

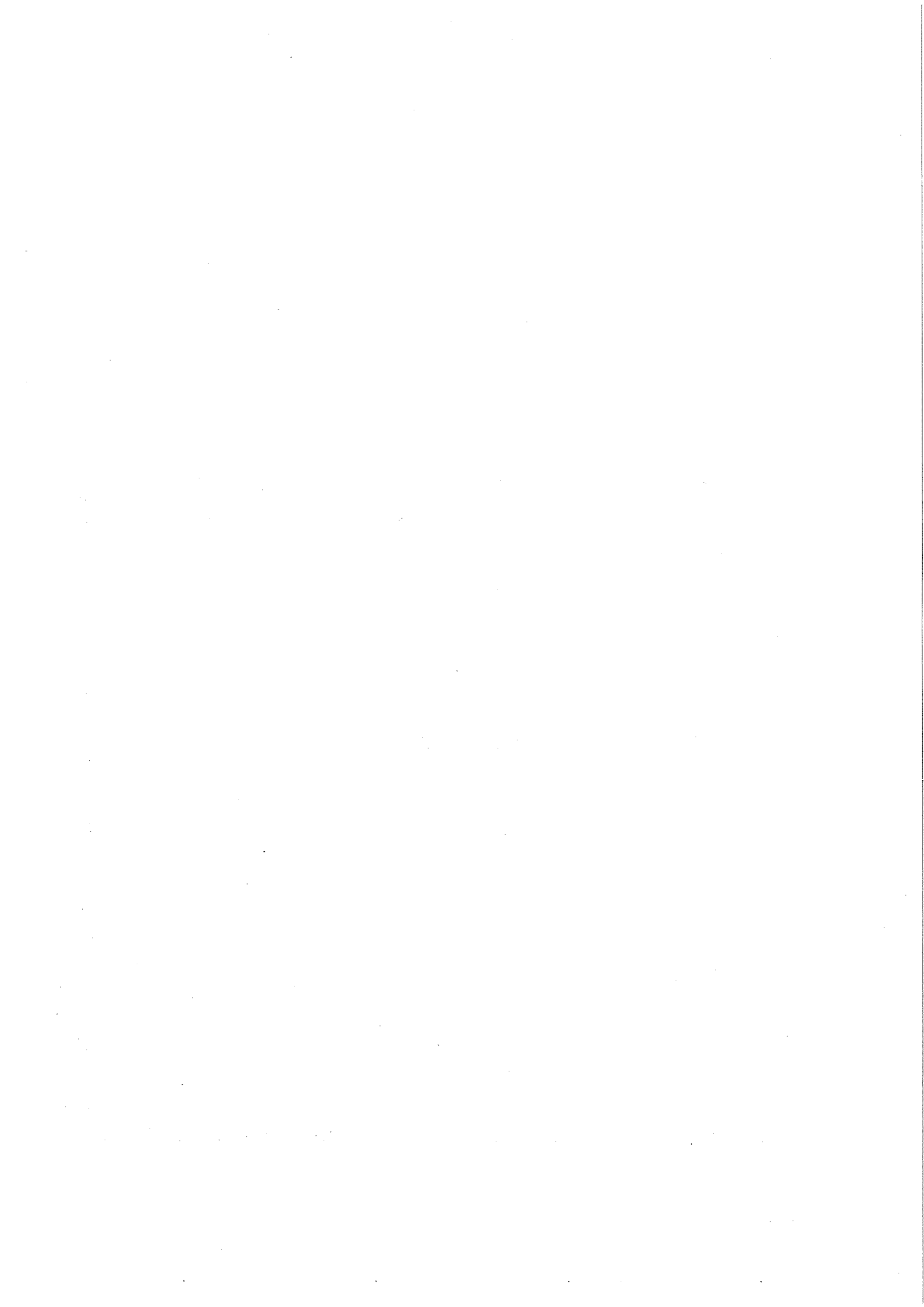
C.E.M. TUITION

Student Name : _____

Review Topic : Trigonometric Equations

(HSC - Paper 1)

Year 12 - 3 Unit



1. Solve $4 \cos x + 4 \sin^2 x = 5$, $-\pi \leq x \leq \pi$

2. Solve $\cos x = \sin 2x$, $0 \leq x \leq 360^\circ$

3. $3\cos 2x = 1 - \sin x$, $0^\circ \leq x \leq 360^\circ$

4. $\tan 2x = 3 \tan x, 0^\circ \leq x \leq 360^\circ$

5. By expressing $\cos 3x$ as $\cos(2x + x)$, show that

$\cos 3x = 4 \cos^3 x - 3 \cos x$. Hence solve $\cos 3x + 2 \cos x = 0$,
where $0^\circ \leq x \leq 180^\circ$.

We shall use $S = \sin x$,
 $C = \cos x$, $S^2 + C^2 = 1$,
 $\cos 2x = 2C^2 - 1 = 1 - 2S^2$,
 $\sin 2x = 2CS$, etc.

- $4 \cos x + 4 \sin^2 x = 5$,
 $-\pi \leq x \leq \pi$
 $4C + 4(1 - C^2) = 5$
 $4C^2 - 4C + 1 = 0$
 $\Rightarrow (2C - 1)^2 = 0$
 $\cos x = \frac{1}{2}$
 $\Rightarrow \pm \frac{\pi}{3}$
- $\sin 2x - \cos x = 0$,
 $0^\circ \leq x \leq 360^\circ$
 $2SC - C = 0$ or $C(2S - 1) = 0$
 $\cos x = 0$ gives $x = 90^\circ, 270^\circ$.
 $\sin x = \frac{1}{2}$ gives $x = 30^\circ, 150^\circ$.
 $x = 30^\circ, 90^\circ, 150^\circ, 270^\circ$
- $3 \cos 2x + \sin x - 1 = 0$,
 $0^\circ \leq x \leq 360^\circ$.
 $3(1 - 2S^2) + S - 1 = 0$
 $6S^2 - S - 2 = (3S - 2)(2S + 1)$
 $= 0$
 $\sin x = \frac{2}{3}$ gives
 $x = 41.81^\circ, 138.19^\circ$
 $\sin x = -\frac{1}{2}$
gives $x = 210^\circ, 330^\circ$
- $\tan 2x - 3 \tan x = 0$,
 $0^\circ \leq x \leq 360^\circ$
 $\frac{2t}{1-t^2} - 3t = 0$
 $3t^3 - t = t(3t^2 - 1)$
 $= 0$
 $\tan x = 0$
 $x = 0^\circ, 180^\circ, 360^\circ$
 $\tan x = \pm \frac{1}{\sqrt{3}}$
 $x = 30^\circ, 210^\circ,$
 $150^\circ, 330^\circ$
- $\cos 3x = \cos(2x + x)$
 $= \cos 2x \cos x$
 $\quad - \sin 2x \sin x$
 $= (2C^2 - 1)C - 2SC \cdot S$
 $= 2C^3 - C - 2C(1 - C^2)$
 $= 4C^3 - 3C$
 $= 4 \cos^3 x - 3 \cos x$

Then to solve: $0^\circ \leq x \leq 180^\circ$

$$\cos 3x + 2 \cos x = 0$$

$$4C^3 - 3C + 2C = 0$$

$$C(4C^2 - 1) = 0$$

$$\cos x = 0 \Rightarrow x = 90^\circ$$

$$\cos x = \pm \frac{1}{2}$$

$$x = 60^\circ, 120^\circ$$

$$x = 60^\circ, 90^\circ, 120^\circ$$