8$$

$$\therefore 2ay = x^2 + 8a^2$$

$$\therefore x^2 = 2ay - 8a^2$$

$$\therefore x^2 = 2a(y - 4a)$$

28) a)

$$(a+b+x)^2 = (-b/a)^2$$

$$= \left( \frac{5}{2} \right)^2$$

$$= \frac{25}{4}$$



$$28) b) (\alpha + 1)(\beta\gamma + \beta + \gamma + 1)$$

$$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \alpha + \beta\gamma + \beta + \gamma + 1$$

$$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma + \alpha + \beta + \gamma + 1$$

$$= -\frac{d}{a} + \frac{c}{a} - \frac{b}{a} + 1$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{5}{2} + 1$$

$$= 4\frac{1}{2}$$

$$c) \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

$$= \beta\gamma + \alpha\gamma + \alpha\beta$$

$$+ \beta\gamma$$

$$= \frac{c/a}{-d/a}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$= 1$$

$$d) \alpha^2 + \beta^2 + \gamma^2$$

$$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$= (-\frac{b}{a})^2 - 2(\frac{c}{a})$$

$$= (\frac{5}{2})^2 - 2(\frac{1}{2})$$

$$= \frac{25}{4} - 1$$

$$= \frac{21}{4}$$

$$29) \text{ let } \alpha = 4 \text{ and } \beta = -2$$

$$\text{Now } \alpha\beta\gamma = -d/a$$

$$\therefore -8\gamma = -8$$

$$\therefore \gamma = 1$$

$$\therefore 3 \text{ roots are } 4, -2, 1$$

$$\text{ie. } (x-4)(x+2)(x-1)$$

$$= (x-4)(x^2+x-2)$$

$$= x^3 - 3x^2 - 6x + 8$$

$$\therefore a = -3 \text{ and } b = -6$$

$$30) \text{ let } P(x) = x^3 - 5x^2 + 7x + k$$

$$\therefore P'(x) = 3x^2 - 10x + 7$$

$$= (3x-7)(x-1)$$

If  $\alpha$  is a double root of  $P(x)$ , then it is also a root of  $P'(x)$ . Because coeff. of  $x^3$  in  $P(x)$  is 1,  $\alpha$  must be 1.

If  $x=1$  is a root of  $P(x)$

$$\text{then } P(1) = 0$$

$$\text{ie. } 1 - 5 + 7 + k = 0$$

$$\therefore k = -3$$

$$31) a) P = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$b) \text{ amp} = 2$$

$$c) \text{ C.O.M. } : x = 0$$

$$d) \text{ I.P. } = \frac{\pi}{4}$$

$$32) x = 2\sin 3t + 3\cos 3t$$

$$\therefore v = 6\cos 3t - 9\sin 3t$$

$$\therefore \ddot{x} = -18\sin 3t - 27\cos 3t$$

$$= -9(2\sin 3t + 3\cos 3t)$$

$$\therefore \ddot{x} = -9x$$

$\therefore$  part. is moving in SHM as acc. in form  $\ddot{x} = -n^2x$

$$b) P = \frac{2\sqrt{3}}{3}$$

$$\text{amp} = \sqrt{2^2 + 3^2}$$

$$= \sqrt{13}$$

$$\text{C.O.M. } x = 0$$

$$33) \ddot{x} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$\therefore -4x = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$\therefore \frac{1}{2} v^2 = \int -4x \, dx$$

$$= -2x^2 + C$$

$$\text{When } v=0, x=2$$

$$\therefore 0 = -8 + C$$

$$\therefore C = 8$$

$$\therefore \frac{1}{2} v^2 = -2x^2 + 8$$

$$\therefore v^2 = -4x^2 + 16$$

Now, since particle's motion is SHM, particle must initially be at an extremity. Since C.O.M. is at  $x=0$ , next at rest at  $x=-2$ .

Max. speed at C.O.M.

$$\therefore \text{speed} = -4x^2 + 16$$

$$\text{when } x=0$$

$$\therefore \text{speed} = 16$$

$$34) \ddot{x} = -4x$$

$$\therefore \frac{1}{2} v^2 = -2x^2 + C \quad (\text{from Q33})$$

$$\text{when } x=6, v=0$$

$$\therefore 0 = -72 + C$$

$$\therefore C = 72$$

$$\therefore \frac{1}{2} v^2 = -2x^2 + 72$$

$$\therefore v^2 = -4x^2 + 144$$

$$\text{when } x=3$$

$$\text{speed} = \sqrt{-36 + 144}$$

$$= 6\sqrt{3}$$

$$\text{acc} = -12$$

$$35) v^2 = 10 + 3x - x^2$$

$$a) \frac{1}{2} v^2 = 5 + \frac{3}{2}x - \frac{x^2}{2}$$

$$\text{Now } \ddot{x} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$\begin{aligned} \therefore \ddot{x} &= \frac{d}{dx} \left( 5 + \frac{3}{2}x - \frac{x^2}{2} \right) \\ &= \frac{3}{2} - x \\ &= -1 \left( x - \frac{3}{2} \right) \end{aligned}$$

