C.E.M.TUITION

Name : _____

Review of Rules & Formulae

Methods of integration & Primitives of $\sin^2 x$ and $\cos^2 x$

Year 12 - Maths Extension 1

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[A] Integration By Substitution Or By Change Of Variable Method

RULE : $y = \int f(u).\frac{du}{dx}\,dx = \int f(u)\,du$....[1]

$$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c \text{.....[2]}$$

Examples :

[1] Find each of the following integrals using the indicated substitution :

[a] $\int \sqrt{6x+1}\,dx$; using $u = 6x+1$

$$\frac{\sqrt{(6x+1)^3}}{9} + c$$

[b] $\int \frac{1}{(3x-2)^3} dx$, $u = 3x - 2$

$$-\frac{1}{6(3x-2)^2} + c$$

[c] $\int \frac{\ln x}{x} dx$; $u = \ln x$

$$\frac{(\ln x)^2}{2} + c$$

[d] $\int \frac{3x}{\sqrt{4-x}}\,dx\,;\ u^2 = 4-x$

$\boxed{-2(8+x)\sqrt{4-x}+c}$

[e] $\int 2x\sqrt{3x-4}\,dx;\ u^2 = 3x-4$

$\boxed{\frac{4}{135}(9x+8)(3x-4)^{\frac{3}{2}}+c}$

[2] Find the following integrals using the suggested substitution.

[a] $\int \cos x \sin^3 x \, dx$; $u = \sin x$

$\boxed{\frac{\sin^4 x}{4} + c}$

[b] $\int \frac{\sin x}{\sqrt{7+\cos x}} \, dx$; $u = \cos x$

$\boxed{-2\sqrt{7+\cos x} + c}$

[c] $\int \sqrt{1-x^2}\ dx;\ x = \sin\theta$

$\frac{1}{2}(\sin^{-1}x + x\sqrt{1-x^2}) + c$

[B] Primitive of $\sin^2 x$ and $\cos^2 x$

Important to know the following results :

$$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\Rightarrow \sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\Rightarrow \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

Also note that :

$$\text{when } x = 2\theta \text{ then } \sin^2 2\theta = \frac{1 - \cos 4\theta}{2}$$

and make use of the Standard Integrals table.

Examples :

Find the following integrals :

[1] $\int \sin^2 2x \, dx$

$$\frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) + c$$

[2] Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\frac{\theta}{2}\,d\theta$

$\frac{1}{2}\left(\frac{1}{2}+\frac{\pi}{3}\right)$

[3] Evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{16}} (\sin^2 2\theta - \cos^2 4\theta)\ d\theta$

$\frac{1}{2}\left(\frac{1}{8} - \frac{1}{4\sqrt{2}}\right)$

(3) Miscellaneous Exercises :

(1) (a) Show that $\dfrac{x+2}{x^2+2x-8} = \dfrac{1}{3(x+4)} + \dfrac{2}{3(x-2)}$

(b) Hence or otherwise, show that $\displaystyle\int \frac{x+2}{x^2+2x-8}\,dx = \ln\left|(x+4)^{\frac{1}{3}}(x-2)^{\frac{2}{3}}\right| + c$

(2) (a) Given that $\cos 2x = 2\cos^2 x - 1$, show that $\cos^4 x = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

(b) Hence or otherwise, find $\int \cos^4 x \, dx$

$\frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$