which is in the form

$$\ddot{x} = -n^2(x - k)$$

$\therefore$  SHM

$$b) \text{C.O.M.} : x = \frac{3}{2}$$

$$\text{Period} = 2\pi$$

$$\begin{aligned} \text{Let } v=0 \therefore x^2 - 3x - 10 &= 0 \\ \therefore (x-5)(x+2) &= 0 \\ \therefore x &= 5, -2 \end{aligned}$$

since a particle moving in SHM is at rest at its extremities, the extremities of this particle must be  $x=5$  and  $x=-2$ . Since C.O.M. is  $x = \frac{3}{2}$ , amplitude is  $3\frac{1}{2}$ .

$$36) v = \frac{1}{2}(x-1)$$

$$\therefore \frac{dx}{dt} = \frac{x-1}{2}$$

$$\therefore \frac{dt}{dx} = \frac{2}{x-1}$$

$$\begin{aligned} \therefore t &= \int \frac{2}{x-1} dx \\ &= 2 \int \frac{1}{x-1} dx \end{aligned}$$

$$\therefore t = 2 \ln(x-1) + C$$

$$\text{when } t=0, x=2 \therefore C=0$$

$$\therefore t = 2 \ln(x-1)$$

$$\text{when } t=2$$

$$2 = 2 \ln(x-1)$$

$$\therefore \ln(x-1) = 1$$

$$\therefore x-1 = e$$

$$\therefore x = e + 1 \text{ metres to right of } 0$$

$$\begin{aligned} 37) \frac{d}{dx} \left( \frac{1}{2} v^2 \right) &= \ddot{x} \\ &= 6x^2 - 4x - 3 \end{aligned}$$

$$\therefore \frac{1}{2} v^2 = 2x^3 - 2x^2 - 3x + C$$

$$\text{when } x=0, v=3$$

$$\therefore \frac{9}{2} = C$$

$$\therefore \frac{1}{2} v^2 = 2x^3 - 2x^2 - 3x + \frac{9}{2}$$

$$\therefore v^2 = 4x^3 - 4x^2 - 6x + 9$$

$$\text{when } x=2$$

$$v^2 = 13$$

$$\therefore v = \pm \sqrt{13}$$

$$38) \int_{3/5}^4 \frac{dx}{1+x^2}$$

$$\therefore I = \left[ \tan^{-1} x \right]_{3/5}^4$$

$$I = \tan^{-1} 4 - \tan^{-1} \frac{3}{5}$$

$$\text{let } A = \tan^{-1} 4 \therefore \tan A = 4$$

$$\text{and } B = \tan^{-1} \frac{3}{5} \therefore \tan B = \frac{3}{5}$$

$$\text{Now } \tan(\tan^{-1} 4 - \tan^{-1} \frac{3}{5})$$

$$= \tan(A - B)$$

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{4 - \frac{3}{5}}{1 + 4 \left( \frac{3}{5} \right)}$$

$$= 1$$

$$\therefore \tan I = \tan(\tan^{-1} 4 + \tan^{-1} \frac{3}{5})$$

$$\therefore \tan I = 1$$

$$\therefore I = \frac{\pi}{4}$$

$$39) a) v = 5000 + Ae^{kt}$$

$$\therefore \frac{dv}{dt} = kAe^{kt}$$

$$\text{But } Ae^{kt} = v - 5000 \quad (\text{from line 1})$$

$$\therefore \frac{dv}{dt} = k(v - 5000)$$

$$b) \text{ when } t=0, v=87000$$

$$\therefore 87000 = 5000 + Ae^0$$

$$\therefore A = 82000$$

$$\therefore v = 5000 + 82000 e^{kt}$$

$$\text{when } t=10, v=129000$$

$$129000 = 5000 + 82000 e^{10k}$$

$$\therefore e^{10k} = \frac{124}{82}$$

$$= \frac{62}{41}$$

$$\therefore 10k = \ln\left(\frac{62}{41}\right)$$

$$\therefore k = \frac{\ln\left(\frac{62}{41}\right)}{10}$$

$$40) T = M + Ae^{-kt}$$

$$\text{when } t=0, M=18, T=80$$

$$\therefore 80 = 18 + A$$

$$\therefore A = 62$$

$$\text{when } t=15, T=68$$

$$\therefore 68 = 18 + 62e^{-15k}$$

$$\therefore 50 = 62e^{-15k}$$

$$\therefore \frac{25}{31} = e^{-15k}$$

$$\therefore k = \frac{\ln\left(\frac{25}{31}\right)}{-15}$$

$$\text{when } T=30$$

$$30 = 18 + 62e^{-kt}$$

$$\therefore 12 = 62e^{-kt}$$

$$\therefore t = \frac{\ln\left(\frac{6}{31}\right)}{-k}$$

$$= 115 \text{ min (nearest min)} \quad 14.$$

$$41) \frac{dA}{dt} = 1.99 \text{ mm}^2/\text{sec}$$

$$V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$

$$\therefore \frac{dV}{dr} = 4\pi r^2 \quad \therefore \frac{dA}{dr} = 8\pi r$$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$$

$$= \frac{1}{8\pi r} \times 1.99$$

$$= \frac{1.99}{8\pi r}$$

$$\therefore \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

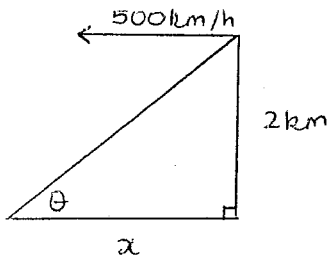
$$= 4\pi r^2 \times \frac{1.99}{8\pi r}$$

$$= 1.99r/2$$

when  $r = 0.6$

$$\therefore \frac{dV}{dt} = 0.597 \text{ mm}^3/\text{s}$$

42)



$$\text{Now } \frac{dx}{dt} = -500$$

$$\text{and } \tan\theta = \frac{2}{x}$$

$$\therefore \theta = \tan^{-1}(2/x)$$

$$\text{let } u = \frac{2}{x} \quad \therefore \theta = \tan^{-1} u$$

$$\frac{d\theta}{dx} = -\frac{2}{x^2}$$

$$\therefore \frac{d\theta}{du} = \frac{1}{1+u^2}$$

$$= \frac{1}{1+\frac{4}{x^2}}$$

$$= \frac{1}{\frac{x^2+4}{x^2}}$$

$$= \frac{x^2}{x^2+4}$$

$$\therefore \frac{d\theta}{dx} = \frac{d\theta}{du} \times \frac{du}{dx}$$

$$= \frac{x^2}{x^2+4} \times -\frac{2}{x^2}$$

$$= -\frac{2}{x^2+4}$$

$$\therefore \frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$$

$$= -\frac{2}{x^2+4} \times -500$$

$$= \frac{1000}{x^2+4}$$

when  $x = 2$

$$\therefore \frac{d\theta}{dt} = 125 \text{ radians/hour}$$

$$43) a) \ddot{x} = 0$$

$$\therefore \dot{x} = A \text{ (where A is a constant)}$$

when  $t=0$ ,  $\dot{x} = V \cos\theta$

$$\therefore A = V \cos\theta$$

$$\therefore \dot{x} = V \cos\theta$$

$$\therefore x = Vt \cos\theta + B$$

(where B is a constant)

when  $t=0$ ,  $x=0$

$$\therefore x = Vt \cos\theta$$

$$\text{Now } \ddot{y} = -g$$

$$\therefore \dot{y} = -gt + C \text{ (where C is a constant)}$$

when  $t=0$ ,  $\dot{y} = V \sin\theta$

$$\therefore C = V \sin\theta$$

$$\therefore \dot{y} = -gt + V \sin\theta$$

$$\therefore y = -\frac{gt^2}{2} + Vt \sin\theta + D$$

(where D is a constant)

when  $t=0$ ,  $y=0 \quad \therefore D=0$

$$\therefore y = -\frac{gt^2}{2} + Vt \sin\theta$$

$$b) \text{ If } x = Vt \cos\theta,$$

$$\text{then } t = \frac{x}{V \cos\theta}$$

$$\therefore y = -\frac{g}{2} \left( \frac{x}{V \cos\theta} \right)^2 + V \left( \frac{x}{V \cos\theta} \right) \sin\theta$$

$$= -\frac{gx^2}{2V^2} \sec^2\theta + x \tan\theta$$

$$\therefore y = -\frac{gx^2}{2V^2} (1 + \tan^2\theta) + x \tan\theta$$

$$c) \underline{y + \frac{gx^2}{2V^2} = 0}$$

$$\therefore -\frac{gt^2}{2} + Vt \sin\theta = 0$$

$$\therefore -t \left( \frac{gt}{2} - V \sin\theta \right) = 0$$

$$\therefore t = 0, \frac{2V \sin\theta}{g}$$

$$\therefore \text{Time of flight} = \frac{2V \sin\theta}{g}$$

$$d) \text{ when } t = \frac{2V \sin\theta}{g}$$

$$x = V \left( \frac{2V \sin\theta}{g} \right) \cos\theta$$

$$= \frac{2V^2 \sin\theta \cos\theta}{g}$$

$$\therefore x = \frac{V^2 \sin 2\theta}{g}$$

$$e) \dot{y} = -gt + V \sin\theta$$

let  $\dot{y} = 0$  for maximum height

$$\therefore t = \frac{V \sin\theta}{g}$$

NOTE: this also proves maximum height attained at mid-flight

Max. height

$$= -\frac{g}{2} \left( \frac{V \sin\theta}{g} \right)^2 + V \left( \frac{V \sin\theta}{g} \right) \sin\theta$$

$$= \frac{-V^2 \sin^2\theta}{2g} + \frac{V^2 \sin^2\theta}{g}$$

$$= \frac{V^2 \sin^2\theta}{2g}$$

In Q's 44-46, the horizontal and vertical components of flight, and cartesian eqn of motion are assumed. In an exam however, they should be derived.

$$44) y = -\frac{gt^2}{2} + vt \sin \theta$$

$$\text{let } y = 0$$

$$\therefore -\frac{10t^2}{2} + 40t \sin 30^\circ = 0$$

$$\therefore -5t^2 + 40t \left(\frac{1}{2}\right) = 0$$

$$\therefore -5t^2 + 20t = 0$$

$$\therefore -5t(t - 4) = 0$$

$$\therefore t = 4 \text{ secs}$$

$$\therefore \text{time of flight} = 4 \text{ secs}$$

$$\therefore \text{when } t = 4$$

$$\begin{aligned} \therefore x &= 40(4) \cos 30 \\ &= 160 \left(\frac{\sqrt{3}}{2}\right) \end{aligned}$$

$$\therefore x = 80\sqrt{3} \text{ metres}$$

$$45) y = -\frac{gx^2}{2v^2} (1 + \tan^2 \theta) + x \tan \theta$$

$$\therefore 2 = \frac{-10(50)^2}{2(80)^2} (1 + \tan^2 \theta) + 50 \tan \theta$$

$$\therefore 2 = \frac{-125}{64} (1 + \tan^2 \theta) + 50 \tan \theta$$

$$\therefore 128 = -125 - 125 \tan^2 \theta + 3200 \tan \theta$$

$$\therefore 125 \tan^2 \theta - 3200 \tan \theta + 253 = 0$$

$$\therefore \tan \theta = \frac{3200 \pm \sqrt{10113500}}{250}$$

$$\therefore \tan \theta = 25.5207, 0.0793$$

$$\therefore \theta = 88^\circ, 5^\circ \text{ (nearest degree)}$$

$$46) y = -\frac{gx^2}{2v^2} (1 + \tan^2 \theta) + x \tan \theta$$

$$\therefore y = \frac{-10x^2}{2(100)^2} (1 + \tan^2 \theta) + x \tan \theta$$

$$= \frac{-x^2}{2000} (1 + \tan^2 \theta) + x \tan \theta$$

$$\text{when } x = 40, y = 2$$

$$\therefore 2 = \frac{-1600}{2000} (1 + \tan^2 \theta) + 40 \tan \theta$$

$$\therefore 4000 = -1600 - 1600 \tan^2 \theta + 8000 \tan \theta$$

$$\therefore 1600 \tan^2 \theta - 8000 \tan \theta + 5600 = 0$$

$$\therefore 40 \tan^2 \theta - 2000 \tan \theta + 140 = 0$$

$$\therefore \tan \theta = \frac{2000 \pm \sqrt{3977600}}{80}$$

$$\therefore \tan \theta = 49.9299,$$

$$\therefore \theta = 89^\circ, 4.009^\circ$$

Now, particle will finish furthest past the wall when  $\theta = 4.009^\circ$

Now particle will hit ground when  $y = 0$

$$\text{ie. } -\frac{gt^2}{2} + vt \sin \theta = 0$$

$$-5t^2 + 100t \sin 4.009^\circ = 0$$

$$\therefore -5t(t - 2 \sin 4.009^\circ) = 0$$

$$\therefore t = 1.3985 \text{ secs}$$

$$\text{when } t = 1.3985 \text{ secs}$$

$$x = 100(1.3985) \cos 4.009^\circ$$

$$\therefore x = 140 \text{ metres (nearest metre)}$$

$\therefore$  Furthest past wall particle will land is ~~100~~ metres past wall.